# Mathematical thought in the Renaissance and the genesis of modern mathematics

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**Abstract**

This article argues that the Renaissance rediscovery of Greek mathematics functioned both as a stimulus to renewal and as a powerful constraint. The classical paradigm was not simply revived: it had to be reorganized and reinterpreted, yet its internal grammar – marked by tensions between form and extension, number and magnitude – continued to shape what could count as an admissible object and a legitimate proof. Francesco Maurolico provides the central case study, through his effort to construct a renewed framework for Archimedean geometry of measure – an effort that did not culminate in a stable synthesis, but instead exposed internal tensions within the classical framework. Galileo is used as a stress test: his attempt to mobilize the Euclidean theory of proportions for the description of motion reveals both the productivity and the limits of that inherited structure.

**Keywords**

Renaissance mathematics, Maurolico, Euclidean–Archimedean tradition, New quantities and the mathematization of nature

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## *1. Introduction*

Which pathways led to the mathematical revolutions of the seventeenth century – to Descartes and Leibniz? This question can now be posed on firmer grounds than was possible a few decades ago, after a sustained wave of studies and critical editions has reshaped our understanding of fifteenth- and sixteenth-century mathematics.

The Renaissance, long described either as a mere interlude between medieval calculatores and early modern science or as a simple revival of ancient models, increasingly appears as a field of tensions shaped by the encounter of two major strands. On the one hand, the humanist recovery of Greek mathematical texts fostered practices of commentary, rewriting, and rigorous philological control. On the other hand, algebraic and abacus-based traditions cultivated symbolic procedures, practical geometry, and techniques of measurement tied to the needs of commerce, engineering, perspective, and mechanics.

These lineages were not separated by a rigid divide. They met in courts and in civic life, where mathematicians and practitioners confronted shared problems and negotiated standards of acceptability across languages, genres, and institutional settings. In such contact zones, classical geometry could be mobilized, adapted, or resisted within technical contexts, and abacus-based techniques could be reformulated, translated, or warranted through classical forms.

A fundamental contribution to the consolidation of these exchanges was made by the advent of printing with movable type. Beyond multiplying circulation, print reshaped the conditions of comparison and stabilization of mathematical texts, helping to sustain communities of reference beyond local settings – a development of special importance for mathematics, whose transmission depends vitally on stable written forms.[1] Over the course of the sixteenth century, this infrastructure of comparison helped to weave the two strands more tightly together: Euclid, Archimedes, and Apollonius were recovered, corrected, and subjected to increasingly demanding demonstrative scrutiny, while algebraic and practical traditions circulated alongside them and pressed on their boundaries. The resulting process was neither peaceful nor linear: it generated productive exchanges, but also revealed limits within the inherited framework.

Against this background, the present special issue approaches Renaissance mathematics not as a juxtaposition of local developments, but as a historically structured field in which practices, texts, institutions, and demonstrative constraints evolved under changing conditions of transmission and authority. The guiding questions of the call – the paths leading toward Descartes and Leibniz, including possible blind alleys as well as durable innovations, the interplay of Greek recovery and abacus-based algebra, and the role of pro-

[1] On the role of print in the formation of scholarly communities, see Elizabeth L. Eisenstein, *The Printing Revolution in Early Modern Europe* (New York: Cambridge University Press, 2005), especially 209–285.

portion in the 'geometrisation' of nature – are thus taken up here as elements of a broader reassessment.

The present article adopts a deliberately more circumscribed perspective within this broader framework.[2] Rather than surveying the entire Renaissance mathematical landscape, it focuses on a structural problem that becomes especially visible within the classical geometric tradition: how the Euclidean-Archimedean framework was reworked, strained, and in some cases pushed to its limits in the course of the sixteenth century.

Within this perspective, Francesco Maurolico occupies a particularly significant position. Deeply committed to the recovery of ancient texts, he does not confine himself to philology, but treats classical sources as material to be reorganized and, where needed, extended in the service of a more coherent mathematical whole. He departs from a strictly philological posture whenever system-building and demonstrative standards pull in different directions. Following Maurolico will allow us to bring into view a set of problems that arise when one attempts to restructure the geometry of measure while remaining within the Euclidean-Archimedean standards of proof.

Are these problems merely Maurolico's, or do they point to a more general limitation of the classical framework as it was reworked in the sixteenth century? Galileo enters here as a stress test – not as a general chapter in the history of his physics, but as an illustrative case from a different field and a different scholarly tradition. The same underlying constraints reappear when proportional reasoning, grounded in the Euclidean theory of proportions, is carried into natural philosophy, where Aristotelian 'qualities' must be stabilized as magnitudes.

## 2. *Tensions in the Classical Mathematical Tradition of the Sixteenth Century*

### Vitality and tension

During the sixteenth century, mathematical activity displays a level of intensity and diversity that resists any simple characterization in terms of continuity or rupture. On the

[2] Interpretations of Renaissance mathematics have often moved along two broad lines: accounts that explain transformations primarily through social settings, institutions, and regimes of circulation, and approaches that foreground practices, epistemic styles, and the material and textual conditions under which knowledge is produced and stabilized. The present contribution does not engage these frameworks at their own scale. Its premise is that such debates may remain under-determined unless they are constrained by an analysis of what, in a given tradition, can count as an admissible mathematical operation. We therefore adopt an analytical perspective centered on demonstrative constraints within the Euclidean-Archimedean framework – a dimension that has received comparatively limited systematic treatment, despite its centrality to sixteenth-century reworkings of classical geometry.

one hand, the period is marked by an increasingly systematic recovery, translation, and assimilation of the *disiecta membra* of Greek mathematics, a process that accelerates and deepens over the course of the century and leads to a renewed and critical engagement with classical texts.[3]

On the other hand, within the culture of the abacus, the manipulation of algebraic techniques reaches a level of sophistication that goes beyond the limits encountered by earlier traditions, including Arabic algebra. Italian algebraists succeed in formulating general procedures for the solution of third-degree equations, while algorithms of commercial arithmetic spread widely across Europe, reshaping practices of calculation and problem-solving in a variety of applied contexts.[4]

The mathematics of this period is not a linear transition toward a new scientific order but a space of sustained tension, in which inherited conceptual frameworks coexist with increasingly ambitious demands for generalization, rigor, and applicability. This tension does not arise from a lack of technical sophistication; on the contrary, it emerges precisely when classical mathematical texts are widely available, actively studied, and critically reworked, and when established procedures are pushed to their limits across a broad range of problems.

**A common constraint**

Despite the variety of sixteenth-century mathematical contexts, problems, and methods, many major renewal efforts remained constrained by the ways in which mathematicians of the period received and made operative the classical tradition, especially the Euclidean theory of proportions, which served as the main framework for relating magnitudes. Within this Renaissance reading of the classical tradition, quantitative reasoning remained tied to proportional comparison between homogeneous magnitudes; this requirement shaped both the admissible objects of mathematical investigation and the forms of reasoning through which they could be treated.

3 Rose remains to date the most comprehensive overview of the humanist rediscovery and assimilation of Greek mathematics and science from Petrarch to Galileo, even if its general thesis is shaped, at points rather explicitly, by the anti-Duhemean and broadly culturalist controversies of the 1970s; cf. Paul Lawrence Rose, *The Italian Renaissance of Mathematics: Studies on Humanists and Mathematicians from Petrarch to Galileo* (Genève: Droz, 1975). For a different reading of Renaissance mathematics, centered on the internal organization of problems and traditions, see Pier Daniele Napolitani, "Il Rinascimento italiano," in *La Matematica. Vol. 1: I luoghi e i tempi*, ed. by Claudio Bartocci and Piergiorgio Odifreddi (Torino: Einaudi, 2007), 237–281.

4 On the development of algebra in sixteenth-century Europe, see Sabine Rommevaux, Maryvonne Spiesser, and Maria Rosa Massa Esteve, eds., *Pluralité de l'algèbre à la Renaissance* (Paris: Honoré Champion, 2012). On the impact and circulation of abacus-based arithmetic and commercial calculation, see Raffaele Danna, *The Craft of Indo-Arabic Numerals: How Practical Arithmetic Shaped Commerce and Mathematics in Western Europe, 1200–1600* (Cambridge, MA: Harvard University Press, 2026).

The central role of proportional reasoning also helps explain why repeated attempts were made throughout the sixteenth century to reform, extend, or reinterpret the classical theory of proportions itself. Such efforts – pursued in different ways by figures such as Maurolico, Guidobaldo dal Monte, Benedetti, Valerio, Galileo, and others – testify to the pressure exerted on it by new classes of problems and magnitudes.

The Archimedean geometry of measure, based on so-called 'exhaustion' procedures and double *reductio ad absurdum*, is connected to the proportional framework. Archimedean methods provide a powerful, flexible toolkit for extending results to broader classes of figures while maintaining strict control over the objects involved. Mathematical entities are generated and justified through specific procedures of construction, comparison, and proof.[5]

A further, closely related resource was provided by the recovery of Archimedes' so-called 'mechanical' works. Treatises such as *On the Equilibrium of Planes* and *On Floating Bodies* show, with exemplary clarity, how a strictly geometric mode of demonstration can be mobilized to articulate conditions of balance, buoyancy, and the determination of centers of gravity. What Archimedes handles in these works are magnitudes that can be directly placed under proportional comparison within a classical setting – weights and distances in statics, weights together with volumes or surfaces in hydrostatics, and, in the case of the *Spirals*, times together with traversed spaces. However, this still does not constitute a universal calculus for physical quantities. By contrast, quantities that later readers would be tempted to reconstruct as derived magnitudes – moment, pressure, specific gravity, velocity – do not figure in Archimedes' analysis at all.

