



# Crossing institutional boundaries: Abacus and scholastic knowledge in fourteenth-century Italy

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## Abstract

This article examines the circulation of abacus mathematics within learned environments of late medieval Italy, drawing on broader historiographical debates on premodern scientific knowledge. Tempering an account that confines abacus mathematics to urban, practical schooling, it documents the presence of abacus knowledge within mendicant studia and learned circles, as well as abacus teachers within university arts faculties in the fourteenth century. Four kinds of evidence are discussed: (1) theologians writing on abacus mathematics; (2) archival evidence from Bologna, Siena, and Perugia; (3) references to convent libraries in abacus texts; and (4) abacus teachers' reports on non-practitioners engaged with abacus mathematics. The paper then examines whether philosophical knowledge informs abacus writings, focusing on scholastic traces in three case studies: Jacopo of Florence's *Tractatus algorismi*, Dardi's *Aliabraa argibra*, and an anonymous Genoese *Tractatus super practicam geometrie*. The article concludes that abacus mathematics was neither socially nor intellectually isolated but embedded within a common culture comprising scholastic knowledge.

## Keywords

abacus mathematics, scholastic knowledge, medieval Italy

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### 1. Introduction: framing a new research question

This article arises from a research question concerning the interaction between philosophy and mathematics in premodern intellectual Europe. Recasting a well-known interrogation articulated by Jens Høyrup in his *In Measure, Number, and Weight* (1994),<sup>1</sup> the question may be formulated as follows: was philosophy merely an epiphenomenon in the historical development of mathematics, or did philosophical discourse contribute to mathematical innovation, perhaps providing an epistemic framework to mathematical practice? I suggest that this question finds a particularly fruitful testing ground in a specific historical and cultural context, namely, fourteenth-century Italy. At that time, cities such as Florence, Siena, Perugia, Bologna, Parma, Pavia, and Padua witnessed the rise and consolidation of a distinctive educational landscape. A unique intellectual atmosphere emerged from the coexistence of varied institutions: the studia associated with mendicant orders, the universities, and the pre-university schools, including abacus schools. This cultural configuration constitutes the principal case study of the present article, defining its chronological focus on the fourteenth century.

Once this context is chosen, the guiding question may be reformulated in more precise terms. One may ask, first, whether the material and institutional conditions for sustained exchanges between philosophical and mathematical domains were in fact present; and, second, whether such exchanges were intellectually meaningful and yielded outcomes of historical significance. The present article argues that the first condition is clearly met and proposes a preliminary exploration of the second, based on an initial analysis of the ways in which practical mathematical texts were framed and introduced. Furthermore, the article investigates how philosophical and mathematical knowledge were reciprocally put to use. More specifically, it asks whether abacus knowledge was mobilized within philosophically oriented contexts, and conversely, whether scholastic knowledge informed the teaching and conceptual frameworks of abacus mathematics. In other words, to what extent did abacus knowledge circulate within learned settings, and how far did university-based philosophical frameworks shape mathematical practice in abacus schools?

To pursue this line of inquiry, I adopt two historiographical perspectives that have proven influential in accounts of premodern scientific knowledge and learning: those of John Murdoch and Edgar Zilsel. Murdoch's position, articulated most programmatically in "From Social into Intellectual Factors: An Aspect of the Unitary Character of Late Medieval Learning" (1975), is directed against separatist interpretations of medieval intellectual life.<sup>2</sup> Murdoch argues instead that late medieval learning, especially in the fourteenth century, exhibited a fundamentally unitary character, both institutionally and intellectual-

<sup>1</sup> Høyrup, *In Measure, Number, and Weight*, especially Chapter 5, "Philosophy: Accident, Epiphenomenal, or Contributory Cause of the Changing Trends in Mathematics."

<sup>2</sup> Murdoch, "From Social into Intellectual Factors."

ly. Institutionally, this unity was secured by the medieval universities; intellectually, it was sustained by shared problems and analytical tools.

Within the university system, at least in Paris and Oxford, which are the focus of Murdoch's study, all advanced students – whether destined for law, medicine, or theology – were required to pass through the faculty of arts.<sup>3</sup> This curricular structure ensured a common training in logic, natural philosophy, and mathematics (arithmetic and geometry). It was this common training that facilitated the transfer of methods and problems across disciplines and faculties. Such institutional unity enabled a reciprocal shaping of theology and philosophy in particular, but also of natural philosophy (physics) and mathematics: philosophical inquiry imported problems from theology and natural philosophy (for example, debates concerning the status of the infinite), while theology and natural philosophy relied on philosophical and mathematical tools to articulate and resolve their own difficulties (for example, the logical distinction between syncategorematic and categorematic terms in dealing with infinities).<sup>4</sup> In this sense, the arts training furnished a shared analytical language that permeated multiple domains of learning, including the scientific one of physics. Murdoch's central conclusion is methodological as much as historical: for the historian, philosophy, theology, and science should not be treated as autonomous or insulated domains, but rather as interdependent components of fourteenth-century intellectual life.

Edgar Zilsel's contribution broadens the analytical horizon by shifting attention beyond the confines of the medieval university and advancing a sociological account of the emergence of modern science. In doing so, Zilsel partially challenges approaches – such as Murdoch's – that explain scientific rationality primarily through internal intellectual developments. In his seminal 1942 article, with the telling title "The Sociological Roots of Science," Zilsel argues that modern science could emerge only once entrenched social barriers between different categories of knowledge holders began to erode.<sup>5</sup> Between approximately 1350 and 1600, the gradual weakening of rigid social and cultural divisions allowed for sustained interaction between elite, leisure-based learned groups and social strata oriented – so to speak – toward practical effectiveness.

As Zilsel emphasizes, late medieval and early modern Europe witnessed an increasing circulation of knowledge between university-trained scholars and humanists, on the one hand, and artisans, craftsmen, instrument-makers, and artists, on the other. Figures such as the sculptor Lorenzo Ghiberti (1377–1455) exemplify this mobility of knowledge across social and intellectual strata, binding 'manual and skilled labor' and 'rational thinking.' A

<sup>3</sup> It is worth specifying that in the case of Italian universities, such as Bologna, the arts training was regarded as preparation particularly for medicine. See, for instance, Masuzawa, "Theology," 1279.

<sup>4</sup> Hence the philosophical tendency to approach Aristotelian doctrines through a "quantifying lens," as emerges from Di Liscia and Sylla, eds., *Quantifying Aristotle*.

<sup>5</sup> Zilsel, "The Sociological Roots."

crucial feature of this new configuration, according to Zilsel, was the growing prominence of quantification: measurement assumed a central role, and mathematics was increasingly deployed in an operational way. This development occurred within what Zilsel characterizes as an “early capitalistic society,” understood as an ensemble of economic practices, labor relations, and social structures that directly affected the production and circulation of knowledge. In such an urban, mercantile, and socially mobile environment, the traditional separation between pure scholars and practitioners weakened, thereby making possible the integration of theoretical reflection and technical practice, thus preparing the ground for the emergence of modern – or rather, post-medieval? – science.

At first glance, Murdoch’s and Zilsel’s perspectives may appear to stand in tension. Murdoch identifies a form of internal coherence *within* late medieval learned culture, whereas Zilsel emphasizes a bridge *between* learned and practical forms of knowledge beginning in the later Middle Ages. Framed in this way, Murdoch may be seen as accounting for the structural unity of medieval intellectual life from within a single milieu, while Zilsel explains why this unity of a single milieu did not yet culminate in modern science. Despite these differences, the two approaches function complementarily in the present study, serving as historiographical presuppositions that frame its central question.

From Murdoch, I retain the view of knowledge as porous rather than rigidly compartmentalized, a condition that allowed logical and mathematical tools to be mobilized across domains, including natural philosophy. From Zilsel, I adopt the insight that science is not an isolated intellectual achievement but a collective and socially situated enterprise, shaped by interactions among diverse cultural actors. From this combined perspective, the question of the interaction between mathematics and philosophy must first transform into a broader inquiry into the interaction between different communities of mathematical and philosophical actors, and ultimately into how the contributions of one discipline allow us to explain the mechanisms and notions of the other.

