



Crafting practical mathematics: The making of an abacus manual

Andrea Papi 

Scuola Superiore Sant'Anna; andrea.papi@santannapisa.it

Abstract

This paper examines an unpublished manual of practical mathematics, the *Libro di ragione* (MS Baldovinetti 229), written in 1461 by the Florentine Matteo Cerretani (1444–1519). Based on a comparative analysis with earlier and later abacus manuscripts, this paper argues that such texts were not passive repositories of techniques, but active sites of knowledge production. Cerretani's manual emerges as the result of processes of selection and reorganization of clusters of mathematical problems – the so-called *ragioni* – drawn from other Tuscan sources and closely tied to commercial practice. Far from being an isolated case, this mode of compilation reflects a broader pattern of knowledge-making within the abacus tradition, one of the main channels through which practical mathematics circulated in late medieval and early modern Europe. Finally, archival evidence further allows us to reconstruct Cerretani's social and cultural networks, shedding light on the *milieu* in which such knowledge was shaped, transmitted, and used.

Keywords

abacus manuals, practical mathematics, Renaissance Florence, knowledge-making

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Introduction: abacus mathematics and its sources

Abacus mathematics was one of the most widespread traditions of practical arithmetic in late medieval Europe, circulating through networks of practitioners.¹ Recent scholarship has emphasized the long-term importance of this culture in shaping economic practices as well as technical and scientific knowledge.² Its surviving sources span the late thirteenth to the sixteenth century, with fourteenth and fifteenth-century Italy as its main center of production.³ In this context, Tuscany – and Florence in particular – played a crucial role in fostering a “culture of precise quantification” closely tied to accounting, measurement, and calculation.⁴

The rise of mathematical education took place outside traditional channels of learning. Specialized schools appeared in major cities as well as in smaller towns and villages, expanding opportunities to acquire practical numerical skills through teaching and hands-on practice.⁵ As the government of Prato declared in January 1498, “knowledge and practice of mathematics are useful and necessary for the everyday activities of men in commerce; without them, society itself can scarcely be preserved.”⁶ Abacus masters were mostly laymen who taught privately or were appointed by public authorities, while also offering their mathematical expertise as consultants in accounting, building practices, and occasionally astrology.⁷ They described their knowledge as an ‘art’, that is, a craft, revealing the epistemological status of abacus mathematics in the period.⁸ Thanks to the survival of numerous archival sources, Florentine abacus schools are relatively well documented. They functioned much like artisanal workshops and were accordingly known as *botteghe d’abaco*: learning there resembled workshop training, where apprentices acquired skills through observation and repeated practice. As abacus manuals show, students learned

¹ Cf. Danna, *The Craft of Indo-Arabic Numerals*, and Høyrup, *The World of the Abbaco*.

² Cf. Kelly and Ó Gráda, “Connecting the Scientific and Industrial Revolutions”; Cormack, Walton, and Schuster, eds., *Mathematical Practitioners*.

³ About 485 practical arithmetic manuals were produced in Italy before 1600. Of these, about half were written in the fourteenth and fifteenth centuries. Cf. Danna, “Elaboration and Diffusion of Useful Knowledge,” 60–70.

⁴ Goldthwaite, “The Practice and Culture of Accounting,” 641–642. Cf. Tognetti, “Una civiltà di ragionieri.”

⁵ Cf. Ulivi, “Scuole d’abaco”; Black, *Education and Society*.

⁶ Papi, “Francescani e matematica,” 197.

⁷ Cf. Biagioli, “The Social Status of Italian Mathematicians.” For a broader discussion of mathematical practitioners and mathematical practice, see Bennett, “Practice and Profession.”

⁸ The introduction to the first printed abacus book, issued in Treviso in 1478, is particularly telling. See Flores D’Arcais, ed., *L’aritmetica di Treviso*, f. 1: “Incommencia una practica molto bona et utile a ciaschaduno chi vuole usare l’arte dela merchadantia, chiamata vulgarmente l’arte de l’abbacho” (Here begins a very good and useful treatise for anyone who wishes to practice the art of commerce, commonly called the art of the abacus).

mathematical craft through the solution of problems – the so-called *ragioni* – drawn from mercantile and artisanal contexts.

To understand this culture and its dynamics, we must consider its main sources: abacus manuals, which offer exceptional insight into the medieval and early modern tradition of practical mathematics. By developing mathematical procedures around problems drawn from everyday commercial and artisanal activities, these manuals transformed mathematical knowledge into an operational tool for acquiring and applying numerical skills. Abacus manuals were social and cultural artefacts in which algorithmic techniques were elaborated in response to concrete practices and, in turn, helped to improve those practices. They were instructional texts whose production formed part of the expansion of technical literatures in the late Middle Ages. Like other vernacular sources – such as mercantile notebooks, recipe collections, and craft books – they raise important questions about the codification and transmission of know-how.⁹ Written by schoolmasters and practitioners, they largely eschewed modern notions of authorship and instead embodied a form of “collective knowledge.”¹⁰ In this sense, abacus manuals sustained practical knowledge across generations while being continually reworked in use.

This paper addresses a simple but often overlooked question: how were abacus manuals made? Focusing on Matteo Cerretani’s *Libro di ragione*, it reconstructs the practices of selection and adaptation through which practical mathematical knowledge was transmitted and reconfigured.¹¹ Cerretani’s manual forms part of a wider network of textual circulation and cultural exchange. After outlining its main characteristics, the paper compares MS Baldovinetti 229 (National Library of Florence) with other Renaissance abacus manuals and integrates archival evidence to reconstruct Cerretani’s socioeconomic and cultural environment. In doing so, it explores the mechanisms through which practical mathematics was produced in vernacular manuals crafted by liminal practitioners such as Cerretani.

Matteo Cerretani and his Libro di ragione: a Renaissance manual of practical mathematics

The ownership note identifies MS Baldovinetti 229 as having been written by the Florentine Matteo Cerretani in Pisa in 1461.¹² The note clearly tells us that “This book belongs to

⁹ On instructional and reference texts see Creager, Grote, and Leong, “Learning by the book.” Cf. Danna’s paper in this volume.

¹⁰ Smith, *From Lived Experience*, 37–39. For a reflection on authorship in practical mathematics texts, albeit in a later context, see Morel, “Five Lives of a *Geometria subterranea*,” 225–228.