Archimedean mechanics exemplifies the tight link between admissible magnitudes and demonstrative procedures that defines the classical paradigm. This coupling is its strength: it enables major extensions of classical results and the systematic unification of previously separate problems. But it also imposes rigid limits: the demand for homogeneous magnitudes, the primacy of proportional relations, and the procedural generation of objects.

As a result, many sixteenth-century innovations take the form of an intensification of classical techniques rather than a reconfiguration of their conceptual foundations. Procedures are refined, generalized, and applied to new domains, but they remain embedded within a framework that is, in crucial respects, still Greek.

## Extension of procedures and ambitions of generality

One of the most striking features of late sixteenth-century mathematics is the growing ambition to extend classical procedures by extracting from Archimedean proofs those fea-

5 On the structural role of Archimedean procedures of measure and their limits, see Ken Saito, "Archimedes and Double Contradiction Proof," *Lettera Matematica International* 1 (2013): 97–104, https://doi.org/10.1007/s40329-013-0017-x.

tures that can be made portable, and then redeploying them as general results for whole classes of objects.[6]

A clear and particularly accomplished example is offered by Luca Valerio's *De centro gravitatis solidorum* (1604). Instead of treating centers of gravity and quadrature case by case, Valerio builds an organized corpus of theorems intended to apply to whole classes of figures, defined by explicit structural properties.

In this setting, a condition that originally serves to make an Archimedean-style argument go through – for instance, the monotone decrease of sections – can be promoted into a definition, so that the existence of suitable inscribed and circumscribed approximations is secured *by construction* and general theorems can replace ad hoc constructions.

The gain is methodological: an approximation theorem can be proved once and for all and then reused across an open-ended range of cases. Valerio's project aims to transform a set of powerful, though heterogeneous, classical arguments into a systematic, reproducible procedure – a true *via regia*, to use Valerio's own expression.

This drive toward generality comes at a conceptual cost. First, the push for procedural generalization increasingly outpaces the availability of genuinely new mathematical objects to which such procedures can be applied. Once the Archimedean corpus has been systematically reworked and extended, further progress often consists in reiteration, variation, or virtuoso recombination, rather than in opening fundamentally new domains of application. The very success of general methods thus reveals a structural limitation: procedures gain autonomy without a corresponding expansion of admissible mathematical objects. It is therefore not surprising that, in the following decades, Bonaventura Cavalieri will further drive the instrumental use of figures and the 'extraction' of quantitative relations from them.

Second, the same generalizing moves weaken the traditional role of geometric form in defining mathematical objects. In Valerio and, even more so, in Cavalieri, figures are

6 This tendency is already visible in mid-sixteenth-century projects of systematic rewriting and methodological consolidation (Maurolico, Commandino) and continues through a broader constellation of authors, extending into the early seventeenth century – notably Kepler (*Nova stereometria doliorum vinariorum*, 1615) and Bartolomeo Sovero (or Barthélemy Souvey, *Curvi ac recti proportio*, 1630). It reaches a particularly mature form in Valerio and will later be pushed further, in a different key, by Cavalieri. For Valerio's *De centro gravitatis solidorum* and the generalization of Archimedean procedures of measure, see Pier Daniele Napolitani and Ken Saito, "Royal Road or Labyrinth? Luca Valerio's De centro gravitatis solidorum," *Bollettino di Storia delle Scienze Matematiche* 24, no. 1 (2004): 71–134, https://doi.org/10.1400/18843. For Cavalieri and the theoretical difficulties of the method of indivisibles, the extended studies by Enrico Giusti, *Bonaventura Cavalieri and the Theory of Indivisibles* (Roma: Cremonese, 1980), and Kirsti Andersen, "Cavalieri's Method of Indivisibles," *Archive for History of Exact Sciences* 31 (1985): 291–367, https://doi.org/10.1007/BF00348519, remain fundamental; for a broader reassessment of seventeenth-century debates on indivisibles, see Vincent Jullien, ed., *Seventeenth-Century Indivisibles Revisited* (Basel: Birkhäuser, 2015), https://doi.org/10.1007/978-3-319-00131-9.

increasingly treated through processes, sections, or families characterized by shared structural properties rather than fixed configurations. This partial dissolution of form is not accompanied by a fully articulated notion of abstract quantity, necessary for the creation of a new mathematical object.

The extension of procedures, the partial erosion of form, and the unresolved problem of abstract quantity thus bring into view a structural difficulty: the classical pattern can be stretched and recombined, but it resists reform from within.

### New magnitudes and the mathematization of nature

If Archimedean mechanics had made it plausible that geometrical proof could articulate physical configurations, sixteenth-century authors soon faced a more demanding question: how to render certain physical *qualities* quantitatively tractable within the Euclidean theory of proportions. The difficulty now shifts from the geometrical description of configurations to the quantitative stabilization of qualities. The issue, in other words, is less a matter of 'applying geometry to nature' than of deciding what may count as a magnitude, and under which rules it can be compared and composed.

Within Book V of Euclid's *Elements*, a ratio is not a numerical object but a relation between magnitudes of the same kind, established and compared through the criterion of equimultiples. This gives the theory its remarkable rigor – allowing, in particular, the comparison of incommensurable magnitudes – while also fixing a strict boundary: quantitative reasoning is limited to relationships among magnitudes of the same kind, and operations on ratios are allowed only by acting on the magnitudes that constitute those ratios.

The consequences are decisive: quantities that, from a modern perspective, are defined through fixed relations between heterogeneous magnitudes – whether by proportionality (space, time, and speed; weight, volume, and specific weight) or by composition (weight and distance in mechanical moment) – cannot be assumed *as such* as autonomous mathematical objects. If they are to enter mathematical reasoning at all, they must be reconstructed indirectly, by reducing them to controlled comparisons between admissible magnitudes (spaces with spaces, times with times, weights with weights). This requirement is not merely a technical limitation, but a structural one: it determines, in advance, the forms under which new physical properties may be stabilized within classical mathematics.[7]

Against this background – that is, against the demand for homogeneous magnitudes imposed by Book V – it is worth recalling that attempts to accommodate physical properties within a proportional framework did not begin with Galileo. Already in the early de-

[7] On the structural role of the classical theory of proportions in early modern mechanics and its consequences for the treatment of physical quantities, see Enrico Giusti, *Euclides Reformatus. La teoria delle proporzioni nella scuola galileiana* (Torino: Bollati Boringhieri, 1993), especially ch. 2, "La teoria delle proporzioni e il linguaggio della natura," 33–56.

cades of the sixteenth century, figures working firmly inside the classical tradition explored different ways of regulating new physical situations through proportional schemata.

In his early writings, Maurolico provides a particularly clear example. Problems concerning equilibrium and *mechanical moments* are handled through composed ratios involving weights and distances; in optics, refraction is approached by comparing angles and lines within a Euclidean setting; illumination is discussed by introducing rays and by relating their 'density' or concentration to the resulting effect.[8] In these cases, the relevant quantity is not treated as a numerical quotient, but is introduced (and regulated) as a legitimate magnitude within a classical setting, endowed with explicit definitions and conditions of comparison. Maurolico thus exemplifies an early and systematic attempt to stabilize new physical magnitudes *within* the proportional framework itself.

A different, more conservative strategy can be observed around the turn of the seventeenth century in Marino Ghetaldi's *Archimedes promotus* (1603). Rather than introducing specific weight as an autonomous magnitude, Ghetaldi constructs a proportional model for the loss of weight experienced by a body immersed in a fluid, keeping the analysis within controlled comparisons between homogeneous magnitudes.[9] Taken together, these cases indicate that 'new magnitudes' did not generate a single research program, but a range of local solutions within the same Euclidean constraint.

They also help clarify a second point: the Archimedean revival does not unfold along a single line. As texts are recovered, edited, translated, and re-used, Archimedes is appropriated in different registers – humanist, technical, mathematical, and philosophical – and the 'mechanical' works, in particular, often require active clarification and reworking.[10]

When the young Galileo engages with Archimedes, he therefore does so within an already articulated landscape. His distinctive profile emerges not from adherence to a sin-

8 On Maurolico's early optical writings (*Photismi*, 1521; *Diaphana*, 1523, reworked in 1553–1554), see Francesco Maurolico, *Optica*, ed. by Riccardo Bellé and Ken'ichi Takahashi, vol. 10, *Edizione Nazionale dell'opera matematica di Francesco Maurolico* (Pisa: Fabrizio Serra, 2017), https://maurolico.it/Maurolico/sezione.html?path=10. His treatment of mechanical moment (*momentum*) likewise dates from the 1520s; see Francesco Maurolico, *Archimedea. Tomus B: De Momentis*, ed. by Riccardo Bellé, Pier Daniele Napolitani, and Beatrice Sisana, vol. 7, *Edizione Nazionale dell'opera matematica di Francesco Maurolico* (Pisa: Fabrizio Serra, 2022), https://maurolico.it/Maurolico/sezione.html?path=7.B.