Building on these premises, the article is structured in two main parts. The first section establishes the material and institutional conditions that made exchanges between mathematical and philosophical actors not only possible but structurally plausible in fourteenth-century Italy. In line with Zilsel’s attention to the circulation of knowledge across social and institutional boundaries, I show that abacus teachers (and the mathematical knowledge they transmitted) were present and operative in settings that were not confined to abacus schools. These include theological studia of mendicant orders and the university arts faculties.

The second section shifts the focus from institutional settings to the structure of mathematical practice as expounded in abacus texts. Although scholarship tends to minimize the role and importance of university knowledge in shaping the work of mathematical practitioners – and with solid reasons – the available evidence invites a more nuanced assessment, as scholastic doctrines surface at various points in these writings. Drawing on

Murdoch's account of the permeability and unity of late medieval learned culture, this section proposes to look at abacus mathematics in light of the shared philosophical training that informed both university and mendicant intellectual environments.

## 2. *The shared paths of philosophical and mathematical actors*

As stated in the previous section, central-northern Italian cities of the fourteenth century constitute a particularly apt case study for addressing the first strand of the overarching research question, namely, the existence of shared contexts in which historical actors operating within philosophical and mathematical domains could interact. Late medieval Italian cities such as Florence, Siena, Perugia, Bologna, Parma, Pavia, Padua, and Venice were vibrant intellectual environments in which the activities of three institutional settings devoted to the education of young adults were closely interlaced within the urban life: the university – comprising, on the one hand, the faculties of arts and medicine, and, on the other, the faculty of law; the studia associated with mendicant orders, broadly comparable to theology or divinity schools; and the abacus schools, which provided instruction in practical mathematics to the offspring of the emerging mercantile class.

Philosophy – understood here as the constellation of doctrines derived from Aristotle in logic, natural philosophy, metaphysics, and moral philosophy – was formally taught within the institutional framework of the faculty of arts.<sup>6</sup> The same contents also entered the foundational education of friars, insofar as theological study presupposed familiarity with philosophical tools and distinctions.<sup>7</sup>

Theoretical foundations of mathematics, including issues that might today be classified under the philosophy of mathematics, such as the ontology of numbers, were also imparted within the faculty of arts as an essential component of quadrivial instruction in arithmetic, geometry, astronomy, and music – with astronomy often occupying a particularly prominent position.<sup>8</sup> Core texts included Boethius' *De arithmetica*, so-called al-

<sup>6</sup> See Bianchi and Randi, eds., *Filosofi e teologi*; and Pedersen, *The First Universities*.

<sup>7</sup> To frame the kind of knowledge imparted within learned mendicant circles, I refer to Roest, *A History of Franciscan Education*, especially 137–146, where the arts and philosophy curriculum is considered.

<sup>8</sup> Studies shedding light on mathematical instruction at European universities, most prominently the university of Paris, include Beaujouan, “L'enseignement de l'arithmétique”; Rommevaux, “Un exemple de *Question*”; and Rommevaux, “L'enseignement de la géométrie.” By contrast, our knowledge of the mathematical teaching at Italian universities before the fifteenth century is comparatively uneven. Useful information is scattered across studies devoted to individual institutions. For instance, for the curriculum of mathematical teaching at the university of Padua, see Siraisi, *Arts and Sciences*, especially 67–77; for the university of Pavia, see the concise note in Azzolini et al., “La facoltà di arti e medicina,” 518; and for the university of Bologna, Rashdall, *The Universities of Europe*, 249–250.

gorism treatises introducing operations with Hindu-Arabic figures (such as that by Alexander of Villa Dei, despite documented contextual resistance), Euclid's *Elements* mostly in Campanus of Novara's Latin version, Campanus' *Theorica planetarum* – among others. For analogous reasons, quadrivial fundamentals also formed part of the educational curriculum within mendicant studia.<sup>9</sup>

Abacus schools, by contrast, specialized in practical mathematics, offering instruction primarily in practical arithmetic, algebra, and practical geometry, with a strong emphasis on commercial applications and a secondary focus on recreational problems, astronomy and calendrical knowledge, and systems of weights and measures.<sup>10</sup> A concrete sense of the range of knowledge circulating within such schools can be obtained from a typical abacus textbook, such as the one produced within the Florentine school – or *bottega* – of Santa Trinita, where Paolo dell'Abaco was active, and preserved in manuscript Florence, Biblioteca Nazionale Centrale, II.IX.57.<sup>11</sup> The contents of this manuscript are diverse and not limited to practical mathematics: alongside Paolo dell'Abaco's *Regoluzze* and the *Trattato di tutta l'arte dell'abaco*, collections of commercial and numerical problems involving procedures for finding unknown quantities, it also includes astronomical and astrological materials, as well as medical texts and preparations of remedies.

It is thus evident that the knowledge circulating within an abacus school was not exclusively mathematical: in the case of Santa Trinita, it appears to encompass medical materials, albeit framed in a markedly practical orientation. This broader intellectual profile is corroborated by an important contemporary testimony, Giovanni di Pagolo Morelli's *Ricordi*, which describes the education of Florentine merchants around 1393. Therein, Giovanni explicitly attributes to schools the reading of classic authors (Vergil and Seneca), major poets (Boethius and Dante), rudiments of rhetoric (Cicero), and elements of philosophy (Aristotle). Supplemented by biblical instruction, this curriculum leaves little room – says Giovanni – for the addition of further, non-superfluous content.<sup>12</sup> Giovanni thus seems to outline what may be described as a proto-humanist education for young

<sup>9</sup> For mathematical knowledge within mendicant contexts, I refer to Chandelier and Robert, eds., *Savoirs profanes*.

<sup>10</sup> Danna, "Figuring out," especially 13–14 and 47; Høyrup, "Abacus Geometry."

<sup>11</sup> This textbook has recently been the subject of detailed analysis by Ceccherini, "Come nasce un libro d'abaco"; cf. Danna, "Una scienza per la rinascita."

<sup>12</sup> Giovanni di Pagolo Morelli, *Ricordi*, 226: "E di poi ài aparato, fa che ogni in dì, un'ora il meno, tu istudi Vergilio, Boezio, Senacha o altri autori, chome si leggie in ischuola [...]. Tu ti potrai istare chon Boezio, chon Dante e chogli altri poeti, chon Tulio che t'insegnierà parlare perfettamente, chon Aristotile che tti insegnierà filosofia: chonoscierai la ragione delle chose [...]. Istara'ti cho' santi profeti nella Santa Iscrittura, legierai e studierai la Bibia [...]; e da questa virtù della scienza tu sarai tanto bene amaestrato e 'nsegnato che non bisognerebbe dire più avanti, ché tutto è di soverchio."

adults.<sup>13</sup> The reference to Aristotle is particularly noteworthy, as it is associated with the unveiling of the nature of things, which in vernacular reads “la ragione delle cose”<sup>14</sup> – an expression that may simultaneously evoke genuinely Aristotelian explanatory aims and concrete abacus procedures oriented toward finding the unknown.

Having outlined the richness of the educational landscape of some Italian cities, I now turn to the question of whether there are positive grounds for assuming intersections among actors traditionally associated with one or another of these institutions – or, at the very least, the possibility that individual actors inhabited multiple contexts, carrying their expertise with them and possibly rearticulating it within new educational environments. Evidence for such intersections can be drawn from mathematical works authored by non-practitioners (§2.1), formal documentary transcriptions relating to city and university life, including salary and payment records (§2.2), and indirect reports by abacus teachers themselves (§2.3).