¹¹ For a recent discussion of the composite and stratified nature of abacus manuals, see Ceccherini, “Come nasce un libro d’abaco.”

¹² The manuscript is on paper, in quarto, 220x148 mm, and consists of 105 folios. Cf. Van Egmond, *Practical Mathematics*, 132–133.

Matteo di Niccolò Cerretani, written in his own hand in 1461 in Pisa. Whoever finds it is asked to return it, for the sake of God and Saint Nicholas, for it is no small labor to produce such a book.”¹³ The manuscript is a typical Renaissance abacus manual, which Cerretani himself calls *Libro dirittamente di ragione* in the initial religious invocation.¹⁴ The analysis below shows that it was not an entirely original composition, but the result of the compilation and reorganization of previous textual materials. It gathers a wide range of problems (*ragioni*), which can be classified according to Van Egmond’s categories of practical mathematical problems. This typology situates the manuscript within the abacus tradition and clarifies its purpose.¹⁵ Table 1 outlines the structure of Cerretani’s manual, indicating the contents and their corresponding folios.

Before tackling the *ragioni*, the manual introduces Indo-Arabic numerals and explains the four operations with fractions, providing the fundamental tools.¹⁶ As later evidence suggests, Cerretani was both the writer and the primary user of the manual, as was often the case with contemporary vernacular practical texts.¹⁷ After this preliminary section, the manual proceeds to the *ragioni*, presenting them progressively from the simplest to the most complex. It gathers problems involving prices, measurements, and currencies;¹⁸ these are followed by problems on companies, bartering, discounts, and metal alloys, solved using the rule of three and the rules of false position,¹⁹ before moving to algebraic and geometrical problems (Fig. 1v–r).²⁰ The final section is devoted to volumes and quantities, silver and gold coins, and additional geometrical problems.²¹ The organization of the contents points to the manual’s didactic purpose, following a fairly coherent structure with problems presented, as noted above, in increasing order of difficulty. Although this structure may have emerged from the recombination of materials accumulated over time, the resulting text displays a degree of coherence that goes beyond a simple collection of

¹³ BNCF, MS Bald. 229, f. 1r: “Questo lib(r)o è di Mateo di Niccholò Cieretani schrito di xua mano l’anno 1461 a Pisa. Chi llo truova lo renda da parte di Dio e di San Niccholò, ch’è faticha a falgli.”

¹⁴ Ibid. The following invocation is similar to those found in other contemporary texts: “In the name of God and of his glorious mother, the humble Virgin Mary [...] and of all the heavenly court of paradise [...] may he grant that I may be able to write or copy this *Libro dirittamente di ragione*.” (Al nome sia [di Dio e della] sua mad[re] gloriosa e umile vergine M[ari]a [...] e di tutta la santa chorte del pa[radiso] [...] choncedimi ch’io possa schrive(re) overo chopiare q(u)esto lib(ro) dirittamente di ragione).

¹⁵ Van Egmond, “Types and Traditions,” 391–428.

¹⁶ BNCF, MS Bald. 229, ff. 1–22v.

¹⁷ Cf. Creager, Grote, and Leong, “Learning by the book,” 6–10.

¹⁸ BNCF, MS Bald. 229, ff. 22v–43.

¹⁹ Ibid., ff. 43–68v.

²⁰ Ibid., ff. 68v–86v.

²¹ Ibid., ff. 87–105.

heterogeneous materials.²² These remarks will gain further significance shortly, as another abacus book comes into play.

Table 1. The structure of Cerretani's manual

CONTENTS	FOLIOS
Preliminary material: introduction; numeration; <i>librettine</i> ; the four arithmetical operations	1r–21v
Business problems: prices and products via the rule of three	21v–22r
Operations with fractions	22v–27r
Business problems: exchange	27r–28r
Business problems: mensuration, prices and products	28r–33v
Recreational problems: money and number problems	34r–v
Recreational problems: series and progressions, and problems of pursuit	35r–37r
Business problems: exchange	37r–40r
Recreational problems: find a number	41r–v
Business problems: interest	42r–43v
Recreational problems: money problems	44v–47r
Business problems: partnership	47r–49r
Business problems: barter	49r–50v
Business problems: discount	50v–52r
Business problems: alligation	52v–63r
Business problems: equation of payments	63r–65r
Business problems: interest	65r–66r
Business problems: equation of payments	66r–68v
Recreational problems solved by algebra	68v–78v
Geometrical problems: measurement of figures and real objects	79r–85v
Recreational problems	86r–105r

Based on the surviving manuscripts, the text appears to have enjoyed some success within the tradition of practical arithmetic, being preserved in three manuscripts that convey a closely similar text. All three are later versions, dating from around 1465 to 1495.²³ Except for the last, written in Pisa by the merchant Andrea Lanfreducci, all are anonymous

²² About abacus manuscript technology, see Ceccherini, “Come nasce un libro d’abaco.”

²³ In chronological order, these are: BMLF, MS Ash. 353; CU, MSS Plimpton 205, 206. Cf. Van Egmond, *Practical Mathematics*, 78, 266–267.

and compiled by different hands.²⁴ Among them, MS Ashburnham 353 stands closest to Cerretani's manual, matching its structure and its wording, whereas MS Plimpton 206 lacks the geometrical section. The close similarity among these witnesses points to the diffusion of a relatively stable textual core within a network of practitioners – a pattern of standardisation common in fifteenth-century abacus production, as the treatise of the Florentine master Benedetto and an anonymous manual from the first half of the century discussed below attest.²⁵

The existence of multiple versions of the same text, together with the long-term circulation of the autograph across different contexts, raises the question concerning the usefulness of these manuals. Since they often contained commercial information – such as prices, currencies, and weights – that was liable to change over time and from one place to another, one may ask why older copies continued to circulate. Yet this variability did not undermine their usefulness. Although current commercial information was undoubtedly valuable, abacus manuals were intended to teach algorithmic techniques, not to provide up-to-date data. What mattered above all was the acquisition of mathematical skills and the ability to apply them across a wide range of practical situations. The persistence of older versions contributed to the stabilization of a shared body of mathematical knowledge, reinforcing the internal coherence of the tradition.