9 See Pier Daniele Napolitani, "La geometrizzazione della realtà fisica: il peso specifico in Ghetaldi e in Galileo," *Bollettino di Storia delle Scienze Matematiche* 8, no. 2 (1988): 139–237.

10 On the complex transmission of the Archimedean tradition between the Middle Ages and the Renaissance, and on the non-linear character of this process of reception, see Beatrice Sisana, "'L''archimedeismo' negli scritti giovanili di Galileo" (PhD diss., Università degli Studi Roma Tre, 2023) and Pier Daniele Napolitani, "Between Myth and Mathematics: The Vicissitudes of Archimedes and His Work," *Lettera Matematica International* 1 (2013): 105–112, https://doi.org/10.1007/s40329-013-0021-1.

gle model, but from an ability to coordinate different registers into a problem-oriented synthesis – a form of *syncretic Archimedeanism*.[11] In Galileo's earliest investigations, and throughout his later work, these constraints remain operative. They are negotiated with success in some domains (notably in treatments of uniform motion), but the same tension will reappear, in different guises, from problems of specific weight and moment to the harder case of accelerated motion.

**Alternative strategies and conceptual reorganizations**

Alongside the intensification of classical procedures discussed above, the late sixteenth century also witnesses the emergence, across different European contexts, of proposals that aim more explicitly at loosening – or even bypassing – the constraints of the classical paradigm. These attempts do not take the form of incremental reform of the Euclidean-Archimedean framework, but explore paths that move beyond it, often without a clear sense of how such paths might ultimately be stabilized.

A first example is provided by Adriaan van Roomen, whose project of a *mathesis universalis*, articulated in the *Apologia pro Archimede* (1597), aims at grounding a general science of quantity no longer tied either to geometry or to number. The ambition is deliberately foundational; yet it remains largely programmatic, and the lack of shared conceptual resources limits its capacity to generate a stable practice.

A different strategy is pursued by Simon Stevin. By redefining the notion of number and introducing decimal fractions in a systematic way (*La disme*, 1585), Stevin effectively weakens the traditional boundary between number and magnitude, thereby extending the range of quantities that can be treated arithmetically. Here too, however, the conceptual consequences of this move are only partially integrated into the classical demonstrative framework.

At a still different level, François Viète's *In artem analyticem Isagoge* (1593) brings to maturity a range of earlier experiments in algebraic symbolism,[12] and shapes them into a general analytic art explicitly connected to the ancient method of analysis and synthesis transmitted by Pappus, whose *Collectiones mathematicae* appeared in print in 1588. Viète's

11 For the main lines of Archimedean reception in the late sixteenth century and their relevance to Galileo's early writings, see Sisana, "'L''archimedeismo' negli scritti giovanili di Galileo." An analysis of Galileo's juvenile studies on the centers of gravity of solids can be found in Riccardo Bellé and Beatrice Sisana, "Galileo Galilei and the Centers of Gravity of Solids: A Reconstruction Based on a Newly Discovered Version of the Conical Frustum Contained in Manuscript UCLA 170/624," *Archive for History of Exact Sciences* 76, no. 6 (2022): 551–588, https://doi.org/10.1007/s00407-022-00289-4.

12 On this process, see Albrecht Heeffer, "From the Second Unknown to the Symbolic Equation," in *Philosophical Aspects of Symbolic Reasoning in Early Modern Mathematics*, ed. by Albrecht Heeffer and Maarten Van Dyck, 57–101, Studies in Logic 26 (London: College Publications, 2010).

symbolism greatly expands the expressive power of reasoning, but it does not, by itself, supply a conceptual idiom capable of replacing classical architecture.

The diversity and the lack of convergence of the examples just mentioned suggest that the difficulty is structural: not merely a matter of extending procedures, but of identifying a common ground on which new objects, new relations, and new standards of legitimacy could be coordinated. In particular, the classical requirement of homogeneous magnitudes and proportional comparability remains a powerful – and selective – constraint, while increasingly general inferential templates tend to outpace the repertoire of admissible objects.

Against this European backdrop – where internal intensification and external displacement coexist without resolution – Maurolico's position becomes especially revealing: it is a deliberately internal response that seeks to make the presuppositions of the classical architecture explicit.

## *3. Maurolico and the Classical Tradition*

### Problems from within Archimedean geometry

Maurolico works with full awareness of the language and methods of Greek geometry as his main tools of mathematical rigor and proof. His engagement with Archimedean mathematics spans his entire career and serves as a constant reference point for his mathematical thought.

This strict adherence to ancient models leads him to address foundational problems, which emerge most clearly in his extensive work of commenting on, rewriting, and systematically reorganizing Archimedean texts. Maurolico's work thus offers a privileged viewpoint on how sixteenth-century mathematics, while formally faithful to the classical tradition, increasingly questioned the conditions under which Archimedean-style arguments could be applied, just as it was beginning – more or less consciously – to move beyond them.

### Before and after 1544: reconstructing Archimedes and confronting his foundations

A first phase of Maurolico's engagement with Archimedean geometry develops under conditions that are, from a textual point of view, severely constrained. A substantial part of his early Archimedean production – including the *De sphaera et cylindro*, the *De dimensione circuli*, the *Libellus de momentis aequalibus*, and the *Quadratura parabolae* – was composed before he had access to the authentic works of Archimedes.[13]

[13] For Maurolico's engagement with Archimedean texts, the chronology of his reconstructions, and the impact of the Basel *editio princeps* of 1544, see specifically the "General Introduction"

The sources available to him prior to the middle of the century offered only a fragmentary and indirect image of Archimedean geometry. They consisted primarily of excerpts transmitted through humanist compilations, medieval reworkings of Archimedean results, and derivative traditions in which demonstrative rigor and foundational explicitness were unevenly preserved.[14] Within this heterogeneous textual horizon, Maurolico's early reconstructions already display a remarkable technical competence, but they also rely on assumptions and argumentative shortcuts inherited from non-Archimedean contexts.

The situation changes decisively with Maurolico's encounter with the Basel *editio princeps* of Archimedes' works in 1544. For the first time, he gains access not only to treatises previously unknown to him – such as the *Spirals* and the *Conoids and Spheroids* – but also to the authentic structure of Archimedean demonstration, including its explicit use of definitions, postulates, and preliminary lemmas. This encounter forces a systematic comparison between Maurolico's own reconstructions and the demonstrative economy of Archimedes himself.

The outcome of this comparison is not uniform. In the case of *Quadratura parabolae* and *De dimensione circuli*, Maurolico's earlier reconstructions turn out to be remarkably accurate, requiring only limited adjustments. By contrast, his *De sphaera et cylindro* and *Libellus de momentis aequalibus* are judged – by Maurolico himself – to surpass the medieval tradition yet still to demand a thorough revision in the light of the authentic text. The *Spirals* are read with close fidelity, though not without significant interventions, whereas in the *Conoids and Spheroids* Maurolico not only redistributes the material into two books and makes explicit several conic-section arguments left implicit by Archimedes, but also develops an original treatment of the relation between sphere and spheroid, based on a systematic comparison of sections.[15] As he would later emphasize in his letter to Juan de Vega (1554), Archimedes now appeared to him as the author of a single, coherently struc-

and the "Introduction to Tomus A" in Francesco Maurolico, *Archimedea. Tomus A: Opera Archimedis Ex Traditione Maurolyci*, ed. by Riccardo Bellé, Pier Daniele Napolitani, and Beatrice Sisana, vol. 7, *Edizione Nazionale dell'opera matematica di Francesco Maurolico* (Pisa: Fabrizio Serra, 2022), XVII–LXVII and 3–34, https://maurolico.it/Maurolico/sezione.html?path=7.

14 On the printed sources and medieval traditions available to him before 1544, see in particular Giorgio Valla, *De Expetendis Et Fugiendis Rebus* (Venezia: Aldus Manutius, 1501), which transmits excerpts from Archimedes and Eutocius; Luca Gaurico, *Tetragonismus Idest Circuli Quadratura* (Venezia: Giovanni Battista Sessa, 1504), based on a corrupted version of William of Moerbeke's translation of *De dimensione circuli* and *Quadratura parabolae*; and the *Liber de curvis superficiebus* attributed to Iohannes de Tinemue (critical edition in Marshall Clagett, *Archimedes in the Middle Ages. Vol. I: The Arabo-Latin Tradition* (Madison: University of Wisconsin Press, 1964), 439–557).

15 On these different outcomes and on the constructive aspects of Maurolico's reworking of the Archimedean corpus see the relevant introductions and notes in Maurolico, *Archimedea. Tomus A*.

tured corpus, while the *Libellus de momentis aequalibus* assumed the status of an original contribution rather than a mere reworking.[16]

From this point onward, Maurolico's engagement with Archimedean geometry is no longer limited to reconstruction or improvement of individual treatises. It acquires a distinctly foundational dimension. The task he increasingly sets himself is not merely to reproduce Archimedean results, but to clarify, systematize, and, where necessary, justify the principles that make those results demonstratively possible. This shift provides the immediate background for his later attempt to articulate an explicit introduction to Archimedean geometry, the *Praeparatio ad Archimedis opera*.