## 2.1 Non-practitioners’ mathematical works: Grazia dei Castellani and Leonard of Pistoia in Florence

The first type of source on which I focus concerns two Florentine figures: the Augustinian friar Grazia dei Castellani (d. 1401) and the Dominican friar Leonard of Pistoia (*fl.* 1338–1342), both active in Florence in the fourteenth century. The former has already received sustained attention from the historian of mathematics Elisabetta Ulivi, while the latter’s mathematical expertise has been examined by Gino Arrighi and Raffaella Franci. I begin by outlining Grazia dei Castellani’s intellectual profile on the basis of Ulivi’s survey.<sup>15</sup> Grazia dei Castellani was a theologian active at the convent of Santo Spirito of the Hermit friars, which he entered between 1356 and 1358 and where he was ordained in 1361.<sup>16</sup> During the 1390s, Grazia served as a professor of theology at the Florentine university: in 1391 and 1392, he was appointed lecturer on Peter Lombard’s *Sentences* and Augustine’s *De civitate Dei*; whereas in 1394, he was further authorized to lecture and comment on the *Apocalypse* in vernacular, at the church of Santo Stefano al Ponte.<sup>17</sup>

In line with these appointments, sources attest to Grazia’s authorship of theological works such as the *Lectiones theologicae* and the *Quaestiones super libros Sententiarum*. These writings should have been preserved within the library of Santo Spirito, but are no longer listed in the convent library catalogue of 1450 and may therefore be lost – it remains pos-

<sup>13</sup> This is how Aurélien Robert defines it; Robert, *Le monde mathématique*, 55.

<sup>14</sup> See footnote 12.

<sup>15</sup> Ulivi, “Su Maestro Grazia.”

<sup>16</sup> *Ibid.*, 114.

<sup>17</sup> *Ibid.*, 121; Garfagnini, “Lo studio a Firenze,” here 116. The establishment of Florence’s university was not a linear process, for the *studium* had been re-established several times prior to Grazia’s appointment. See Zorzi, “Lo studium nel XIV e nel XV secolo.”

sible that they survive among the Conventi Soppressi holdings of the Biblioteca Nazionale in Florence.<sup>18</sup> Be they lost or hidden, these are the kind of texts that demanded of their author not only theological preparation, but also to engage with deeply philosophical issues.

Beyond this already substantial activity as theologian and lecturer, further evidence points to Grazia's involvement in scientific and mathematical production closely connected to the abacus tradition. To begin with, Grazia is mentioned in Giovanni Gherardi's (1362–1442) *Paradiso degli Alberti*, a collection of novellas inspired by Boccaccio's *Decameron* that offers vivid insights into the Florentine intellectual milieu. There, Grazia is explicitly described as “theologian and mathematician,”<sup>19</sup> alongside another major contemporary figure at the intersection of philosophy and mathematics, Blasius of Parma (1347–1416).

More decisive evidence of Grazia's mathematical expertise is provided by a group of texts attributed to him that fall squarely within the domain of what may be termed business mathematics. These writings are incorporated into two fifteenth-century treatises on practical arithmetic, preserved in Florence, Biblioteca Nazionale Centrale, Pal. 573, and Vatican City, BAV, Ottob. lat. 3307. They include *chasi del chatain* (a discussion of the rule of double false position used to approximate solutions to first- and second-degree equations), *chasi sopra compagnie*, *chasi notabili al meritare e scontare* (dealing with interest, discounting, and partnership problems), as well as a vernacular translation of a treatise on practical geometry, *De visu*.<sup>20</sup>

The content and form of these texts clearly situate them within the abacus teaching milieu. According to Ulivi, however, these writings were not intended for direct pedagogical use; rather, they may reflect Grazia's own early education in an abacus school prior to his entrance into the convent of Santo Spirito. Yet, they would have been composed after Grazia had been ordained. While there is no direct evidence that Grazia himself taught in Florentine abacus schools during his time at the convent, a significant indirect testimony is provided by a Florentine abacus teacher, possibly Benedetto di Antonio and surely a disciple of Domenico Vaiaio. In his *Practicha d'arismetricha* (ca. 1460–1463), Benedetto explicitly states that he acquired insight into particularly subtle mathematical matters

<sup>18</sup> Negri, *Istoria degli scrittori fiorentini*, 315. Gutiérrez, “La biblioteca di Santo Spirito.”

<sup>19</sup> Giovanni Gherardi, *Il Paradiso degli Alberti*, III, 7–9, 164–165: “Similmente ancora per simile modo ci era maestro Biagio da Parma, universal filosofo e metamatico più che altro che quella età si avesse. [...] Eraci ancora il preclaro e famoso teologo e metamatico maestro Grazia, non meno d'ingegno divino che umano.” To give a further sense of Grazia's embeddedness within Florence's cultural scene, I will here recall that Santo Spirito was chosen by Boccaccio, according to his will, as a repository for his texts after his death. The Santo Spirito friar Martino da Signa, who was appointed executor of Boccaccio's will, also witnessed Grazia's religious vows.

<sup>20</sup> One of these texts from the Pal. 573 has been edited in Maestro Grazia de' Castellani, *Chasi sopra chonpagnie*.

(“chasi sottili”) precisely from our Grazia, alongside other renowned abacus teachers.<sup>21</sup>

Leonard of Pistoia's life is less documented. Nevertheless, Silvana Vecchio's entry in the *Dizionario biografico degli italiani* provides sufficient details to situate him within the Dominican convent of Santa Maria Novella. Leonard appears to have served as lecturer in logic in the Dominican convent in Cortona in 1338, before moving to Lucca in 1339 to pursue further studies in philosophy, and eventually settling in Florence at the Dominican *studium generale* of Santa Maria Novella, where he completed his theological training around 1340.<sup>22</sup> His career clearly places him within the same scholastic framework as Grazia: he emerges as a theologian with a strong grounding in philosophy and logic, as reflected in his cursus.

Leonard is the author of several theological works, including the *De praescientia et praedestinatione divina* – long attributed to Thomas Aquinas until the nineteenth century – the *Speculum Christianae religionis* (a compendium on Christian doctrine closely aligned with Aquinas' *Summa theologiae* and *Summa contra gentiles*), the *Tractatus de decem praeceptis*, and the *Super oratione dominica* (a study of the act of praying and an exegesis of the *Our Father*).

His intellectual production, however, is not confined to theology. Leonard also authored a series of mathematical texts: four short treatises corresponding to the traditional quadrivial disciplines – arithmetic, geometry, music, and astronomy – transmitted under the collective title *Mathematica* in the manuscripts Florence, Biblioteca Nazionale Centrale, Conv. soppr., J.V.7, and Biblioteca Riccardiana, ms. 1169.<sup>23</sup> Although the *Mathematica* aligns with the scholastic tradition of quadrivial teaching, it also reveals notable influences from algorism and abacus traditions, particularly in his treatment of arithmetic and geometry. This combination led Arrighi to rank Leonard of Pistoia among the most significant mathematicians of the fourteenth century.<sup>24</sup>

Remarkably, the arithmetic and geometry sections of the *Mathematica* do not present purely speculative taxonomies of natural numbers or geometrical shapes. Instead, they of-

<sup>21</sup> Arrighi, “Il codice L.IV.21,” 155, where a transcription of Benedetto's mention of Grazia is provided. Cf. Ulivi, “Su Maestro Grazia,” 122; and Black, *Education and Society*, 368. On Benedetto di Antonio, Ulivi concludes that he is likely the same author of the arithmetical and geometrical works transmitted in manuscripts Florence, BNC, Pal. 573 and 577, Siena, Biblioteca degli Intronati, L.IV.21, and Vatican City, BAV, Ottob. lat. 3307. The *Practicha d'arismetricha* would have undergone four stages: it would have been composed before 1460, revised in 1460 in manuscript Pal. 573, revised again in 1463 in manuscript L.IV.21, and finally, in 1465, it would have been united with the *Praticha di geometria* (present in Pal. 577) within manuscript Ottob. lat. 3307. See Ulivi, “Benedetto da Firenze,” 59–60.

<sup>22</sup> Vecchio, “Leonardo da Pistoia.”

<sup>23</sup> Gino Arrighi transcribed selected parts of the *Mathematica*. See Arrighi, *L'Arithmetica di fra Leonardo*; Arrighi, “Fra Leonardo da Pistoia”; and Arrighi, “La *Cosmimetria* di fra Leonardo.” See also Franci, “L'insegnamento dell'aritmetica nel medioevo,” 128–129.

<sup>24</sup> Arrighi, “Fra Leonardo da Pistoia,” 382.

fer practical procedures for computation and measurement – methods for operating with integers and fractions, as well as techniques for determining dimensions such as height, width, and depth through the use of instruments. In the arithmetic section, Leonard introduces the *ars numerandi*, explicitly identifying it with *algorismus* in its vernacular form; for Franci observes that this section also includes the extraction of cube roots.<sup>25</sup> Although Leonard's *Mathematica* likely originated in a conventual teaching context, its practical orientation suggests connections with urban mathematical instruction. This line is further validated by the fact that, as will be recalled below in §2.3, Leonard's treatment of integers and fractions was later taken up by the author of the *Practicha d'arismetricha*.