These later versions suggest that abacus manuals could move beyond their original context of production, reaching individuals engaged in diverse activities and facing comparable computational demands. What might seem a single compilation thus reflects a wider environment of exchange, in which texts functioned as flexible tools within networks of practitioners rather than as fixed and bounded works. Abacus manuals were not primarily aimed at a single social group, but belonged to a broader social *milieu* of practitioners – apprentices, merchants, artisans, and teachers –, the so-called *strato culturale intermedio*, that is, an amphibious sphere of cultural activity situated between artisanal practices and learned culture, and operating across vernacular and Latin.²⁶

Matteo Cerretani belonged to this social and cultural context. Like many *marchands écrivains*, he also worked on manuscripts containing literary and philosophical texts.²⁷ Matteo was born in 1444 into a family with a long-standing tradition originating from the Flo-

²⁴ On Lanfreducci family, see Petralia, *Banchieri e famiglie mercantili*, 191–192.

²⁵ On the basis of the surviving manuscripts – at least eighteen – Benedetto's abacus treatise appears to have been the most widely circulated in the fifteenth century. Cf. Danna, *The Craft of Indo-Arabic Numerals*, 131–132.

²⁶ Maccagni, "Leggere, scrivere e disegnare," 636–638. Cf. Biagioli, "The Social Status of Italian Mathematicians."

²⁷ The following manuscripts can be attributed to Cerretani's workshop: BMLF, Pluteo 90 sup. 135¹; *ibid.*, Laurenziano Gaddi 106; BNCF, MS Fond. Naz. II V 98. I borrow the expression *marchands écrivains* from Bec, *Les marchands écrivains*.

rentine countryside.²⁸ His father, Niccolò, was a prominent politician who held key offices for the Florentine republic, such as *Gonfaloniere di giustizia*, *Priore*, *Gonfaloniere di compagnia*.²⁹ Like his father, Matteo joined the *Arte dei Mercatanti* or *Calimala* in 1471, and around the same time married Ginevra Cambi, daughter of the merchant banker Bernardo. The couple had several children and lived in a house at Canto alla Paglia, near the Baptistry of St. John, together with Matteo's brothers, Paolo and Giovanni, and their respective families.³⁰ Matteo held several public offices: he served as one of the *Buonuumini*, as *Gonfaloniere di compagnia*, and as *capitano* or *podestà* in various towns of the Florentine dominion.³¹ He also held the offices of *Camerlengo del Monte comune* and *Signore della Zecca*, both of which required numeracy.³² This invites the question of whether writing a mathematical manual may itself have enhanced an individual's suitability for such appointments. This hypothesis becomes even more plausible if we consider that abacus manuals are rich in problems involving currencies, exchange rates, and alloy calculations, which were relevant to the circulation of coins and the minting practices. For instance, a typical problem concerning alloying metals (*allegare*) sounds like the following one: "A man has a quantity of metal with an alloy of 10 ounces 16 denari per pound, and he wishes to make a coin alloy of 8 ounces 20 denari per pound; I ask how much copper must be added per pound."³³

Like many young Florentines of his social background, Matteo probably received training in practical mathematics.³⁴ It is nevertheless useful to formulate some hypotheses about it, given the availability of multiple channels of cultural transmission and learning in the Renaissance. When he wrote the manual, Matteo was seventeen. On the basis of the surviving evidence on manuscript ownership, he appears to have been the youngest known writ-

²⁸ Originally from the village of Cerreto, at the foot of Monte Morello, the family settled in Florence at the end of the twelfth century.

²⁹ Niccolò married Averarda Alberti. Niccolò features as a protagonist in Leon Battista Alberti's dialogue *De iciarchia*. Cf. Boschetto, "Note sul *De iciarchia*," 203–215.

³⁰ In 1480, the family numbered fourteen members: see ASFi, Monte Comune, Copie del Catasto, 91, ff. 827r–828r; *ibid.*, 93, ff. 147r–148r.

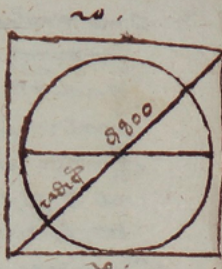
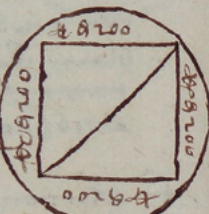
³¹ Among the offices held by Matteo were the *podesterie* of Buggiano (from 20 May 1482) and Montepulciano (from 1 March 1492), the *vicariati* of Certaldo (from 1 July 1495) and Casentino, the *capitanati* of Livorno (from 8 April 1500 and again in 1515), and the *castellania* of the Fortezza Nuova in Volterra (from January 1512). Cf. Papi, "Fra ragioni ed esperienze," chapter 4.

³² On 1 July 1493, Matteo Cerretani began his term as *Camerlengo*, which he held until January of the following year. He also served as *Signore della Zecca* on two occasions, in 1499 and 1518, when he was responsible for the minting of gold coinage. Cf. *ibid.*

³³ BNCF, MS Bald. 229, f. 58r: "Uno à moneta a llegha di 10 once 16 d(enari) p(er) lib(r)a, vuole fare moneta a llegha di 8 once 20 d(enari) p(er) lib(r)a. Adomando q(u)anto rame bisongnerà giugnere p(er) lib(r)a."

³⁴ On the prominent role of mathematical training in Florence, see Tognetti, "Una civiltà di ragionieri," 237–242, and Black, *Education and Society*, 1–42.