**Implicit assumptions and demonstrative legitimacy: the case of *De sphaera et cylindro***

The foundational difficulties that Maurolico gradually comes to recognize are particularly clear in his treatment of *De sphaera et cylindro*. In its original Archimedean form, the treatise opens with a carefully articulated set of definitions, postulates, and preliminary lemmas, whose function is not merely formal, but constitutive of the validity of the demonstrations that follow. By contrast, the medieval tradition through which Maurolico first accessed this material transmits many of the results while leaving their foundational underpinnings largely implicit.

This difference becomes evident when Maurolico's early version of *De sphaera et cylindro*, completed in 1534, is compared with the authentic Archimedean text. Maurolico's reconstruction relies in several places on demonstrative patterns inherited from the medieval *Liber de curvis superficiebus*,[17] which makes systematic use of assumptions concerning the existence of surfaces equal to a given surface,[18] as well as principles of inclusion governing the comparison of curved surfaces.[19]

16 On the letter to Juan de Vega and its role in Maurolico's redefinition of his Archimedean project, see the "General Introduction," § 3.3.1, in Maurolico, *Archimedea. Tomus A*. For the text: *Edizione Nazionale dell'opera matematica di Francesco Maurolico*, available at https://maurolico.it/Maurolico/sezione.html?path=2.A.

17 Marshall Clagett was the first to demonstrate Maurolico's dependence on the medieval text attributed to a certain Iohannes de Tinemue; see *Archimedes in the Middle Ages. Vol. III: The Fate of the Medieval Archimedes in the Sixteenth Century* (Philadelphia: American Philosophical Society, 1978), 798–812. On the unidentified author to whom this work is ascribed in some manuscripts, see Wilbur R. Knorr, "John of Tynemouth Alias John of London: Emerging Portrait of a Singular Medieval Mathematician," *The British Journal for the History of Science* 23, no. 3 (1990): 293–330.

18 "Rationis causa. Presentis demonstrationis hypothesis, et tota sequentium theorematum series, lineam rectam curva et superficiem rectam curva esse aequalem sibi postulat admitti" (Clagett, *Archimedes in the Middle Ages. Vol. I*, 452, 15–17).

19 "Superficies inclusa maior sit superficie includente, quod est impossibile" (Clagett, *Archimedes in the Middle Ages. Vol. I*, 458, 96–97). It should be noted that the medieval formulation is less

A revealing example is provided by Maurolico's proof of theorem X, asserting that the surface of the sphere is equal to the rectangle constructed on the diameter of the sphere and the circumference of its greatest circle. The argument proceeds by a double *reductio ad absurdum*, based on the inscription and circumscription of solids of revolution obtained by rotating polygons, and on the comparison of concentric spheres. For the proof to be legitimate, however, two assumptions are required, and Maurolico uses both of them as if they were immediately acceptable, inserting them into the flow of the proof as intuitive claims. First, he asserts that, if the surface generated from a given line is not equal to the surface of a given sphere, it will be equal to the surface of some greater or smaller sphere – that is, he presupposes the existence of a spherical surface equal to any given surface. Second, he assumes as impossible that an included surface should be greater than the surface that includes it, thereby relying on a principle of inclusion to guarantee that the surface of an inscribed solid cannot exceed that of the sphere containing it. In Maurolico's 1534 text, these assumptions are simply taken over as local working hypotheses, with no attempt to formulate them as general principles grounded in a preliminary system of definitions and postulates.

When confronted with the Archimedean text after 1544, however, Maurolico read *De sphaera et cylindro* as a highly structured model of demonstrative organization. In Archimedes' own formulation, the comparison of curved surfaces does not rest on ad hoc premises introduced only within the course of the proof, but on principles explicitly stated at the outset, together with precise conditions – such as concavity in the same direction – under which comparisons are admissible. Read in this way, the Archimedean text changed the status of the assumptions that had functioned as local working hypotheses in Maurolico's 1534 reconstruction: they could no longer remain merely implicit without raising questions of demonstrative legitimacy.

The contrast between these two approaches brings into focus a problem that is not merely textual, but structural. Demonstrative procedures that function within a given argumentative context cannot be freely generalized unless the principles on which they depend are explicitly stated and justified.

In the extant text of Maurolico's *De sphaera et cylindro* one can in fact discern traces of a first attempt to accommodate these requirements within the original framework of the treatise. Additional lemmas and local adjustments point to a partial reworking carried out in the light of the Basel edition, but the result is a composite text in which older demonstrative habits and new foundational demands still coexist in a somewhat unstable way.[20]

precise than the Archimedean one, in which the condition that the two lines or surfaces be concave in the same direction is explicitly required.

20 On the composite character of the surviving version of *De sphaera et cylindro* and on the traces of this reworking see Riccardo Bellé, "Il processo compositivo del *De sphaera et cylindro ex traditione Maurolyci*," *Bollettino di Storia delle Scienze Matematiche* 42, no. 2 (2022): 311–337.

The examination of the other Archimedean treatises transmitted by the Basel edition – in particular the *Spirals* and the *Conoids and Spheroids* – confirms that the difficulties brought to light by *De sphaera et cylindro* are not confined to a single work, but concern more generally the foundational presuppositions of Archimedean geometry.

### The genesis of the *Praeparatio ad Archimedis opera*: foundational needs of a 'general mathematics'

The shift described above from a local legitimacy of procedures to explicit foundations is described by Maurolico himself, reflecting on the work he undertook after gaining access to the complete Archimedean corpus:

> Having examined all these works, I endeavoured to make their content easier to understand by adding many lemmas, by proving many results that Archimedes had left implicit, and, moreover, by treating the centres of solids in his investigation of equal moments, a topic that he had passed over.[21]

This passage makes clear that Maurolico conceives his task not simply as one of transmission or clarification, but as an intervention aimed at rendering explicit what had previously remained tacit within Archimedean demonstrations.

The need for such an intervention becomes particularly evident when Maurolico turns back to his own earlier work and explicitly acknowledges that, in his *De sphaera et cylindro*, he had adopted what he now calls a *facilior via*, relying on assumptions that might appear *inconcessibilia* if left without justification:

> In my treatise *De sphaera et cylindro* I followed an easier path. In order that no one may think that I assumed inadmissible principles when I supposed that, for any given surface, there exists a spherical, conical, or cylindrical surface, under a given height, or the surface of a spherical segment, equal to it, we shall here demonstrate those very principles. Likewise, given two surfaces, we shall show that there exists a surface similar to one of the given and equal to the other; and, given two solids, that there exists a solid similar to one of the given and equal to the other. Since this requires the determination of two mean proportionals, we shall treat this problem following the tradition of the ancient philosophers, as Eutocius has recorded in his Commentaries.[22]

21 *Praeparatio*, Proemium, § 18, in Maurolico, *Archimedea. Tomus A*, 86: "Haec ego omnia cum vidissem, conatus sum ad faciliorem intellectum multa lemmata adiicere, multa ab Archimede omissa demonstrare, tum in aequalium momentorum negotio centra solidorum tractare, rem ab illo praetermissam."

22 *Praeparatio*, Proemium, § 19, in Maurolico, *Archimedea. Tomus A*: "In Libello De sphaera et cylindro usus sum faciliori via. In quo ne quis arbitretur me inconcessibilia principia postulasse

It is not the rejection of the classical framework, but the recognition that demonstrative validity cannot rest on inherited schemas alone. Procedures that were previously secured by local argumentative practice must now be grounded in explicit principles that specify the admissible operations and the conditions under which they apply. Composed in 1550, at a moment when Maurolico also presents Archimedes as the author of a unified corpus, the *Praeparatio* is thus conceived as a preliminary work in which the foundations of Archimedean geometry can be stated and examined in a systematic way.

**Postulates and the regulation of comparison**

A first and decisive step in Maurolico's attempt to make his vision of the geometry of measure fully coherent consists in the explicit formulation of a small but systematic set of postulates, designed to support both the *via facilior* adopted in his *De sphaera et cylindro* and, more generally, his reorganization of Archimedean geometry. The *Praeparatio* is not, in this respect, a mere preface, but a genuine attempt to lay down the principles on which his demonstrative procedures are to rest. Among these postulates, a central place is occupied, on the one hand, by those regulating admissible comparisons between magnitudes and, on the other, by the principle that reduces any ratio between magnitudes of the same kind to a ratio between lines – a principle that underlies the existence theorems discussed in the next subsection.

Within this framework, the postulates of inclusion play a clarifying rather than innovating role. In his *De sphaera et cylindro*, as well as in its medieval antecedent, principles of inclusion were already at work, but only in the form of local assumptions attached to specific arguments. In the *Praeparatio*, Maurolico rewrites them as general postulates governing the comparison of surfaces and solids. They state, in general terms, that a surface or solid which circumscribes or includes another is greater than the surface or solid that is inscribed or included, provided that the figures involved are concave in the same direction. What is crucial here is not the novelty of the claim itself – which closely follows Archimedean usage – but its elevation to the status of an explicit principle.

By making such conditions explicit, Maurolico addresses a structural difficulty already encountered in his earlier work. Demonstrations based on inscription and circumscription presuppose principles of inclusion in order to rule out certain inequalities as impossible. The *Praeparatio* turns this inherited practice into a general rule: comparisons between curved surfaces and solids are declared legitimate only when the relevant

si cuilibet superficiei aliquam sphaericam aut conicam sive cylindricam sub data celsitudine aut sphaericae portionis superficiem aequalem esse supponam, demonstrabimus et hic ipsa principia; item datis duabus superficiebus superficiem esse uni datarum similem et alteri aequalem datisque duobus solidis aliquod solidum esse uni datorum simile et alteri aequale. Ad quod cum necessaria sit duarum mediarum proportionalium inventio, id ipsum problema ex veterum philosophorum traditione tractabimus, ut Eutocius memoratus in Commentariis scripsit."

inclusion conditions are satisfied.