Grazia dei Castellani and Leonard of Pistoia thus emerge as friars, theology lecturers, and experts in abacus mathematics. The available evidence concerning their career and writings points to figures who operated across multiple intellectual contexts and contributed within each of them. In this respect, they exemplify the convergence of scholastic and practical mathematical traditions – much like later figures such as the Franciscans Mariotto Guiducci and Luca Pacioli, Grazia and Leonard stand at the intersection of the convent and the abacus school.<sup>26</sup>

## 2.2 City- and university-life records in Bologna, Siena, and Perugia

If Grazia dei Castellani provides evidence of early abacus knowledge entering and being reworked within mendicant circles – albeit not with the aim of instructing others – other types of sources delineate a different, yet closely related, trajectory of knowledge transfer, namely, from the abacus school to the arts faculty (and back again). A small number of examples drawn from the Universities of Bologna, Siena, and Perugia will substantiate this claim. In what follows, I look at the *Rotuli dei lettori, legisti e artisti* for Bologna, at the university *Chartularium* for Siena, and the city statutes for Perugia.

The *Rotuli*, that is, the salary registers of the University of Bologna, record two cases that are particularly relevant for the present inquiry.<sup>27</sup> Beginning in the academic year 1387–1388, the faculty of arts appointed masters to lecture not on astrology, but explicitly “ad lecturam abbachi.” This designation recurs consistently in the payment records of the fourteenth century. In 1387–1388, the subject was taught by one Nicola Ottaviani (also referred to as de Abbacho or de Bononia) alongside Antonio Bonini Biliotti of Florence.<sup>28</sup>

<sup>25</sup> Franci, “L'insegnamento,” 129.

<sup>26</sup> I refer to Papi, “Francescani e matematica”; and Ulivi, “Luca Pacioli.”

<sup>27</sup> Andrea Papi meticulously reconstructed the tradition of abacus teaching in Bologna up to Scipione Dal Ferro, covering the fourteenth century while focusing especially on the fifteenth, for which the documentary record is considerably richer. See Papi, “Prima di Scipione Dal Ferro.”

<sup>28</sup> *Rotuli dei lettori*, vol. 4, 14: “AD LECTURAM ABBACHI. M. Nicolaus de Abbacho cum salario librarum quinquaginta. M. Antonius Boninj Belliotti libras centum. Ita tamen quod predicti magistri non possint nec debeant diebus feriatis ire ad mensurandum, salvo quod dictus magister

Antonio was permitted to teach at the university while simultaneously holding a civic appointment granted by the city of Bologna, which involved supervising masonry, land surveying, and account auditing. The salary entry specifies that he alone was authorized to carry out such activities on non-teaching days (“diebus feriatis”).<sup>29</sup>

What emerges from this notice is that Antonio’s activity was not confined to academic instruction but extended directly into the urban and administrative life of the city. Notably, during the same years in which Nicola Ottaviani and Antonio Bonini Biliotti lectured on abacus mathematics (“ad lecturam abbachi”) – that is, between 1387 and 1393<sup>30</sup> – the arts faculty also counted among its members one of the most prominent figures in contemporary philosophy, teaching astrology and logic: Blasius of Parma.

A further instructive case appears in the academic year 1395–1396, when two teachers were again appointed “ad lecturam abbachi”: Antonio Bonini Biliotti, who continued to combine university teaching with civic duties, and Jacopo of Betania. The *Rotuli* describe Jacopo’s appointment in particularly revealing terms:

**AD LECTURAM ABBACHI** [...] Magister Iacobus de Betania electus ad dictam lecturam sceu docendum Abbachum ... et teneatur publicas retinere scollas et abbachum docere in dicta civitate Bononie ac videre et chalchullare ac in saldum reducere rationes, nec non mensurare et assagiare res quaslibet ad commune Bononie spectantes et reactare herologium communis et accedere ad videndum laboreria et hedificia communis Bononie que videri et revideri expediens foret et que oppoteret reparari et eiam de novo fieri in quacumque parte quociens et quandocumque foret opportunum.<sup>31</sup>

This document makes clear that, in 1395, the boundaries between university teaching, civic service, and pre-university mathematical education were far from rigid. Jacopo was appointed as a university lecturer in mathematics, yet his duties explicitly included calculating and settling accounts, maintaining the city clock, inspecting workshops and public buildings and – crucially – continuing to teach abacus mathematics in the city of Bologna. In other words, the same individual was expected to operate simultaneously within the university, the communal administration, and the abacus school system. At that time, one of Jacopo’s teaching colleagues in the arts faculty was Angelo of Fossombrone, a master renowned for his command of the Oxford Calculators’ approach to natural philoso-

Antonius teneatur et debeat mensurare res spectantes ad commune Bononie et videre et calculare rationes dicti communis cum expediens fuerit, ut patet in litteris transmissis per dominos Antianos dicto magistro.” Cf. Mazzetti, *Repertorio di tutti i professori*, 46.

<sup>29</sup> Antonio Bonini Biliotti’s twofold investment was first mentioned by Ulivi, “Scuole e maestri d’abaco,” 124.

<sup>30</sup> *I Rotuli dei lettori*, vol. 4, 16.

<sup>31</sup> *Ibid.*, 18. Bold here and in other quotations is always mine.

phy and sophismata (particularly as developed by William of Heytesbury).<sup>32</sup>

Up to this point, I have deliberately used the generic expression “teaching mathematics” to describe the appointments of Antonio Bonini Biliotti, Nicola Ottaviani, and Jacopo of Betania despite the fact that their official designation was “ad lecturam abbachi” – sometimes being referred to simply as “arithmetic.”<sup>33</sup> Should this label be taken to indicate the formal establishment of a chair of practical mathematics proper at Bologna at the end of the fourteenth century? I do not wish to press this claim without nuancing it. For the statutes of the arts faculty of 1405 continue to organize mathematical instruction around distinct geometrical and astrological/astronomical speculative components.<sup>34</sup> More cautiously, however, the evidence does warrant a different take: specialists trained in practical mathematics – figures who elsewhere functioned as city abacus teachers – could and did lecture within the arts faculty. Also noteworthy is the case of Jacob the German (lit. *de Alamania*), who was appointed not only to lecture on arithmetic *and* astrology, *but also* on moral philosophy.<sup>35</sup>

A closely comparable situation can be documented at the arts faculty of the University of Siena, and at an even earlier stage, that is, in the first half of the fourteenth century. From the 1320s, abacus teachers appear to have been embedded within university teaching, hired explicitly because of their mastery and expertise (“ars et doctrina”).<sup>36</sup> More specifically, the university *Chartularium* reveals that between 1328 and 1341 the teaching of mathematics within the arts faculty was entrusted to two masters trained in the abacus tradition: Giovanni di Puccio (also called Morroccho) and Lapo di Neri (or di Ranieri) from Florence – and Tommaso di Davizo from Florence, at the passing of the latter.<sup>37</sup>

Giovanni di Puccio was appointed in 1328, with an annual salary of 25 lire, paid in two semiannual installments. This salary remained stable until the final year of his service in

<sup>32</sup> Ibid., 19.

<sup>33</sup> For instance, *ibid.*, 11, where Antonio Bonini’s activity is defined as “ad lecturam arismetrice.”

<sup>34</sup> *Statuti delle università*, 276: “In astrologia in primo anno primo legantur algorismi de minutis et integris, quibus lectis, legatur primus geometrie Euclidis cum commento Campani. Quo lecto, legantur tabule Alfonsi cum canonibus. Quibus lectis, legatur theorica planetarum. In secundo anno primo legatur tractatus de sphaera, quo lecto, legatur secundus geometrie Euclidis, quo lecto legantur canones super tabulis de linerijs. Quibus lectis, legatur tractatus astrolabij Mes[sa]chale. In tertio anno, primo legatur Alkabicius, quo lecto legatur Centiloquium Ptolomej cum commento haly. Quo lecto legatur tertius geometrie, quo lecto, legatur tractatus quadrantis. In quarto anno primo legatur quadripartitus totus, quo lecto, legatur liber de urina non visa. Quo lecto, legatur dictio tertia almagestj. Dictis annis completis, et completis dictis libris in dicto termino, fiat circulus, et redeatur ad lecturam primj anni postea ad lecturam secundi anni, et sic per ordinem.” Cf. Rashdall, *The Universities of Europe*, 249–250.