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er and owner of this type of book, which circulated primarily among abacus masters and practitioners. By that age a young Florentine would normally have completed his schooling and entered an apprenticeship.³⁵ In this sense, the writing of an abacus book may be understood as an exercise in consolidating and reworking the mathematical knowledge acquired as a pupil. It was widely understood that mathematical skills, painstakingly acquired during schooling, could easily be lost without regular practice.³⁶ Copying and rewriting mathematical problems formed part of the learning process itself. Writing was therefore not simply a means of recording mathematical knowledge but one of the most effective ways of acquiring fluency in practice, internalizing procedures, and assessing one's own competence. As Lorraine Daston has argued, "understanding an algorithm means working through it, step by step, with the hand as well as the head."³⁷ By crafting his manual, Matteo Cerretani produced a valuable *aide-mémoire* for the everyday activities in the market and in public offices, while also creating a tool that embodied his expertise. Unlike other abacus manuals, however, Cerretani's book soon passed out of the family's possession, passing into several hands between Florence and Rome.³⁸ Notes added to the first folio attest to its history until the early seventeenth century. In 1502, one of Matteo's sons, Andrea, donated the book to Bernardo Rustici. By 1586, it had come into the possession of Giovanni Scala, and by 1595 it is recorded in Rome as belonging to Giulio Fagni. In the early seventeenth century, the manuscript was owned by the Florentine master Giovanni Silli.³⁹

Cerretani's manuscript was by no means exceptional. Its production and subsequent circulation exemplify key features of the abacus tradition, particularly its distinctive forms of authorship and transmission. Abacus books exhibit little or no authorship, instead constituting a "voice of collectivity" that reflects the cumulative and collective making of practical knowledge.⁴⁰ Individual manuals therefore differed less in the procedures they

³⁵ Cf. Cherubini, "Frammenti di quaderni," 236–242, and Ulivi, "Scuole d'abaco."

³⁶ Contemporaries recognised the demanding nature of abacus training. Referring to his son, an apprentice in Francesco Datini's company, ser Lapo Mazzei urged Datini to allow the boy to continue his mathematical studies, warning that "if he is removed from the abacus now, it will not take a month for him to forget everything, because in that subject, unless one becomes firmly grounded, everything is lost" (Camesasca, ed., *Epistole*, 261). The experience of Guerrieri di Tribaldo Rossi confirms this concern: after seven months of study, an interruption of just one month was enough for him to lose much of what he had learned, and on returning to school he was reassigned to the *librettine* class (see Klapisch-Zuber, "Le chiavi fiorentine di Barbablù," 766–769).

³⁷ Daston, *Rules*, 89. Cf. Danna, Locatelli, "Dalla teoria alla prassi della matematica."

³⁸ Ownership notes are particularly valuable for understanding the purposes and circulation of these sources; they can also shed light on the personalities of their owners and compilers. While some compilers and owners seem eager to guard these texts jealously, others clearly sought their wide diffusion.

³⁹ BNCF, Bald. 229, f. 1r.

⁴⁰ Smith, *From Lived Experience*, 37.

contained than in the ways those procedures were configured. Their aim was not to build a deductive science but to provide useful tools: collections of *ragioni* accompanied by the rules and algorithms required to solve them. The authority of these rules and algorithms rests not on formal demonstration but on their effectiveness in practice. Like recipes, the *ragioni* were presented as sequences of operations to be followed: it was only by repeatedly working through them that pupils gradually assimilated the underlying rules, learned to recognize analogies between different problems, and thereby acquired the ability to transfer the same procedures to new cases.⁴¹ The practical knowledge embodied in these manuals likewise occupied an intermediate epistemic position between handicraft and science, much like the mechanical arts. Abacus manuals thus codified practical mathematics by organising and applying procedures, not through systematic exposition or theoretical demonstration. Yet this mode of learning fostered a distinctive form of generalization: problem solving followed an inductive logic that moved not from particular cases to general principles, but from one particular case to another.⁴²

Let us consider, by way of example, a typical *ragione* drawn from Cerretani's manual. It concerns the so-called rule of three and its application to a business problem involving the price of a given quantity of goods.

When you have to do some *ragione* in which there belong three things, I say that you must take the thing that you ask or wish to know, and then take that which is not similar to the thing you wish to know. And having done this, you must set the one against the other, and the sum that it makes you must divide by the other part, that is, the thing that remains, and what comes out will be the result of what you ask. And below I will give you an example so that you will understand it.

Eight braccia of cloth are worth 11 florins; how much will 97 braccia be worth? You must do thus: the thing that we ask, that is, what 97 braccia will be worth; the thing not similar to the braccia is 11 florins; and therefore you must multiply 11 by 97, which makes 1067 florins; and this 1067 you must divide by 8, and it comes to 133 $\frac{3}{8}$; and so much will 97 braccia of cloth be worth, when 8 braccia are worth 11 florins.⁴³

⁴¹ Cf. Maccagni, "Leggere, scrivere e disegnare," 642–644, and de Beaune, Hilaire-Pérez, and Vermeir, "L'analogie, les techniques et le savoirs."

⁴² Cf. Daston, *Rules*, 57–58, 76–81, 96–97.

⁴³ BNCF, MS Bald. 229, f. 21v: "Quando ài a fare alchuna ragione nella quale s'apartengha 3 chose, dicho che debi p(r)endere la chosa che domandi, overo vuoi sapere, e poi p(r)endere quella che non è simigliante a quella chosa volgi sapere. E fatto questa debi metter l'una chon tro all'altra, e quella soma che fa debi partire nell'altra parte, overo chosa che t'è rimasto, e quello che ne verà sarà l'efetto di quello domandi e qui apresso te ne darò essenpro p(e)r modo lo 'ntenderai. Otto b(r)accia di panno valgliono 11 fl, che verano 97 b(racci)a[?] Devi chosi fare: la chosa che nnoi dimandiamo sie che varanno 97 b(racci)a; la chosa non simigliante a le

According to Michael Baxandall, fifteenth-century practitioners faced a “formidable confusion” of problems arising from everyday commercial life. The rule of three provided a common computational procedure that reduced this diversity to a common proportional structure, allowing the fourth quantity to be calculated from three known ones. Indeed, “in late medieval and early modern commercial arithmetic, the most frequently used of all reckoning rules was the Rule of Three.”⁴⁴ In the present example, the problem can be expressed as a simple proportion: if 8 *braccia* of cloth are worth 11 florins, what is the value of 97 *braccia*? In modern notation:

$$\frac{8}{11} = \frac{97}{x}$$

The unknown is therefore obtained from the following operation:

$$x = \frac{97 \times 11}{8}$$

Cerretani's Manual in Context

The comparative analysis that follows suggests that the *Libro di ragione* should not be understood simply as a repository of algorithmic procedures. Rather, its organisation reveals how inherited mathematical materials could be selected and recombined into a coherent manual for teaching and practical use. Historical and comparative evidence indicates that Cerretani's manuscript drew on previous texts, one of which can be identified as a school manual written by an abacus master.