This move has an important methodological consequence. It does not extend the domain of admissible objects, nor does it introduce new forms of reasoning. Rather, it regulates the use of existing procedures by fixing in advance the conditions under which they may be applied. The postulates of inclusion thus function as boundary conditions for Archimedean-style arguments, delimiting the domain within which operations of comparison retain demonstrative force.

**Existence theorems and the reduction of ratios to lines**

Alongside the regulation of admissible comparisons, the *Praeparatio* introduces a second foundational move, which proves even more far-reaching: the systematic use of existence theorems grounded on the reduction of relations between magnitudes to relations between straight lines. This principle, stated explicitly among the postulates at the outset of the work, establishes that for any two magnitudes of the same kind there exist two lines standing in the same ratio:

> Quibuslibet duabus eiusdem generis magnitudinibus esse duas lineas proportionales (*Praep.* Postulatum 1, § 21).

On this basis, relations between surfaces, solids, and other magnitudes can be treated indirectly through relations between segments. Subsequent propositions then assert the existence of lines, squares, circumferences, and spherical surfaces satisfying prescribed proportional relations.[23]

A particularly instructive example is provided by the proof of Proposition XV, which asserts the existence of a circumference equal to any given straight line. Given a line *A*, Maurolico constructs a circle *BED* on an arbitrary diameter *BD*. If the circumference *BED* is not equal to the given line *A*, he invokes the existence of a fourth proportional *CF*, such that *BED:A=BD:CF*. The line *CF* is then taken as the diameter of the circle whose circumference is equal to the given line *A*.[24]

[23] See *Praeparatio,* in Maurolico, *Archimedea. Tomus A*. For instance, Prop. V: “Esse aliquam lineam ad quam data linea datam habet rationem”; Prop. VII: “Esse aliquod quadratum ad quod datum quadratum datam habeat rationem”; Prop. XV: “Cuivis datae lineae alicuius circuli peripheriam esse aequalem”; Prop. XX: “Cuilibet datae superficiei alicuius sphaerae superficiem esse aequalem.”

[24] The existence of this fourth proportional is guaranteed by *Praeparatio*, Prop. V, which in turn rests on Postulate 1.

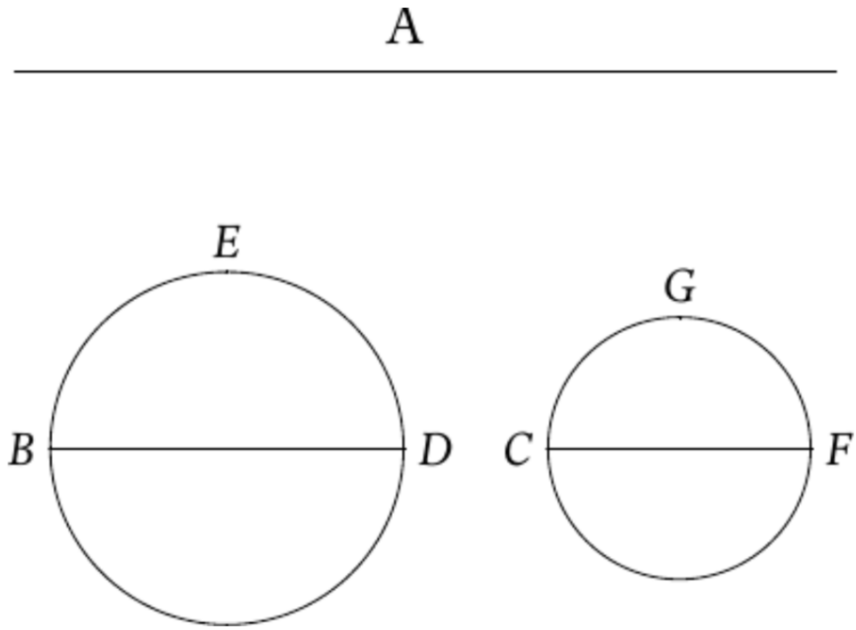


Figure 1 - *Praeparatio*, Proposition XV.

What matters here is not the introduction of new constructions, but the change in their status. In Archimedean practice, analogous results typically function as local tools, introduced to secure inequalities or approximations required for a specific proof – as in *De sphaera et cylindro* I.2 or in *Spirals* 4. As Reviel Netz has emphasized, such propositions do not aim to establish the 'existence' of objects in any philosophical sense, but to furnish instruments for obtaining inequalities between magnitudes and ratios, which can then be used to prove geometrical equalities.[25]

In the *Praeparatio*, by contrast, the corresponding results are elevated to the status of general existence theorems. They guarantee in advance the availability of objects satisfying prescribed proportional conditions, thereby allowing demonstrations to proceed without repeated appeal to ad hoc constructions.

This shift has a clear methodological effect. By reducing questions about the relations between heterogeneous magnitudes to questions about ratios between lines, Maurolico creates a uniform medium in which existence and comparison can be handled systematically. At the same time, this strategy does not dissolve the classical constraints under

25 Reviel Netz, *The Works of Archimedes. Vol. I: The Two Books on the Sphere and the Cylinder* (Cambridge: Cambridge University Press, 2004).

which those relations operate. The requirement of homogeneity is preserved, and proportional comparison remains the fundamental mode of reasoning. What changes is the level at which these constraints are enforced: no longer locally within individual proofs, but globally, through explicitly stated principles.

The result is a far-reaching reorganization of the classical Archimedean framework. The reduction of ratios to ratios between lines does not introduce a genuinely abstract notion of quantity independent of geometrical determination. Instead, it provides a general mechanism for translating relations between diverse magnitudes into a form compatible with classical proportional theory. In this sense, the existence theorems of the *Praeparatio* both extend the reach of Archimedean methods and reveal their internal limits: procedures acquire a new degree of generality, yet the ontology of admissible mathematical objects remains essentially unchanged.

At the same time, the very attempt to treat arbitrary magnitudes via proportional relations between lines brings into focus a new tension around the notion of quantity – a tension that Maurolico would try to address in various ways in the years following the composition of the *Praeparatio*, as the next section will show.

## *4. Quantity, Number, and Ratio: an Unfinished Synthesis*

### The Archimedean Dossier: a project that existed – and why it did not close

From the *Praeparatio ad Archimedis opera* (1550) onward, Archimedes remains a stable point of reference in Maurolico's mathematical horizon. This does *not* mean that his post-1550 writings are all devoted to Archimedes; rather, it means that Archimedean geometry continues to set a standard of demonstrative legitimacy and to provide a privileged testing ground for what counts as an admissible operation on magnitudes.

The documents that allow us to reconstruct Maurolico's attempt at a *comprehensive* and *definitive* arrangement of his Archimedean work, however, are of a specific kind and fall within a limited cluster: programmatic statements (above all the letter to Juan de Vega, drafted in 1554 with later additions dated 1556), short foundational essays written in June 1554 (the *Sermones*), the construction of a general operational framework for calculation in Book II of the *Arithmeticorum libri duo* (completed in 1557, printed in 1575), and the late compendium of Euclid's Book V (*Compendium Quinti Elementorum*, 1567). Taken together, these texts leave little doubt that a coherent project existed: Maurolico did not merely annotate Archimedes; he sought to reorganize the Archimedean corpus within an architectonic view of mathematics and of its demonstrative legitimacy.[26]

[26] For the set of documents relevant to the evolution of Maurolico's Archimedean work, see the Introduction in Maurolico, *Archimedea. Tomus A*, LIII–LV.

The dossier 'fails to close' not because Maurolico loses interest in Archimedes or lacks technical control. The dossier fails to close because, in the very act of trying to stabilize an Archimedean synthesis (and to keep it coherent with the foundational stance already taken in the *Praeparatio*), Maurolico is driven to a prior task: to make explicit a general framework for quantity, ratio, and proportion – a framework he does not find ready-made in Greek mathematics, neither in Euclid nor in Archimedes. It is this foundational detour, generated from within the Archimedean agenda itself, that repeatedly deflects the revision and prevents it from settling into a final synthesis.

That Maurolico himself perceived the connection is already clear in the *Sermo de divisione artium* (June 1554), where he explicitly adds to Euclid's postulates and common notions precisely the principles he had placed as postulates in the *Praeparatio* – principles governing inclusion, proportionality, and the existence of equal, greater, or smaller magnitudes. He notes that they are required "not only in elementary demonstrations, but also in Archimedean and other demonstrations," and that "upon these, as upon foundations, all propositions are constructed in successive demonstration." [27]

**In search of a framework: the de Vega letter and the *Sermones***

The documents produced between 1554 and 1556 – above all the letter to de Vega and the *Sermones* of June 1554 – are best read not as isolated occasional pieces, but as Maurolico's attempt to build a theoretical framework capable of hosting a heterogeneous body of work. By the mid-1550s, his scholarly output extends well beyond Archimedes: it includes Apollonius, spherical geometry, and a highly ambitious reorganization of arithmetic. What is missing is a context in which these domains can be integrated while preserving demonstrative legitimacy. The de Vega letter sketches this context at a programmatic level; the *Sermones* deepen it by identifying the notions that must carry the load – above all quantity, ratio, and proportion.