<sup>35</sup> *I Rotuli dei lettori*, vol. 4, 11.

<sup>36</sup> See below the reference to Giovanni di Puccio’s teaching for the year 1329.

<sup>37</sup> Zdekauer, *Lo studio di Siena*, 14–15; Moscadelli, “Maestri d’abaco a Siena,” 208. Cf. Ulivi, “Scuole e maestri d’abaco,” 126.

1342. The records further indicate that Giovanni occasionally placed his expertise at the disposal of the city: in 1336 and again in 1340, he is recorded as acting as a surveyor. Of particular relevance for the present argument is the consistency with which Giovanni is designated as “magistro abbaci” and remunerated for teaching abacus, abacus geometry, and arithmetic within the arts faculty. A selection of salient entries from the *Chartularium* is reproduced below – the year of reference and emphasis are my additions.

- [1329] Die lune XVI. novembris. Item XII. l., X. s. magistro Iohanni Pucci, vocato Magistro Morroccho, **Magistro abbaci**, pro suo salario **occasione sue artis et doctrine**, pro tempore sex mensium inceptorum in kalendis maii proxime preteritis, ad ratione viginti quinque l. in anno, ut apparet per eius stantiammentum.<sup>38</sup>
- [1330] Item Magistro Iohanni Pucci, magistro abbachi, pro salario VI. mensium, per instantiammentum dominorum Novem – XLVI l., X s.<sup>39</sup>
- [1336] Die lune VIII. decembris. Item I. l. magistro Morroccho de albaco, pro mensurando murum communis porte Montis Guatiani qui fuit XLVII. cannis et quarto.<sup>40</sup>
- [1339] Mezzedima XXX, di giugno. [...] Anco a maestro Giovanni di Puccio chiamato Morocho, maestro in **giometria d'albacho**, per suo salario di sei mesi cioè da chalende giennaio passato fino al di d'oggi, come apare per stanziamento fatto per mano di ser ... – XII. l., X. s.<sup>41</sup>
- [1340] Maestro Giovanni di Puccio, **maestro in arismethica**, de' avere per suo salario di detti sei mesi – XII. l., X. s.<sup>42</sup>
- [1341] Maestro Giovanni Pucci Mochi, die avere, e' quali dodici l., diecie s. furo per lo suo salario di sei mesi **per insengiare l'abacho** – XII l., X. s.<sup>43</sup>
- [1342] Maestro Giovanni detto Morroccho, **dottore et maestro in arismetica et geometria**, per li nostri sei mesi – XII l., X. s.<sup>44</sup>

Alongside Giovanni, another abacus teacher such as Lapo di Neri operated at the arts faculty between 1330 and 1339, the year of his death, under more favorable economic conditions: he received an annual salary of 42 lire. Lapo belonged to a Florentine family of abacus practitioners, his father being Maestro Chiaro Neri (or Ranieri).<sup>45</sup> According to the *Chartularium*, Lapo was appointed to teach abacus and geometry and, like Giovanni,

<sup>38</sup> *Chartularium Studii Senensis*, vol. 1, 381.

<sup>39</sup> *Ibid.*, 419.

<sup>40</sup> *Ibid.*, 420. Cf. *ibid.*, 483.

<sup>41</sup> *Ibid.*, 468.

<sup>42</sup> *Ibid.*, 490.

<sup>43</sup> *Ibid.*, 494.

<sup>44</sup> *Ibid.*, 503.

<sup>45</sup> Black, *Education and Society*, 227.

Lapo combined university teaching with service to the city of Siena.<sup>46</sup> Of particular importance is a record from 1334 documenting his parallel activity at the university and in urban abacus schools during the year 1333. For six months (October–April) Lapo would have lectured (“lettura e ’nsegnatura”) on mathematics at the arts faculty; and for a further, partially overlapping six-month period (January–June), he would have taught in the city’s abacus schools – the university appointment being more generously remunerated.

- [1334] Maestro Lapo del maestro Ranieri de ll’abaco die avere per **lettura e ’nsegnatura la detta scientia per VI. mesi**, e’ quali si finiscono in kalende aprile MC-CCXXXIII – XV. l. **E die avere per le scuole del detto abaco per VI. mesi**, e’ quali cominciano in kalende gennaio MCCCXXXIII – VI. l.<sup>47</sup>
- [1339] Anco a maestro Lapo del maestro Ranieri da Fiorenza, **maestro in geometria dell’abacho**, per suo salaro di quattro mesi cioè da chalende otobre anno mille trecento trenta otto in fino a chalende febraio anno detto per ciò che allora passò di questa vita, a ragione di quarantadue l. l’anno, come appare per stanziamento fatto per mano di ser ... – XIII l.<sup>48</sup>

At the death of Lapo di Neri, Tommaso di Davizo, another “abacus and arithmetic teacher” from Florence, took on the mathematical teaching at the faculty of arts, serving between 1339 and 1341, for 25 lire per semester.

- [1340] Maestro Tomagio del maestro Davizo da Fiorenza, **maestro d’abacho e arismeticha**, per suo salario detti sei mesi – XXV. l.<sup>49</sup>

A brief remark on the nomenclature of Giovanni’s, Lapo’s, and Tommaso’s teaching is in order. In Giovanni’s case, as in Lapo’s, the subject of instruction is variously designated as abacus, abacus geometry, and arithmetic; similarly, Tommaso is recorded as teaching both abacus and arithmetic. This fluctuation – also attested for other abacus masters ac-

<sup>46</sup> In 1330, for instance; see *Chartularium*, vol. 1, 387.

<sup>47</sup> Ibid., 402.

<sup>48</sup> Ibid., 468.

<sup>49</sup> Ibid., 490. This information is confirmed in the entry from the Archivio di Stato di Siena (Concistoro 1, c. 42r) reported in Ulivi, “I Davizzi-Corbizzi,” 56. The entry reads: “In nomine Domini amen. Anno Domini millesimo CCCXXXVIII, indictione VII, die XX mensis februarii. [...] Magister Thomasus Magistri Davizi de Corbizis de Florentia, magister theoricus et practicus in arte iometrie et arismetrice, nuper venit ad Civitatem Senarum causa docendi in dicta civitate in facultate et scientia supradicta, et audita fama laudabili sue doctrine et scientie in utraque facultate supradicta, et volentes eidem causam dare ut residentiam faciat in dicta civitate ad docendum in facultatibus supradictis, et super dicta materia tenentes colloquium et tractatum.”

tive within the arts faculty, such as Bartalo di Morroccho (years 1343–1345 and again in 1357),<sup>50</sup> Manno di Boccoccio (years 1344–1348),<sup>51</sup> Pietro Parutino da Montepulciano (years 1349, 1351–1352),<sup>52</sup> and Leonardo del Bianco (year 1354)<sup>53</sup> – suggests that, at the University of Siena, the term *abacus* gradually gained a foothold in the designation of mathematical teaching. The most plausible explanation for this semantic shift lies in the sustained presence of abacus practitioners within a university milieu that had previously been dominated by specialists trained primarily in the classical traditions of speculative arithmetic and geometry.<sup>54</sup>

A comparable development can be observed at the faculty of arts of the University of Perugia. In this case, the evidence derives from municipal statutes regulating the organization of the university, which, although less specific regarding individual masters and salaries, nevertheless provide insight into both the nomenclature of mathematical teaching and the qualifications expected of instructors. For instance, in 1389, when the number of professors and masters was to be determined, the statutes prescribed the appointment of a single master to teach “arithmetic or geometry or abacus,” as the following passage indicates:

In **arimetrica seu geometria vel abbacho** eligatur et eligi possit unus ad legendum et docendum in dicto Studio quantumcunque doctoratus insignis non sit pollens, dum tamen sit sufficiens et expertus.<sup>55</sup>

The provision further specifies that, unlike masters in other disciplines, teachers of abacus (as well as of grammar) were not required to hold a doctorate, but solely to be “sufficient and expert” in their field – a point also emphasized by Stefania Zucchini.<sup>56</sup> The municipal

<sup>50</sup> *Chartularium*, vol. 1, 506, 509–510, and 556. Bartalo, Giovanni di Puccio’s son, would also serve the city of Siena as a surveyor in 1345, alongside his colleague Manno di Boccoccio. See *ibid.*, 512.