When he wrote his manual, Matteo was in Pisa and probably working as an apprentice to his uncle Rosso Cerretani, who was active in the leather trade.⁴⁵ The Cerretani family had enduring ties with Pisa that spanned several generations, including Matteo's grandfather, father, uncles, and brother.⁴⁶ Among them, Niccolò Cerretani, Matteo's father, maintained particularly close connections with the city. He served as captain of Pisa in 1466 and was repeatedly involved in organising the voyages of the galleys: first as *conductor* of the ships sent to Flanders and England (October–November 1441), then as patron of the

b(racci)a sie 11 fl e però debi multiprichare 11 vie 97, fa 1067 fl, e questo 1067 à a partire in 8, vienne 133 3/8 e ttanto varanno 97 braccia di panno valendo 8 braccia 11 fl.”

⁴⁴ Baxandall, *Painting and Experience*, 96; Daston, *Rules*, 57.

⁴⁵ See ASFi, Catasto 825, f. 51r. Rosso was most likely already in Pisa before 1451: see ASFi, Catasto 715, I, f. 231v. By 1469, Rosso was still living in Pisa in a “house with a garden and a small attached dwelling,” which he held from the friars of Nicosia, paying an annual rent of 12 pounds of candles: see ASFi, Monte Comune, Copie del Catasto 91, f. 127r; ASPi, Gabelle dei contratti, reg. 16, f. 40v.

⁴⁶ In 1427, Matteo's grandfather was appointed *Provveditore della gabella*, a post he held for two years (see Boschetto, “Note sul De iciarchia”). Matteo's brother Giovanni taught civil law from 1483 to 1488 (see Verde, *Lo Studio fiorentino*, 451–453).

galleys that sailed from the port of Livorno in April 1443 for Catalonia and Sicily, afterwards as *Console del Mare* (1458); and finally, as captain of the galleys sailed for Flanders and England (October–April 1459–1460) and for Catalonia and Sicily (March 1470).⁴⁷ Against this background, Matteo – born in 1444 – was probably educated in Pisa, where several abacus masters were active.⁴⁸ It is therefore to the manual of one of these masters that we must turn our attention.

The most likely candidate is the manual written by the Pisan master Cristofano di Gherardo Cevoli, a source well known to scholars of abacus mathematics. It conveys a didactic programme probably adopted in his school and a vernacular translation of Fibonacci's *Practica geometriae*.⁴⁹ Its first section, the *Libbro d'abaco*, gathers a collection of *ragioni* with a stronger didactic emphasis.⁵⁰ An ownership note allows the manuscript to be dated to 1441.⁵¹ Cristofano began his career as master later, after serving in the Pisan army; he probably taught mathematics until his death, which occurred between 1454 and 1455.⁵² Matteo Cerretani may have attended his school or studied under one of his pupils, since abacus manuals “had several generations of users as their audience.”⁵³ The available evidence, however, points more strongly to a connection through Matteo's father, Niccolò, within the Pisan maritime *milieu*.⁵⁴ Three annotations in Cristofano's manuscript indicate that the master's son, Dino, took part in two maritime expeditions in 1442 and 1443, making it plausible that he came into contact with Niccolò Cerretani.⁵⁵ This provides a plausible explanation for how Niccolò – and, through him, Matteo – may have gained access to Cristofano's manual.

Comparison with Cristofano's manual demonstrates that Cerretani's *Libro di ragione* drew extensively on the master's collection of *ragioni*. The sequence of problems is in fact almost identical in the two manuscripts, as shown in Table 3. Table 2 illustrates both the

⁴⁷ Cf. Boschetto, “Note sul De iciarchia,” 205–206.

⁴⁸ Cf. Ulivi, “Su Leonardo Fibonacci,” 271–282.

⁴⁹ Fibonacci, *La pratica di geometria*.

⁵⁰ BRF, MS Riccardiano 2186, ff. 9r–80v. For a description see Van Egmond, *Practical Mathematics*, 147–148. Cf. Danna, *The Craft of Indo-Arabic Numerals*, 101.

⁵¹ BRF, MS Ricc. 2186, f. 9r. Cf. Fibonacci, *La pratica di geometria*, 8. The date in the manuscript follows the Pisan style; thus, 1 May 1442 corresponds to 1 May 1441 in modern reckoning.

⁵² Ulivi, “Su Leonardo Fibonacci,” 278. But I found a certain “Cristofoni Ceuli, civis et mercator pisanus,” involved in an act dated 1467: see ASPi, Gabelle dei contratti, reg. 16, f. 17r.

⁵³ Danna, *The Craft of Indo-Arabic Numerals*, 103.

⁵⁴ About this *milieu* see Mallett, *The Florentine Galleys*.

⁵⁵ BRF, MS Ricc. 2186, f. 131v. For a transcription of the annotations, see Fibonacci, *La pratica di geometria*, 9. Although the year is only partially legible and was read by Arrighi as 1448, the chronology adopted here is based on the identification of the officials named in the annotations – Giovanni Bandini and Giovenco (rather than Francesco, as transcribed by Arrighi) della Stufa – with the Florentine galley voyages reconstructed by Mallett, *The Florentine Galleys*, 159.

close textual affinity between the two manuscripts and their principal differences, which are chiefly linguistic, reflecting the Pisan vernacular of the former and the Florentine vernacular of the latter. It reproduces two corresponding *compagnia* problems concerning partnership contracts.