Maurolico uses the letter to frame his engagement with classical mathematics in terms of foundations and demonstrative legitimacy: before offering corrections or easier ways of proving, he insists on the need to "lay down the necessary foundations."

In the *De quantitate sermo noster*, Maurolico proposes to consider quantity *simpliciter*, prior to its division into the specific species treated by arithmetic or geometry. Since quantity as such cannot be divided without being associated with qualities that differentiate its species, Maurolico introduces the idea of an undivided subject for an undivided mathematics – a *mathematica prima* concerned with operations and relations applicable

[27] *Sermo de divisione artium*, §§ 61–62: "His et illud addendum existimo …Quae tria non solum elementariis, sed etiam Archimedeis et aliorum demonstrationibus saepe usu veniunt. Super his enim tamquam fundamentis propositiones omnes successiva demonstratione construuntur." (*Edizione Nazionale dell'opera matematica di Francesco Maurolico*, available at https://maurolico.it/Maurolico/opera.html?path=2.C.1.1).

to quantity in general: multiplication and division, ratio and proportion, commensurability and incommensurability. The point is not metaphysical. It is methodological: without such a level of generality, the kind of operational mathematics Maurolico is developing in the 1550s – above all in the second book of the *Arithmetica* – cannot claim the universality it demands.

An idea already hinted at in the de Vega letter is taken up again in the *De quantitate* and explicitly grounded: time, weight, mechanical moment, and *vis* are discussed as magnitudes that can be treated as continuous and compared *proportionaliter*. Time is framed as a continuous flux tied to an indivisible instant; weight and lightness, though 'relative qualities', are reduced to quantitative comparison by proportion; *momentum* is introduced as a distinct *potentia* exemplified by the balance and anchored in his doctrine of equal moments; and *vis* is treated as a further *potentia* whose instances can likewise be compared proportionally.[28]

At this point, ratio becomes the bottleneck. In the *Sermo de proportione*, Maurolico introduces two distinct axes of classification: *rational/irrational* (relative to a *quantitas rationalis posita*) and *known/unknown*. Here *irrational* does not mean *unknown*, since 'knowledge' is a matter of *denominability* (by commensurability or by a law derived from it), not of commensurability as such.

On the first axis, Maurolico extends to *every* quantity a Euclidean-style classification into *rational* and *irrational* species. A quantity counts as rational if it is commensurable with the posited rational quantity either *in magnitude* or at least *in power*; it counts as irrational when it is commensurable in neither of these ways. Here "in power" is not a shorthand for the Euclidean quadratic case: Maurolico explicitly allows commensurability "in the first, second, or whatever power" and remarks that Euclid says nothing about quantities that are rational only in the cube, from which further families of irrational magnitudes and ratios could in principle be generated.[29]

On the second axis, every ratio is, in the first place, either *known* or *unknown*. A ratio is *known* when it can be brought 'into some knowledge' by being bounded or stabilized *in numerical terms* – either directly by commensurability, or by some rule of relation that ultimately derives from commensurability. Incommensurability does not by itself block knowledge. An irrational ratio may still be known as long as the magnitudes can acquire

[28] *De quantitate sermo noster*, §§ 78–88 (*Edizione Nazionale dell'opera matematica di Francesco Maurolico*, available at https://maurolico.it/Maurolico/opera.html?path=2.D.1.1).

[29] On the open-ended scope of *in power*, see *Sermo de proportione*, § 126 ("*per potentiam primam, secundam, vel quotamcumque*"). On the extension beyond Euclid's Book X (including 'cube-rational' quantities) and the potential proliferation of further species, see §§ 133–134; the appended table after § 137 (*Rationales, In potentia, Cubice, Mediales*) makes the intended grading explicit. (*Edizione Nazionale dell'opera matematica di Francesco Maurolico*, available at https://maurolico.it/Maurolico/opera.html?path=2.D.1.2).

'names' and be numerically signified through such a rule. Maurolico's own example is telling: the ratio between the side of a pentagon and the diameter of the circle circumscribed about it is irrational, yet it is nonetheless a *known* ratio, precisely because both terms can be signified in numerical language (through their denomination and the laws that connect them).

By contrast, an *unknown* ratio is defined only privatively: it falls outside the domain of all denominable relations, admits no numerical naming, and is therefore *indemonstrable* in the required sense. The ratio of circumference to diameter is presented as the paradigmatic case – already hinted at in the de Vega letter. 'Unknown', in this sense, is not simply 'incommensurable': it is *anonymous, nullo pacto nominabilis*.

The upshot is sharper than a mere commensurable/incommensurable divide: the projected universality of the framework turns on a boundary between ratios that can be numerically denominated – even when irrational – and ratios that resist denomination altogether. At that point, universality becomes conditional rather than absolute. This tension will structure Maurolico's later attempt to make *quantitas generalis* operative and, eventually, his return to Book V.

### From concept to practice: *quantitas posita* and the arithmetization of magnitudes

The *Sermones* articulate an ambition, but they do not yet deliver a usable technique. To claim that mathematics may treat *quantitas simpliciter* is one thing; to show how such a notion can be made operative in concrete demonstrations is another. In Maurolico's case, the bridge between the two is built around a single, deliberately instrumental device: the *posita* magnitude (or quantity), chosen *ad libitum* as a term of reference – and, in the explicit language of Book II of the *Arithmetica, named unitas* – used to *signify* magnitudes by numerical terms and thus make calculation on magnitudes demonstrably practicable.

A first, still local, manifestation of this device appears in the *Appendix* to the *Data* (7 April 1554), that is, only a few weeks before the *Sermones*.[30] There Maurolico makes explicit a principle that is easy to miss if one reads the *Sermones* in isolation: "no magnitude is given except with respect to a posited magnitude" (*nulla magnitudo datur, nisi respectu positae magnitudinis*). If one wants to perform constructions that amount to extracting roots or establishing mean proportionals, one must first fix a reference magnitude. In the geometrical setting of the *Data*, this reference is still typically a straight line, because it must be inserted into a construction. The point, however, is already unit-like: the posited magnitude functions as a general pole of comparison, in a way that is formally analogous to the unit in arithmetic.

30 Veronica Gavagna and Rosario Moscheo, "I *Theonis Datorum libelli duo* di Francesco Maurolico," *Bollettino di Storia delle Scienze Matematiche* 22, no. 2 (2002): 267–348. In Edizione Nazionale: https://maurolico.it/Maurolico/sezione.html?path=4.3.

By fixing a *magnitudo posita* as a term of reference, he shows how to construct the square root – and, mutatis mutandis, the cube root – of an arbitrary given magnitude. The procedure itself remains classical: for the square root he relies on the standard Euclidean construction via a mean proportional (*Elements*, VI.13 / II.14), while the cube root is obtained by inserting two mean proportionals. But the same move is then generalized and made systematic in Book II of the *Arithmeticorum libri duo*.[31]

Here Maurolico states, in explicitly programmatic terms, that arithmetic is the instrument of all calculation and that every magnitude can be 'signified' through numbers; geometry, he adds, has a twofold practice – one by drawing, one by calculation – and the latter is derived from the former 'as from a source.' This distinction is introduced to account for procedures in which one searches, *by numbers*, ever closer to an irrational or even unknown magnitude. On this basis he announces a *novum demonstrandi genus*, tied not to the line as a privileged species, but to *general quantity* as the proper subject of a *mathematica prima*.

The operational core of this reorientation is the definition of *quantitas posita*. Maurolico characterizes it as the quantity freely chosen as the common measure for quantities of the same kind, and, as he puts it, "denominated *ab unitate*" – that is, it is *called unitas*, just as the unit is the common dimension of numbers. Everything that follows depends on this: at the definitional level, a magnitude is said to be *signified* numerically insofar as its relation to the posited quantity can be expressed through numbers in canonical forms. If the magnitude is a multiple of the posited one, it is signified by a single number; if it contains one or more parts of it, it is signified by a pair of numbers, numerator and denominator. In this sense, a fraction is not merely a numerical object; it is the canonical form under which a magnitude becomes arithmetically tractable.

Book II, however, makes clear that 'numerical terms' are not exhausted by integers and fractions. The practice Maurolico sets out to *demonstrate* explicitly includes operations on irrational quantities: extractions of square and cube roots, and, more generally, the handling of irrationals in calculation. The foundational definitions begin with the simplest signifying forms, but the operational dossier treats such irrational expressions as legitimate numerical terms, and develops rules for their use in computation.

Once magnitudes are 'signified' in this way, operations can be extended from numbers to magnitudes. This is most transparent in the definition of multiplication between quantities, which Maurolico defines through a proportional condition: a quantity *c* is the

[31] On the *Arithmeticorum libri duo* (Venezia, 1575), see Jean-Pierre Sutto, "Francesco Maurolico, mathématicien italien de la Renaissance (1494–1575)" (Thèse de doctorat, Université Paris VII–Denis Diderot, 1998), 317–384 (in particular pp. 350–370 regarding the arithmetic of *general quantity*). For the text of Book II: Edizione Nazionale, available at https://maurolico.it/Maurolico/sezione.html?path=6.A.1.

product of *a* and *b*, if *c* stands to *b* as *a* stands to the *posita P*.[32] The definition shows how ratios are used to *control* the extension of numerical operations to magnitudes: multiplication becomes an operation on magnitudes only insofar as a proportional relation to *P* can be established and handled.