<sup>51</sup> *Ibid.*, 511–532 *passim*.

<sup>52</sup> *Ibid.*, 533, 536–537. Cf. Denley, *Teachers and Schools*, 133.

<sup>53</sup> *Ibid.*, 540. Cf. Denley, *Teachers and Schools*, 123.

<sup>54</sup> It is perhaps worth mentioning here another significant piece of evidence from Tuscany, more precisely from Florence and the bottega of Santa Trinita. As argued by Ulivi and recalled by Høyrup, abacus master Giovanni di Bartolo (1364–1440) also taught astrology at the Florentine studium. See Ulivi, “Benedetto da Firenze,” 39, n. 41; and Høyrup, *The World of the Abbaco*, 232, n. 299. Further details on Giovanni di Bartolo are in Arrighi, “Il codice L.IV.21,” 156.

<sup>55</sup> Bellini, *L’università a Perugia*, 71 (III, Scons 1389.3). Cf. Bellini, *L’università a Perugia*, 77 (VIII, Scons 1389.8): “Predicta tamen non vendicent sibi locum in magistris gramaticalibus et magistro abbaci quoquo modo qui possint eligi licet non sint doctores et salarium consequi ordinatum.”

<sup>56</sup> Zucchini, *Università e dottori*, 103.

statutes additionally indicate that such a master could receive a salary of up to fifty gold florins.<sup>57</sup> Despite this relatively favorable remuneration – higher, for instance, than that of grammar masters – the position appears to have been difficult to fill: as Zucchini notes, the chair of abacus at Perugia remained vacant during the final decades of the fourteenth century.<sup>58</sup>

A preliminary conclusion of this section is therefore that abacus teachers circulated within university settings to the point of reshaping the terminology used to designate mathematical instruction. Within the arts faculties, they appear to have taught alongside masters of astrology/astronomy – sometimes prominent figures, such as Blasius of Parma in Bologna – likely contributing foundational training in arithmetic and geometry that prepared university students for more advanced, philosophically engaged study of astrology/astronomy. The question of the language of instruction – whether Latin or vernacular – remains open.<sup>59</sup> What is clear, however, is that these teachers, operating alongside colleagues lecturing on Aristotle, Peter Lombard, and logical texts, embody in concrete terms the institutional permeability and cross-domain circulation of knowledge emphasized by Zilsel.

### 2.3 Indirect reports by abacus teachers: between Florence and Venice

Additional evidence further attests to exchanges between learned settings and abacus schools. Abacus teachers themselves occasionally refer to libraries within convents that preserved mathematical works, as well as to learned individuals who, although not formally recognized as abacus masters, engaged with problems characteristic of practical mathematics.

A particularly instructive case is provided by a Florentine abacus teacher, the above-mentioned disciple of Domenico called Vaiaio. Although the *Practicha d'arismetricha* dates from the middle of the fifteenth century – thus slightly beyond the chronological scope of the present study – it draws extensively on earlier abacus traditions. In particular, it makes use of works by figures discussed above (§2.1) as non-practitioners, such as Grazia dei Castellani and Leonard of Pistoia.<sup>60</sup> At the conclusion of a section devoted to arithmetic, the author recommends consulting Boethius and Euclid, alongside Leonard of Pistoia, “whose works are available at the library of the Santa Maria Novella con-

<sup>57</sup> Bellini, *L'università a Perugia*, 93 (XVI, Scons 1389.16): “Item uni magistro eligendo per conservatores monete Studii sapientes ad legendum abbacum pro suo salario anno quolibet, solvi possint et debeant usque in summam quinquaginta florenorum de auro.”

<sup>58</sup> Zucchini, *Università e dottori*, 104.

<sup>59</sup> Cf. Papi's take in Papi, “Prima di Scipione Dal Ferro,” 236.

<sup>60</sup> See Arrighi, “La matematica a Firenze”; and Arrighi, “Nuovi contributi.” Both articles are reprinted in Arrighi, *La matematica*, respectively at 209–222 and 161–194.

vent.”<sup>61</sup> This reference is significant for two reasons. First, it indicates that abacus teachers engaged directly with writings produced within mendicant and scholastic contexts when these were deemed mathematically relevant. Second, and more importantly, it suggests that the contents of conventual libraries – at least in the case of Santa Maria Novella – were accessible to abacus practitioners. The fact that the author of the *Practicha d'arismetricha* confidently directs his readers to such a repository implies not only familiarity with its holdings but also the practical possibility of consulting them.

A further instructive testimony, also preserved in the *Practicha d'arismetricha*, concerns a figure from fourteenth-century Venice: Marco Trevisano (d. 1378). Although not a friar but a lay nobleman, Trevisano appears to have been deeply influenced by Franciscan intellectual culture, as is evident from his *De macrocosmo*. In this treatise, theological and mathematical concerns – such as the eternity of the world and the status of unity as a number – intersect with questions of natural philosophy, including the atomic constitution of matter.<sup>62</sup>

Trevisano also seems to have maintained contacts with Florentine abacus circles. According to the report transmitted by Vaiaio's disciple, in 1372 a certain “Marcho Trevigiano” addressed a mathematical problem to a group of Florentine teachers, requesting its solution.<sup>63</sup> The problem, which belongs to the domain of recreational mathematics, concerns the measurement of the Earth's circumference and presupposes familiarity with procedures for summing the sequence of natural numbers and their corresponding cubes.<sup>64</sup>

What makes this testimony particularly significant is the profile of Trevisano himself. As a learned layman operating within a scholastic and Franciscan intellectual horizon, he nonetheless turned to practitioners of abacus mathematics to resolve a technical problem. This episode, thus, points to the use of mathematical knowledge beyond

<sup>61</sup> Arrighi, “Nuovi contributi,” 165, where the passage from the *Practicha d'arismetricha* (preserved in manuscript Florence, Biblioteca Nazionale Centrale, Pal. 573, f. 29v) is transcribed: “Chome ò detto, molte altre nature et misterj che ànno e' numerj potrei trattare; ma, se di ciò vuojo bene essere 'sperto, leghasi Boetio et Uclide e 'l frate Lionardo pistoiese frate predicatore; e lle sue opere sono in Sancta Maria Novella.”

<sup>62</sup> Robert, *Le monde mathématique*, chapters 3 and 5.

<sup>63</sup> Arrighi, “Nuovi contributi,” 165; Ulivi, “Masters, Questions and Challenges,” especially 658 (there, Trevisano is acknowledged as an abacus teacher from Treviso); and Robert, “Les mathématiques franciscaines,” 142–143.

<sup>64</sup> The problem is preserved in manuscript Florence, Biblioteca Nazionale Centrale, Pal. 573, f. 138r, and is the following: two individuals depart from the same point on the Earth, proceeding in opposite directions with velocities that increase constantly according to a given proportion; the question posed by Marco Trevisano concerns the time required for them to meet again at the point of departure. An English translation of the problem is provided in Ulivi, “Masters, Questions and Challenges,” 658.

strictly pedagogical settings and highlights a concrete instance of interaction between learned culture and practical mathematical expertise.

### 3. *Abacus mathematics and scholastic knowledge*

Having established that abacus knowledge was not confined to the institutional premises of abacus schools, but that its actors were in contact with – and at times active within – arts faculties and mendicant circles, the groundwork is now laid for the next analytical step. This section draws explicitly on Murdoch’s characterization of medieval learning as porous, in order to examine how abacus mathematics can also be situated within a shared scholastic epistemic framework. To this end, I analyze three cases in which philosophical notions and doctrines are demonstrably present within abacus texts, thereby revealing a permeability between mathematical and philosophical domains. The first is drawn from Jacopo of Florence’s *Tractatus algorismi*, the second from master Dardi’s *Aliabraa argibra*, and the third from a geometric treatise by an anonymous Genoese teacher.