Table 2. Partnership problem

Cristofano, f. 20r
Two men do a partnership together and find that they have made a profit of £100. The first invested 17, the second 41. I ask how much each should receive. You must proceed as follows: add together the capital of the partnership, namely 17 and 41, which makes 58, and this is the divisor. Now, to determine the share of profit due to each. To find what is due to the first, you must multiply his capital by the profit, that is, 17 by 100, which makes 1700; divide this by 58, and it yields £29 s. 6 d. 2 14/29, and this is what is due to the first, namely to him who invested 17. Now see what is due to the second: multiply 41 by £100, which makes 4100; divide this by 58, and it yields £70 s. 13 d. 9 15/29, and this is what is due to the second. Thus we have that the first receives £29 s. 6 d. 2 14/29 and the second £70 s. 13 d. 9 15/29. If you wish to check it, add together £29 s. 6 d. 2 14/29 and £70 s. 13 d. 9 15/29, and you will find that they make exactly £100, which is correct. ⁵⁶
Cerretani, f. 47r
Two partners do a partnership and find that they have made a profit of £100. The first invested 17, the second 41. I ask how much each should receive. You must proceed as follows: add together the capital of the partnership, namely 17 and 41, which makes 58, and this is the divisor. Now, to determine what is due to the first, you must multiply what he invested by what has been gained, that is, 17 by £100, which makes £1700; divide this by 58, and it yields £29 s. 6 d. 2 14/29, and this is what is due to the first, that is, to him who invested £17. Now see what is due to the second: you must multiply what he invested by what they have gained; thus multiply 41 by £100, which

⁵⁶ BRF, MS Ricc. 2186, f. 20r: “Due homini fan(n)o compagnia i(n)sieme e trovansi avere ghuadagnato £ 100. Il p(r)imo misse 17, il secondo 41. Adima(n)do quanto toccherà per uno. Dei fare cosi: agiungie i(n)sieme il co(r)po della compagnia, cioè 17 e 41, fa 58, e questo è il pa(r)titore. Ora, p(er) vedere quello toccha di ghuadagnio a ciaschuno. Ora, p(er) vedere q(ue)llo che ne toccha al p(r)imo, dei mult(iprichare) lo suo capitale contra lo ghuadagnio, cioè 17 via 100, che fa 1700, e q(ue)sto pa(r)te i(n) 58, viene £ 29 s. 6 d. 2 14/29, e cotanto toccha al p(r)imo, cioè a colui che misse 17. Ora vedi quello che toccha al s(econd)o: mult(ipricha) 41 via £ 100 fa 4100, pa(r)te i(n) 58 ne viene £ 70 s. 13 d. 9 15/29, e tanto toccha al s(econd)o. Et abbiamo che al primo toccha £ 29 s. 6 d. 2 14/29 e al s(econd)o toccha £ 70 s. 13 d. 9 15/29. Se la vuoi p(ru)ova(re) giu(n)gie i(n)sieme £ 29 s. 6 d. 2 14/29 co(n) £ 70 s. 13 d. 9 15/29 e troverai di punto faran(n)o £ 100 e sta bene.”

makes £4100; divide this by 58, and it yields £70 s. 13 d. 9 15/29, and this is the profit due to the second. Thus we have that the first receives £29 s. 6 d. 2 14/29 and the second £70 s. 13 d. 9 15/29. If you wish to test it, add together what falls to each and see whether it amounts to £100.⁵⁷

The problem concerns the division of profits between two partners according to shares (*Adomando quanto tocherà per uno*). Here too, proportion plays a central role. One notes the profit (100 £) and the investment of each partner (17 £ and 41 £), then calculates the capital of the partnership by summing the investments (58 £), and finally determines the profit due to each partner by means of a simple proportion, that is, in modern notation:

$$\frac{17}{58} = \frac{x}{100} ; \frac{41}{58} = \frac{x}{100}$$

The first forty-one folios of Cerretani's manual correspond very closely to Cristofano's *Libbro d'anbaco*, both in the sequence of *ragioni* and in their textual formulation. At the same time, the selective reuse and adjustments point to a process of appropriation and reorganisation rather than mechanical copying. Alternative algorithms are occasionally introduced: for instance, two methods are provided to calculate the florin value of £3250, whereas Cristofano uses a different one.⁵⁸ Elsewhere proofs (*prove*) are added – not formal demonstrations but practical procedures for checking the correctness of a result – as in a problem involving the loan of a sum of money: «The £ is lent at 3¼ d. per month; I ask, in how much time do £128 yield £12 16 s. 8 d.[?]».⁵⁹ In problems involving alloying and the Florentine *popolino*, additional explanations absent from Cristofano's text are introduced.⁶⁰ Cerretani also makes occasional copying errors, as in a problem on the equation of payments, where he writes £100 instead of 1000, while keeping the other figures unchanged.⁶¹ Moreover, some *ragioni* are omitted, such as the problem concerning the “man who lends to another a chest full of grain.”⁶²

⁵⁷ BNCF, MS Bald. 229, f. 47r: “2 chompag(n)i fanno chompangnia e truovansi ghuadagnato 100 £. El primo misse 17, el sichondo misse 41. Adomando quanto tocherà per uno. Desi chosi fare: raggiunni insieme el chorpo della chompangnia, cioè 17 e 41, fa 58, e questo è 'l partitore. Ora, p(er) vedere quanto tocha al p(rim)o debbi multip(r)ichare quello misse chontro a quello à ghuadagnato, cioè 17 vie 100 £, fa 1700 £ e questo parti in 58, viene 29 £ 6 s. 2 d. 14/29 e chotanto tocha al primo, cioè a chi misse 17 £. Ora, vedi quanto toccha al sechondo: debi multip(r)ichare quello misse chontra a quello à ghuadagnato, p(er)ò multip(r)icha 41 vie 100 £, fa 4100 £, e questo parte in 58, viene 70 £ 13 s. 9 d. 15/29 di denari, e chotanto tocha di ghuadang(n)o al sichondo. Abbiamo che al p(rim)o tocha di ghuadang(n)o 29 £ 6 s. 2 d. 14/29, e al sichondo tocha di ghuadang(n)o 70 £ 13 s. 9 d. 15/29. E se lla vuoi p(r)ovare raggiungi insieme quello tocha a ciaschuno e vedi se ffa 100 £.”

⁵⁸ BNCF, MS Bald. 229, f. 39v–40r; BRF, MS Ricc. 2186, f. 17v.

⁵⁹ BNCF, MS Bald. 229, f. 42r.

⁶⁰ Ibid., ff. 58v, 62r–v.

⁶¹ Ibid., f. 66r.

⁶² BRF, MS Ricc. 2186, f. 35r.