At this point the tension that will dominate the rest of the dossier becomes unavoidable. The entire apparatus presupposes that the ratio between a magnitude and the posited magnitude is itself a ratio that can be 'named' and treated as *cognita*. Maurolico can allow a very wide family of such cases – including incommensurable ratios, provided they are denominable *per magnitudinem* or *per potentiam quotamcumque*. But the procedure breaks down as soon as the relevant ratio is *incognita* in the stronger sense: *anonymous*, *nullo pacto nominabilis*. The *quantitas generalis* of the *Arithmetica* therefore comes with a built-in restriction: it is general with respect to *species* of magnitude (time, weight, line, surface, and so on), but not with respect to the *ratios* that arise within those species.

This is exactly the point at which the project begins to look like a structural detour rather than a straight road. The Archimedean corpus that Maurolico was trying to reorganize is saturated with comparisons that are controlled by proportional reasoning; and the promise of a *mathematica prima* is to turn that reasoning into a transferable calculus of quantity. Yet the moment the calculus is grounded on *posited quantity* and on numerical signification, it inherits the boundary drawn in the *Sermo de proportione*: not every ratio is a legitimate object of calculation. The more Maurolico pushes toward a general operational language, the more sharply the distinction between denominable ratios and anonymous ones becomes structurally decisive. It is against this background – and not merely as a late, isolated exercise – that his return to Book V must be read.

## Recasting Book V: the 1567 compendium and its unresolved leap

The late *Compendium* of Book V is Maurolico's most explicit attempt to neutralize the friction that had become unavoidable in the *Sermones* and in his arithmetization of magnitudes: how to reason *on ratios* without remaining trapped in the full Euclidean apparatus of equimultiples – and, at the same time, how to treat ratios as objects capable of carrying foundational work in a projected *mathematica prima*.[33]

32 In a schematic modern shorthand: $a \times b = c \Leftrightarrow a : P = c : b$.

33 Maurolico's *Compendium of Book V* belongs to the broader set of *Elementorum compendia*, in which he produced a synopsis of the first ten books of Euclid's *Elements*. This enterprise should be situated within Maurolico's collaboration, in the last years of his life, with the Jesuits of Messina, a collaboration that aimed at producing a complete mathematical *cursus* for teaching purposes. On this context, see Rosario Moscheo, *I gesuiti e le matematiche nel secolo XVI. Maurolico, Clavio e l'esperienza siciliana* (Messina: Società Messinese di Storia Patria, 1998). On the *Compendium of Book V* in particular, see Jean-Pierre Sutto, "Le Compendium du 5$^{e}$ livre des Éléments d'Euclide de Francesco Maurolico," *Revue d'histoire des Mathématiques* 6 (2000): 59–94.

The core move is definitional, and it is best read as a deliberate step toward the autonomy of ratio. Proportion is no longer grounded in the classical comparison of equimultiples of the terms, but is recast as an *equality of ratios*.[34] In Maurolico's compendium, ratios are first sorted by 'name': two ratios may be said to be equal if they share the same denomination. When they do not, equality is decided by a systematic comparison with *named* ratios: $r_1 = r_2$ if, for every named ratio $r$, $r_1$ and $r_2$ occupy the same position with respect to $r$ (i.e. whenever $r_1 \lesseqqgtr r$, then also $r_2 \lesseqqgtr r$). In this way, the criterion of equality no longer passes through the four terms that instantiate the ratios. The ratio itself is treated as the primary object of comparison, endowed with an order structure relative to a privileged class of ratios.

The crucial claim is Proposition 3: "every ratio of magnitudes is either named, or it falls between named ratios." Read on its face, this looks like the decisive closure Maurolico needs. It suggests that non-named ratios can always be bracketed by named ones and therefore controlled by comparison. In a framework built on denominability, this is the natural substitute for the excluded equimultiple machinery: one secures the governance of arbitrary ratios by anchoring them between ratios that can be named.

But this is precisely where the *Compendium* shows its limits. As Sutto notes,[35] the proof is exceptionally short and relies on the unargued existence of the magnitudes required to produce the bracketing ratios. Establishing that existence would demand substantial work on multiples (or an Archimedean-type lemma), or else a return to Euclid's missing material on proportion; yet the *Compendium* has eliminated the relevant equimultiple apparatus and does not provide substitute lemmas of comparable strength. Proposition 3 asserts the very control over arbitrary ratios that the project needs, but it does so by presupposing, rather than reconstructing, the labour that makes such control legitimate.

Even if we set aside this logical vulnerability, a more fundamental barrier still persists. The *Compendium*'s attempt to autonomize ratios *could* have led toward a recasting of number as the general surrogate for ratio – but Maurolico cannot take that step. 'Number' remains, for him, ultimately anchored in the classical idea of a plurality of units, even when

[34] The idea of defining proportion in terms of an equality of ratios goes back to Campanus's edition of the *Elements* (Book V, Def. 4), and Maurolico had already adopted it in the *Sermo de proportione*, § 94: "Videtur igitur ratio esse quodammodo relatio. Et tamen conferuntur rationes quo ad aequalitatem vel inaequalitatem, unde proportio est rationum aequalitas sive identitas." Already in 1554, therefore, Maurolico was thinking in terms of a definition of proportion that would "autonomize" ratios; not by chance, immediately after the passage quoted he proposes to use the Euclidean algorithm: "Posset et proportionalia aliter diffiniri sic: tunc enim bini hic termini binis illic positis proportionales esse dicuntur, cum quoties hic toties illic minor de maiori auferri potest, et quoties hic toties illic residuum de minori; itemque et quoties hic residuum de ablato, toties illic residuum de ablato, itaque in infinitum" (§§ 117–118).

[35] Sutto, "Le Compendium du 5e livre des Éléments d'Euclide de Francesco Maurolico," 73–74.

numerical practice is widened to include fractions and radical expressions. This is stated with particular clarity in the *Arithmetica*, where he insists on the need to distinguish the two branches of geometry precisely because, "by means of numbers, we track ever closer the term or limit of an irrational or unknown magnitude" (*per numeros, irrationalis aut ignotae magnitudinis terminum seu limitem magis ac magis propinquantes vestigamus*) – for instance, in approximating the side of the equilateral triangle or square inscribed in a circle from a given diameter. The procedure licenses an asymptotic numerical pursuit; it does not produce a number that can *signify* the ratio that remains *incognita* in the stronger sense: *nullo pacto nominabilis*.

The result is a structural loop. Book II of the *Arithmetica* needs ratios to define operations on magnitudes via the *posita/unitas*; but a fully general arithmetic of *quantitas generalis* would require a theory of ratio that is autonomous enough to handle, as legitimate objects of calculation, precisely those relations that escape numerical denomination. The *Compendium* moves in that direction by treating ratios as objects with comparison and composition; yet, without re-importing the justificatory work it set aside, and without introducing a numerical surrogate capable of signifying all ratios, it cannot complete the leap.

In this sense, the *Compendium* does not close the Book V problem; it relocates it. And the relocation is fully consistent with the broader picture suggested by the *Sermones* and by the arithmetization of magnitudes: the more Maurolico leans on ratios to secure a calculus of *quantitas generalis*, the more sharply the internal boundary between denominable ratios and *incognitae* becomes structurally decisive.

## 5. *Labyrinths*

### Maurolico: *quantitas generalis* and a structural loop

On the basis of the *Sermones* and the *Compendium*, and taking into account the previously discussed arithmetization of quantities, a single project becomes apparent, in which ratio is asked to mediate between two demands that are not easily reconciled. On the one hand, ratios are pushed toward a measure of autonomy – they must be comparable, orderable, and composable in ways that make them usable across domains. On the other hand, they must remain classically grounded as relations among magnitudes, governed by the constraints that guarantee demonstrative legitimacy. But a *quantitas generalis* needs ratios as the operative pivot that links the *posita/unitas* to multiplication, division, and the transport of procedures.

This is what we will call a *labyrinth*: a chain of internally justified dependencies in which each step is rational and locally productive, yet the sequence as a whole forces the argument back into the very structure it was trying to reorganize. A labyrinth, in this sense, is a productive impasse: it yields clarifications and workable local devices, but it does so by sharpening the boundary conditions of the classical grammar rather than by dissolving them.

In Maurolico's case, the loop tightens around a single requirement: numerical denomination. The operative programme can advance as long as ratios can be brought under a form of naming that makes them usable in calculation; but the same programme also brings into focus relations that resist such naming altogether. The mismatch is generated from within the attempt to turn proportional comparison into a transferable calculus of general quantity, while staying inside the conditions that make ratio a legitimate object of demonstration within the Euclidean–Archimedean structure.

## Galileo as a test case: physical qualities and proportion

What becomes of the Book V constraints when proportional reasoning is used to render physical 'qualities' quantitatively tractable? Galileo's work provides an ideal testing ground, and we appeal to it here only in this limited sense.