Very little is known about Jacopo of Florence. He is the author of one of the earliest extant abacus textbooks, the *Tractatus algorismi*, composed around 1307. Its editor, Jens Høyrup, places Jacopo between the late thirteenth and early fourteenth centuries and has suggested that he may have traveled to Montpellier to deepen his knowledge of abacus mathematics.<sup>65</sup> A closer examination of the prologue of the *Tractatus* allows us to identify the epistemic framework within which abacus mathematics was situated – one plausibly shaped by the intellectual environments of convents and universities. This line of inquiry is further supported by Høyrup’s observation that Jacopo’s *Tractatus* appears to originate from “an environment where scholars and mathematical practitioners were in contact.”<sup>66</sup>

Signs of this intellectual provenance are evident in the introduction to the *Tractatus*, where Aristotelian distinctions are invoked and treated as shared background knowledge. Particularly emblematic is the distinction between natural and scientific knowledge, described respectively as “senno naturale” and “scienza accidentale”: the former depending on divine grace, the latter on human reason.

Conciossia cosa che tucte quelle cose che la humana generatione de questo secolo sanno et possono sapere, si fanno per duy principale vie, le quale vie sonno queste. La prima si è senno, et la seconda si è la scienza. E ciascheuna di queste due vie si à secho duy gentile et nobile compagne. L’una si è gratia di Dio. Et l’altra si è cognoscenza per ragione. [...] È vera cosa che l’omo po nominare lo senno et la scienza, l’uno senno naturale, et l’altro scienza accidentale.<sup>67</sup>

<sup>65</sup> Høyrup, *Jacopo da Firenze*, 7; cf. Ulivi, “Scuole e maestri d’abaco,” 133.

<sup>66</sup> Høyrup, *Jacopo da Firenze*, 6 and 47.

<sup>67</sup> Jacopo of Florence, *Tractatus algorismi*, 1.2, in Høyrup, *Jacopo da Firenze*, 193–194.

The distinction between knowledge acquired naturally and knowledge acquired contingently is characteristic of scholastic epistemology. Rather than indicating direct dependence on a specific author, it reflects a widely shared conceptual framework. A close parallel can be found, for instance, in the work of the Florentine Dominican Remigius dei Girolami, a contemporary of Jacopo. Having studied in Paris during the years of Thomas Aquinas and later taught theology at Santa Maria Novella, Remigius authored the *Divisio scientie*, a text belonging to the genre of introductions to, and divisions of, philosophy.<sup>68</sup> Composed at the very end of the thirteenth century, likely for the Dominican studium, the *Divisio* reflects the scholastic culture transmitted from the Parisian university milieu.<sup>69</sup>

In this work, Remigius rejects certain common classificatory schemes – such as the division between science of things and science of signs, or between speculative and practical science – and instead proposes a more fundamental distinction between knowledge derived from God and knowledge that is properly human. Philosophy and its branches, including mathematics, belong to the latter category and depend on human reason.<sup>70</sup>

Such parallels point not to direct influence of Remigius on Jacopo, but to a shared pool of scholastic doctrines from which abacus practitioners could draw and within which abacus mathematics could be situated and understood. The broader division of knowledge thus functions as a received epistemic framework that legitimized mathematical practice. Its relevance is further confirmed by its recurrence in later abacus writings: Giovanni dei Danti di Arezzo, in his algorism treatise of 1370, adopts a comparable distinction, as does the anonymous author of the *Libbro delle ragioni d'abacho* (Florence, Biblioteca Nazionale Centrale, Conv. soppr., G.7.1137) at the end of the fourteenth century.<sup>71</sup>

A second case illustrating the presence of scholastic knowledge within abacus contexts is provided by another master, Dardi, and more precisely by his treatise entirely devoted to algebra, composed in vernacular around 1344 and translated into Hebrew by Mordecai Finzi in 1473.<sup>72</sup> In the opening section of his *Aliabraa argibra*, Dardi introduces his treatise

<sup>68</sup> Panella, “Nuova cronologia remigiana” (online version: <https://www.e-theca.net/emiliopanella/remigio2/re1200.htm>); Gentili, “Girolami, Remigio de”; for Remigius within the conventual context of Santa Maria Novella, I refer to Pegoretti, “Lo ‘studium’ e la biblioteca,” especially 110–111.

<sup>69</sup> The *Divisio scientie* is edited in Panella, “Un’introduzione,” 27–126.

<sup>70</sup> Remigius dei Girolami, *Divisio scientie*, 2–3, in Panella, “Un’introduzione,” 83–85.

<sup>71</sup> The text of Giovanni dei Danti di Arezzo is preserved in manuscript Florence, Biblioteca Medicea Laurenziana, Plut. 30.26. Jens Høyrup identified eight additional manuscripts from the fifteenth century that reproduce the same distinction, based on Jacopo’s introduction. These are not considered here, as they fall outside the chronological scope of the present study; see Høyrup, *Jacopo da Firenze*, 46.

<sup>72</sup> Further details to situate Dardi can be found in Van Egmond, “The Algebra”; Franci, “The History of Algebra,” especially 182–185; and in the edition of Dardi’s work, that is, Maestro Dardi, *Aliabraa argibra*, 1–21.

tise by indicating four facets (*respettij*) of it: the title of the book, the author, the subject matter, and the utility.

Perché ongni libro che è fatto de' essere fatto per 4 respetti k primo sie lo titulo del libro, lo sicondo la intentione di coluj che chonpone, lo terso la materia dela quale elli tratta, lo quarto e ultimo l'utilità che seghue dell'opera.<sup>73</sup>

While Jens Høyrup has plausibly connected these four elements to the four Aristotelian causes,<sup>74</sup> they are equally intelligible within the long-established tradition of the *accessus ad auctores* – the prefatory framework in which the title, author, subject matter, and utility of a text were discussed before its contents, a convention whose Latin lineage runs roughly from Boethius onward.<sup>75</sup> Besides this way of introducing his mathematical work, Dardi made his connection with the scholastic milieu clear when explicating the utility of his work, namely, to become a good geometer and arithmetician, whose speculative guises, he says, are ultimately part of natural philosophy.

L'utilità che seghue del libro, la quale utilità sie che quellj che intenderà bene le sue regole sinprici e conposte sarà buon arismetico et etiandio geometro le qual cose specchulativamente sono parte di filosofia naturale.<sup>76</sup>

While the inclusion of mathematical disciplines within natural philosophy does not find any direct correspondence in any scholastic divisions of philosophy, it is nevertheless meaningful that Dardi felt the need to specify that our understanding of nature, or rather, natural philosophy itself, encompasses mathematics.

The final case that spotlights the import of scholastic doctrines within abacus texts is the anonymous author of the *Tractatus super practicam geometrie*. Composed in Latin by a Genoese teacher and later translated into the vernacular, the text circulated in the fifteenth century through figures such as the Milanese Luca de Nigris, who was active in Genoa in 1455.<sup>77</sup> The introductory section of the *Tractatus* is particularly revealing, as it mobilizes a series of scholastic doctrines to frame the discipline of geometry. Four examples may be briefly highlighted.

First, geometry is defined as the science of static magnitudes, a formulation that clearly echoes Boethius' fourfold division of the mathematical sciences, in which geometry

<sup>73</sup> Maestro Dardi, *Aliabraa argibra*, 37.

<sup>74</sup> Høyrup, *The World of the Abbaco*, 231–232.

<sup>75</sup> For an overview of this practice, see Quain, “The Medieval *Accessus ad Auctores*.”

<sup>76</sup> Maestro Dardi, *Aliabraa argibra*, 37.