Yet there is more. It is possible to trace the connection between these manuscripts further back in time, to an anonymous manuscript compiled in Lucca in the mid-fourteenth century.⁶³ References to this city in several problems of Cerretani's manual and Cristofano's first prompted comparison with this text, already edited by Gino Arrighi.⁶⁴ However, as shown in Table 3, the body of the Lucca manuscript was reorganized. For instance, some bartering problems in the final part of the Lucca text (ff. 73–75) appear in the core sections of Cristofano's and Cerretani's manuals (ff. 20v–22; ff. 48v–52). Conversely, the latter include sections absent from the Lucchese manual, such as problems on the equation of payments (ff. 89r–92r), discounts (92v–93v), and valuation of silver and gold in different places (ff. 94v–95r). A further divergence concerns the figures themselves, which do not always match: for example, a problem may give 12 denari instead of 14, or assign a different value to the florin. Proofs found in Cerretani and Cristofano are sometimes absent from the Lucchese manuscript, and vice versa, as in the section on the application of the rule of double false position (ff. 31r–32v).⁶⁵

Another similar text can be placed in Pisa between the Lucchese manuscript and Cristofano's, suggesting an intermediate stage in the transmission. This is MS 41.38, preserved in the Archiepiscopal Library of Pisa and written in the early fifteenth century.⁶⁶ The manuscript contains a text closely related to those examined here, especially to Cristofano's manual, and thus to Cerretani's. Set against these sources, MS 41.38 is more extensive in some sections, most notably in the section on the equation of payments (ff. 63r–77v), while omitting others, such as the *Ragione di marchi d'ariento* and the *Ragione d'oro*.⁶⁷ Since this manuscript belonged to the Rossellini, a Pisan family with connections to Lucca, it may have provided one channel by which problems of the Lucchese tradition entered the Pisan milieu.

As the evidence discussed here points primarily to a Tuscan network linking Lucca, Pisa, and Florence, it also reveals a broader pattern within the abacus culture. Practical mathematics developed cumulatively by successive acts of selection and recombination, not through linear textual descent.⁶⁸ Abacus manuals were therefore not transmitted as fixed texts, but continuously reworked by reconfiguring clusters of problems already circulating within the tradition. Viewed through the lens of artisanal epistemology – that is, a

⁶³ BSL, MS 1754. For a description see Van Egmond, *Practical Mathematics*, 163–164.

⁶⁴ Arrighi, ed., *Scuola lucchese. Libro d'abaco*.

⁶⁵ BNCF, MS Bald. 229, ff. 46v–47v; BRF, MS Ricc. 2186, f. 21r–v.

⁶⁶ BAP, MS 41.38. Cf. Galoppini, “Ti mosterò simiglianti ragioni.” The manuscript also includes a *pratica di mercatura*, currently being edited for publication by Laura Galoppini and Andrea Bocchi.

⁶⁷ BRF, MS Ricc. 2186, f. 41r–v; BNCF, MS Bald. 229, ff. 94v–95r.

⁶⁸ A comparable case of circulation can be observed in the transmission of a Florentine abacus manual between Florence and Faenza, suggesting that similar processes of transmission and adaptation extended beyond the Tuscan context. Cf. Papi, “Note su un libro d'abaco.”

form of knowledge grounded in practice and in the effectiveness of procedures rather than theoretical demonstration – these findings shift attention from questions of originality, innovation, and authorship to the social practices that shaped and transmitted mathematics.⁶⁹ The comparison thus serves as a heuristic tool. Its value lies not in reconstructing a straightforward line of textual descent, but in identifying shared textual units whose different configurations reveal processes of selection, recombination, and adaptation. Table 3 presents these parallel sections across the manuscripts of Cerretani, Cristofano, Rosselmini, and Lucca; folio references are provided for each section.

Table 3. Parallel sections

Lucchese 1350 ca.	Rosselmini 1410 ca.	Cristofano 1441	Cerretani 1461
			1r–21r
1r–7r		9r–14r	21v–31v
	≈ 6v–13v	14r–16r	32r–37r
≈ 11v–33v	≈ 25r–44v	16r–21r	37v–48r
≈ 73r–75r	≈ 57v–63r	21r–22r	48v–52r
37r–44v	45v–57v	22r–25v	52v–62v
	≈ 63r–77v	25v–28r	63r–68r
≈ 81r–v	77v–95r	28r–32v	68v–78v
	95r–105r	32v–38r	79r–88v
		38r–41v	89r–95r
			95v–105r

Arranged chronologically, the table makes the cumulative development of the tradition more readily visible. Earlier textual blocks are generally retained, while later manuscripts tend to introduce additional sections and recombine inherited material. Cerretani's manual illustrates this cumulative dynamic particularly well. While the other manuscripts lack an introductory section, Cerretani's manual includes one (ff. 1r–21r). It opens with a presentation of the Indo-Arabic numerals, followed by the multiplication tables – the so-called *librettine maggiori e minori* – and the operations of multiplication and division. This section resembles that of other manuscripts from the first half of the fifteenth century. It is particularly close to the introductory section of the *Ragioni appartenente all'arismetricha*, a Florentine text that circulated widely from around the 1430s.⁷⁰ This text represents a paradigmatic

⁶⁹ Cf. Smith, "Epistemology, Artisanal."

⁷⁰ The earliest surviving manuscript, anonymous, is preserved in BNCF, MS Magl. XI 119. Other manuscripts transmit more or less the same text, including BMLF, MSS Ashb. 343, Ashb. 608; BL, MSS Add. 8784, Add. 10363; and BNF, Ital. 463. Cf. Van Egmond, *Practical Mathematics*, 375.

Florentine abacus manual, whose standardised structure influenced later texts which were organized as follows: a presentation of the positional numeral system, followed by operations and fractions, sections dealing with practical applications of rules, and additional sections involving algebra and geometry.⁷¹ The explanation of the Indo-Arabic numerals found in this text was especially influential and appears in numerous fifteenth-century manuals: “Since there are nine figures in the abacus, by which whoever knows them will easily come to know the others, and since they must be learned, I shall first set out these nine and write above them what they are.”⁷² The presence of this introductory section makes the manual more self-contained, as it includes elementary notions and techniques that masters could otherwise leave implicit. Unlike Cristofano’s manual, Cerretani’s makes these basic notions explicit, thereby providing a more comprehensive point of entry into the text. This addition reflects the ways in which inherited materials could be reorganized and extended to create a more comprehensive pedagogical handbook.