Galileo's entry into 'new magnitudes' is the outcome of a late-Renaissance formation shaped by Archimedean texts, Euclidean proportion theory, and the problem-oriented culture of mixed mathematics. In the writings of the Pisan period – the *De motu antiquiora* and the early work surrounding the *Bilancetta* – and still in *Le meccaniche* (early 1590s), Galileo is more interested in identifying the conditions under which a quality can be handled *as* a magnitude within proportion theory than in defining new physical quantities. The result is a *syncretic Archimedeanism*: a willingness to mobilize Archimedean-style devices (balances, moments, hydrostatic situations) together with Euclidean proportion schemata.

This juvenile laboratory also clarifies why the classical grammar can look, at once, powerful and fragile. It is powerful because it offers a stable and reliable way of operating with a target concept through comparisons among homogeneous magnitudes – spaces with spaces, times with times, weights with weights. But it is also fragile because this stabilization relies on a chain of mediations that may break down either when the phenomenon calls for a derived or compound quantity (whose magnitude-status and admissible operations are not secured within the classical ontology – for instance, specific weight), or when it calls for a practicable handle on variation in time, where no finite-interval comparison can directly stabilize the relevant magnitude. In this sense, Galileo's later successes and impasses are different paths inside the same Italian garden, where the hedges are set by Book V.

When this requirement – that ratios bind homogeneous magnitudes – is carried into natural philosophy, it shapes the approach to physical phenomena. Galileo enters this terrain with a clear working rule, already prepared by his early meditations: a physical quality becomes mathematically tractable only insofar as it can be stabilized as a magnitude governed by proportion. In several domains the rule is productive. Uniform motion is the cleanest case: in *De motu aequabili* (conceived around 1609; published in 1638) 'speed' is not introduced as a numerical quotient, but as a magnitude that can be compared under explicit constraints, by transporting into kinematics a classical proportional scheme.[36] Once the direct and inverse proportionalities are fixed (spaces with spaces for equal times; times with times for equal spaces; etc.), Galileo can move beyond the narrowly Archimedean template that guided his earliest investigations and state operative rules in the idiom of Book V – compounded ratios and inversions do the technical work, while each step remains anchored in comparisons among homogeneous magnitudes.[37]

A comparable reconstruction is available in hydrostatic contexts, where weights, volumes, and specific weights (*gravità in specie*) become manageable only through controlled comparisons (weights of equal volumes; volumes corresponding to equal losses of weight in immersion), as in the *Discorso intorno alle cose che stanno in sull'acqua* (1612).[38] In these cases the Euclidean-Archimedean pattern is stressed, but it can work, because homogeneity is preserved at every step.

The point of maximum stress is accelerated motion, because the problem now demands an operational grasp of variation. Here the 'new magnitude' is the instantaneous speed – a quantity that does not persist for any finite time and therefore resists direct stabilization. Galileo's solution is a characteristic mediation: he anchors the instant to a measurable effect, interpreting instantaneous speed as a 'moment' or efficacy at impact (the *percossa* on a yielding medium). The strategy gives him a handle, but at a price. In the 1604 letter to Sarpi the search for a *principio totalmente indubitabile* leads to a false linear relationship between the 'grades of speed' ($v$) and distance ($d$) from the starting point: $v \propto s$. This move illustrates a deeper mechanism of the proportional framework: monotone dependence is naturally pressed into (linear) proportionality. When the correct dependence is later identified ($v \propto t$), the structural difficulty is displaced rather than removed: one still needs a surrogate that ties the evanescent instant to admissible magnitudes. The ensuing 'integration' of motion is therefore not a natural calculus but a tortuous chain of transports. Since the theory of proportions does not allow one to multiply heterogeneous

36 Winifred L. Wisan, "The New Science of Motion: A Study of Galileo's *De Motu Locali*," *Archive for History of Exact Sciences* 13 (1974): 102–306, especially 281–286. More generally, for pre-classical conceptions of speed and the difficulties surrounding its definition and measurability, see the studies collected in Pierre Souffrin, *Écrits d'histoire des sciences.* L'Âne d'or (Paris: Les Belles Lettres, 2012).

37 Giusti, *Euclides Reformatus*, ch. 2; and Enrico Giusti, "Ricerche galileiane: Il trattato *de Motu aequabili* come modello della teoria delle proporzioni," *Bollettino di Storia delle Scienze Matematiche* 6, no. 2 (1986): 89–108.

38 Napolitani, "La geometrizzazione della realtà fisica: il peso specifico in Ghetaldi e in Galileo," esp. 190–232.

magnitudes, Galileo does not sum infinitesimal spaces of the form $v \cdot dt$; he sums 'grades of speed' (figures, triangles of velocities), obtaining a macroscopic velocity that must then be re-related to space through further proportional steps. The procedure yields results, but it does not dissolve the conceptual knot – and the final demonstrations inevitably rely on bridges that are as much rhetorical as they are mathematical.[39]

### A shared labyrinth – and a cut through it

Late-Renaissance mathematics can be pictured as a garden whose most carefully tended feature is, paradoxically, a labyrinth. The paths are themselves orderly, even elegant: Euclid's Book V fixes what it means to compare magnitudes; Archimedean geometry shows how far one can go by inscription, circumscription, and controlled approximation; composed ratios and proportional schemata make procedures portable across an expanding repertoire of problems. Yet the very discipline of this layout ensures that certain turns have no exit. The garden is not 'Italian' in the sense of a free play of perspectives; it is 'Greek', in the sense that its rules of admissibility are built into the terrain.[40]

Maurolico enters this space from a position that is, in a strict sense, more primitive. At the beginning of the sixteenth century the Archimedean corpus is not yet a stable object; it must be recovered, reconstructed, and organized. Maurolico does precisely this – and in doing so he tries to give Archimedean geometry of measure a more transparent foundation. His postulates of inclusion, his systematic reduction of ratios to ratios between lines, and, later, the device of a *quantitas posita* are all rational, locally productive moves: they aim to secure demonstrative legitimacy while making procedures transferable and operational. But the price of this internal strengthening is that ratio becomes the bottleneck. The more Maurolico leans on an operative notion of *quantitas generalis*, the more decisive becomes the boundary between ratios that can be denominated (and hence inserted into calculation) and ratios that remain *incognitae* in the stronger sense. The late return to Book V attempts to neutralize the friction by 'autonomizing' ratios, finding a way to control arbitrary relations. Maurolico, in short, helps build the labyrinth and simultaneously discovers why it closes.

39 On Galileo and the laws of motion, see the introduction to Galileo Galilei, *Discorsi e dimostrazioni matematiche intorno a due nuove scienze attinenti la meccanica e i movimenti locali*, ed. by Enrico Giusti (Torino: Einaudi, 1990).

40 The metaphor should not be taken to imply that this was the only cultivated ground of sixteenth-century mathematics. Around the Euclidean-Archimedean garden there developed algebraic and abacus-based traditions, ultimately indebted to Arabic mathematics. The point here is more limited: the garden considered in this article is the one whose rules of admissibility continued to govern the treatment of magnitudes, even when other mathematical plants were already growing along the walls of the garden and beginning to test the strength of those boundaries.

The same constraint will reappear, in a different key, once the Archimedean paradigm is carried into natural philosophy.

Galileo, unlike Maurolico, does not need to construct Archimedes. He inherits a late-Renaissance Archimedean landscape already populated by texts, commentaries, and problem-oriented practices, and he coordinates them into a distinctive synthesis. In several domains, this syncretic Archimedeanism proves extraordinarily effective: where the relevant qualities can be stabilized through controlled comparisons among homogeneous magnitudes, the Book V grammar can be made to carry real operative weight. But the hard case is accelerated motion. Here the demand is not merely to compare magnitudes but to grasp variation operationally, and the Euclidean restriction becomes structural: the very quantities one would like to treat as autonomous objects (instant, instantaneous speed, 'efficacy') must be reconstructed indirectly. The results are genuine, but the route remains tortuous; and precisely at the point where the new physics seems to require a general calculus, the classical proportional framework forces Galileo back into the same kind of detour that Maurolico had met from within geometry.

That both men, moving from radically different formations and aiming at radically different ends, are driven toward reforms of the theory of proportions is therefore not an accident but a symptom.

In this sense, Maurolico and Galileo occupy two distant entrances to a shared labyrinth. The first tries to secure the classical architecture from within, making explicit what Archimedean practice had left tacit and pushing toward a general operational treatment of quantity; the second tries to extend the same architecture outward, using it to render nature mathematically tractable. Their impasse is not merely personal, nor is it reducible to a lack of technical ingenuity. It belongs to a broader late-Renaissance configuration: Benedetti's reforms of proportion, Guidobaldo's mechanics, Valerio's generalization of Archimedean procedures, Cavalieri's instrumental dissolution of form – all are, in different ways, attempts to stretch the same grammar beyond the point where it can remain both rigorous and fully operative.

Viète marks a clear break: his symbolism makes algebra portable into geometry; but *even* Viète remains bound by the *lex homogenorum* and by proportional schemata that continue to regulate admissible comparisons when algebra is made to speak about magnitudes.

If these developments can be read, retrospectively, as anticipations, this is because they mark the outer limits of classical grammar, not because they disclose a hidden path out of it.

In the seventeenth century, mathematicians would find a way out of this labyrinth, not by following its paths more cleverly – this labyrinth, in fact, had no way out – but by applying new tools that allowed them to 'cut through the hedges' instead of walking along them. It is from this cutting-through that the journey of modern mathematics begins.

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