<sup>77</sup> Giusti, “Un trattato,” here 213–214 and 217 for the translation. Giusti provides the transcription of the text preserved in manuscript Vatican City, BAV, Vat. lat. 4606.

concerns immobile magnitudes, in contrast to astronomy, which studies magnitudes in motion.<sup>78</sup>

Second, the treatise presents the objects of geometry and its specific kinds of magnitudes – lines, surfaces, and solids – as capable of being considered in two ways: either as embedded in matter or as abstracted from it. This distinction presupposes an Aristotelian conception of mathematical properties as subsisting within material things and of mathematics as a science grounded in abstraction, that is, in the cognitive process by which material entities are considered under their quantitative aspects alone.<sup>79</sup> Such a view, widely disseminated in university and mendicant contexts – as exemplified by Remigius dei Girolami's *Divisio scientie*<sup>80</sup> – can be understood as the received epistemic framework within which mathematical practice was situated. The Genoese author integrates this Aristotelian account with the Boethian classification of mathematical sciences in order to distinguish between two branches of geometry: a theoretical one, concerned with magnitudes insofar as they are abstracted from matter and motion, and a practical one, concerned with magnitudes as embodied and thus subject to change.<sup>81</sup>

A third doctrinal element concerns units of measurement. The smallest unit is identified with the atom, described as an indivisible corpuscle comparable to fine dust visible in a beam of light. While this description draws on ancient atomistic imagery, the insistence on the impossibility of actual division aligns with the prevailing Aristotelian rejection of atomism and resonates with scholastic discussions of *minima naturalia*.<sup>82</sup> For the atom, specifies the Genoese, cannot be divided “in act,” and hence it can serve as the smallest unit of measure.<sup>83</sup> The atom, thus, functions not as a physical constituent in a

<sup>78</sup> *Tractatus super practicam geometrie*, in Giusti, “Un trattato,” 220 (hereafter *Tractatus*): “Geometria est sciencia figurarum immobilis magnitudinis secundum se, ad differenciam astrologie sive astronomie, que est sciencia de motibus magnitudinis mobilis.” Cf. Boethius, *De institutione arithmetica*, I, 1, 4, 7.

<sup>79</sup> On Aristotelian abstraction and its early reception in the Latin-speaking world through Boethius, see Crialesi, “The Status of Mathematics.”

<sup>80</sup> Remigius dei Girolami, *Divisio scientie*, 9, in Panella, “Un’introduzione,” 89–90: “Cum enim ars [mathematica] existens in materia sensibili abstractibilis tamen ab ea per considerationem sit quantitas [...] oportet quod mathematica sit de quantitate.”

<sup>81</sup> *Tractatus super practicam geometrie*, in Giusti, “Un trattato,” 220–221: “Et quia contingit magnitudinem, scilicet lineam, superficiem et corpus dupliciter considerari, scilicet prout sunt in materia et prout intelliguntur abstracte sine materia, propter hoc geometria dividitur in theoreticam et practicam, quia secundum quod abstrahuntur sunt immobilia et de hiis determinat theoricam. Sed prout sunt in materia sunt mobilia, quia tactu variabilis rei permutatur Boetio testante, et de hiis determinat practica quia sensibilia sunt.”

<sup>82</sup> I refer to Zambiasi, “The Doctrine of *minima naturalia*.”

<sup>83</sup> *Tractatus super practicam geometrie*, in Giusti, “Un trattato,” 222: “Hinc inde quatuor modi sive principales gradus sunt mensurandi terras et agros, videlicet attomus, punctus, uncia et pes. Quorum attomos indivisibilis est et dicitur ab a quod est sine et tomos quod est divisio, et sine

strict sense, but as a conceptual limit for measurement.

Finally, the treatise invokes methods of geometrical reasoning derived from Galen's *Tegni*: analysis (*resolutio*), synthesis (*compositio*), and dissolution of the definition (*diffinitiva*).<sup>84</sup> This reference suggests familiarity with the epistemological frameworks associated with the arts faculty, where such methods were discussed in commentaries on Galen.<sup>85</sup>

The density of these scholastic references in the opening of the *Tractatus super practica geometrie* led its editor, Enrico Giusti, to question whether its author should be understood primarily as an abacus teacher well versed in scholastic learning or as a scholar engaged in geometrical inquiry typical of abacus circles. In either case, the text offers clear evidence of an attempt to frame abacus geometry within a structured epistemic context shaped by scholastic thought. Significantly, this framework was not retained in later transmission: the vernacular translator chose to omit the introductory section altogether, thereby setting aside its explicit philosophical grounding.

#### 4. Conclusions

Returning to the initial research question, it is now possible to assess both the material reality and the intellectual significance of the exchanges under consideration, and to situate them within broader historiographical narratives concerning premodern scientific knowledge.

The evidence presented here shows that abacus teachers operated within learned environments traditionally associated with philosophical and theological training, thereby creating concrete sites of interaction rather than merely hypothetical points of contact. Particularly instructive are the cases of Jacopo of Betania at the arts faculty of the University of Bologna and of Lapo di Neri at the University of Siena. Both were authorized – and remunerated – to teach abacus mathematics at the university level while remaining active in urban pre-university abacus instruction. These examples indicate that abacus practitioners were not only integrated into academic settings but were also capable of preparing students for more theoretically oriented forms of mathematical learning within the arts curriculum.

divisione in hoc quod ponitur pro mensura, qui atomos est ad instar minimi pulveris seu sintirule qui consideratur in spera solis fulgente per fenestram. Et talle corpus nunquam potest actu dividi, quia igitur atomus appellatur.”

<sup>84</sup> Ibid., 222: “Et quia mensura recta et iusta est sub ordine et ratione et similiter omnis doctrina iuxta et recta, ut testat Galienus in principio *Tegni* cum dicit: Tres sunt omnes doctrine que ordine habentur, scilicet compositiva, resolutiva et diffinitiva; unde similiter modus triplex est, scilicet compositio, resolutio et diffinitio. Unde modus processus in hoc opere continetur sub aliquo istorum trium, sicut patebit inferius.”

<sup>85</sup> Ragland, “Knowing Diseases,” 172–173 and 183; see also Agrimi and Crisciani, “Medicina e logica.”

Alongside these figures stand individuals who were not primarily identified as mathematical practitioners but nevertheless contributed to the transmission of abacus knowledge. References to convent libraries and to mathematical works authored by friars point to such forms of interaction. Leonard of Pistoia, a Dominican at Santa Maria Novella, authored a *Mathematica* accessible to abacus practitioners through the convent's library, while Grazia dei Castellani, an Augustinian friar at Santo Spirito, produced a series of texts in abacus mathematics taken up by Vaiaio's disciple in the following century. Grazia, in particular, exemplifies the convergence of mathematical, philosophical, and theological interests, emerging as a figure operating at the intersection of multiple institutional and intellectual contexts: the university studium, the mendicant convent, and the abacus school. Such configurations give concrete expression to Zilsel's account of the erosion of social and epistemic boundaries in the late Middle Ages, enabling knowledge to circulate beyond its traditional domains. The emergence of abacus training and nomenclature within university settings is itself indicative of this process.

A closer examination of abacus texts further confirms that this tradition was not intellectually isolated. Its practitioners were aware of the scholastic culture of their time, and their works reflect this context. It is therefore neither incidental nor negligible that texts such as Jacopo of Florence's *Tractatus algorismi*, Dardi's *Aliabraa argibra*, and the *Tractatus super practica geometrie* incorporate Aristotelian and other philosophical elements. Jacopo articulates a division of knowledge that closely parallels widely accepted scholastic frameworks, such as that presented in Remigius dei Girolami's *Divisio scientie*. This interconnection – consistent with Murdoch's account of the porosity of late medieval learning – becomes even more pronounced in the Genoese geometrical treatise, where Aristotelian, Boethian, and Galenic doctrines inform the treatment of geometry, its objects, units of measurement, and methods of reasoning. Dardi, for his part, frames the contents of his algebraic treatise within the long-established philosophical practice of the *accessus ad auctores*.

Taken together, these elements suggest that philosophical resources were not external to abacus mathematics but could be mobilized to interpret and legitimize its procedures. The interaction between philosophical and mathematical knowledge in fourteenth-century Italy thus appears as a feature grounded in both the institutional configurations and the shared intellectual frameworks of the period. Abacus mathematics functioned as a form of *knowledge in use* within the university setting, acting as a hinge between practical training and more advanced, theoretically oriented disciplines such as astronomy/astrology. At the same time, scholastic knowledge was not merely external to this practice but actively mobilized by certain abacus teachers, who, at least in the fourteenth century, framed their subject matter through established philosophical and epistemological categories.

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