In some sections, the manuscript departs from earlier models by adding new material. In particular, it contains a series of recreational problems found neither in Cristofano’s text nor in the other manuscripts considered here. These problems deal with a variety of situations, such as finding an unknown number or quantity, dividing numbers or sums of money, and similar tasks. A typical *ragione* from this final miscellaneous section (ff. 95v-105r) concerns the division of an inheritance, a problem with a long history that already appears in the collection of Alcuin of York and subsequently became widespread in European mathematical literature.⁷³ Although miscellaneous sections are also found in the other manuscripts, the problems collected here do not correspond to those preserved in the surviving witnesses examined above. This divergence suggests that the final section

⁷¹ Cf. Danna, *The Craft of Indo-Arabic Numerals*, 142–143.

⁷² BNCF, MS Magl. XI 119, f. 10r: “Con ciò sia cosa che sono nove figure nell’abacho p(er) le quali chi conoscie q(u)elle agievolmente conoscerà poi l’altre e anche si vogliono inparare, io inporrò p(r)ima quelle nove e di sopra iscriverò quelle che sono.”

⁷³ Cf. Alcuino, *Problemi per rendere acuta la mente dei giovani*, 20–21; Van Egmond, “Types and Traditions.” BNCF, MS Bald. 229, ff. 104v-105: “A man, at the point of death, [leaves money to his sons.] To the youngest he says: ‘Go to the chest and take 1 florin, and also one ninth of what remains.’ To the next he says: ‘Go and take 2 florins, and one ninth of what remains.’ To another he says: ‘Go and take 3 florins, and one ninth of what remains.’ And so on, for as many sons as he has, increasing each time by 1 florin and one ninth of what remains, as with the others. In the end, each is found to have received the same amount. I wish to know how many sons there were and how much money he had.” (E’ gli è uno che viene a morte, e à figliuoli. Al minore dicie: va’ in sul chasone piglia 1 fl. di q(u)egli vi sono el 1/9 di ciò che vi rimane. E all’altro dicie: va’, tone 2 el 1/9 di ciò che vi rimane. E all’altro dicie: va’, tone 3 el 1/9 di ciò che vi rimane. E cchosi a quanti figliuoli gli resta viene di ghrado in ghrado chresciendo 1 fl., el 1/9 di ciò che vi rimane chome gli altri insieme si truova tanti d. l’uno q(u)anto l’altro. Voglio sapere q(u)anti figlioli e q(u)anti d. aveva.)

was not simply drawn from the tradition examined here but reflects a further stage of selection and rearrangement. It remains unclear whether these recreational problems were introduced by Cerretani himself or reached him through intermediary sources. Their pedagogical purpose nevertheless appears quite clear, since they assemble a miscellaneous set of exercises that draw on techniques developed throughout the manual.

Against this comparative background, Cerretani's manual appears less as an isolated work than as one instance of a wider textual tradition. As one instance of this corpus, it reveals how mathematics circulated through the transmission of earlier materials. From this perspective, the historical significance of these manuscripts lies less in their originality or innovation than in what they disclose about the cumulative processes through which mathematical practices were transmitted and reconfigured.⁷⁴ The comparison further suggests that practical mathematics emerged less from the invention of new procedures than from the compilation and adaptation of inherited ones. In this sense, writing an abacus manual was itself a mode of knowledge-making: a practical activity of organising, refining, and making mathematical know-how ready for use.

Concluding remarks

The case study highlights how abacus manuals were produced and transmitted across generations of practitioners, pointing to their broad and enduring diffusion. These manuscripts were not passive records but emerged from successive acts of compilation that recombined problems into new textual configurations. What emerges is a tradition sustained by interconnected communities of practitioners, where problems, techniques, and solutions were exchanged across trading zones. As has recently been observed for practical mathematics manuals more generally, writing a manual often meant bringing together, selecting, and improving existing solutions to a common set of problems.⁷⁵ Cerretani's manuscript exemplifies this process. Its significance lies in the particular textual form through which it organized a body of mathematical knowledge into a coherent manual for learning and calculation. Much as an artisan arranges the tools of his craft, the compiler reshaped earlier materials, producing a text that functioned simultaneously as a tool for calculation, a support for learning, and a marker of expertise.

Seen in this light, abacus manuals appear less as derivative compilations than as crafted artefacts. Their construction cannot be separated from their use: writing problems were themselves ways of doing mathematics. The persistence of shared structures and recurring problems does not point to inertia, but to a pedagogical setting oriented towards the acquisition of transferable techniques and the recognition of patterns across heterogeneous

⁷⁴ Cf. Danna, *The Craft of Indo-Arabic Numerals*, 14.

⁷⁵ Ibid., 290–291, and Morel, “Five Lives of a *Geometria subterranea*,” 250–251.

situations. Abacus mathematics thus emerges not as a simplified form of knowledge, but as a distinct form of mathematical practice, grounded in use, shaped by concerns for effectiveness, and embedded in specific social and cultural contexts. Its procedures relied not on formal demonstration, but on analogy, iteration, and the flexible application of rules. Cerretani's *Libro di ragione* therefore offers more than a local case study. It provides a window onto a wider tradition of practical mathematics, in which knowledge was crafted by assembling, adapting, and transmitting common procedures. Reconsidering these sources in such terms means recognising abacus manuals not as marginal by-products, but as central sites of practical knowledge, where doing mathematics meant, above all, making it through writing.

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 Monte Comune, Copie del Catasto, regg. 91, 93
- ASPi = Pisa, Archivio di Stato
 Gabelle dei contratti, reg. 16
- BAP = Pisa, Biblioteca Arcivescovile
 MS 41.38
- BL = London, British Library, MSS Additional 8784, 10363
- BMLF = Firenze, Biblioteca Medicea Laurenziana
 MS Ashburnhamiano 353
- BNCF = Firenze, Biblioteca Nazionale Centrale
 MS Baldovinetti 229
 MS Magliabechiano XI 119
- BNF = Paris, Bibliothèque nationale de France, Italien 463, <https://archivesetmanuscrs.bnf.fr/ark:/12148/cc9536h?collect>
- BRF = Firenze, Biblioteca Riccardiana
 MS Riccardiano 2186, <http://teca.riccardiana.firenze.sbn.it/index.php/it/?view=-show&myId=945afe40-f4a5-4357-b6c9-acec54d4c693>
- BSL = Lucca, Biblioteca Statale
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- CU = New York, Columbia University
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