The *Olympic Competition* of Thomas Digges: The measurement of parallax and the development of observational methods in astronomy

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Abstract

A few months after the appearance of the supernova SN1572, Thomas Digges wrote a Latin book entitled *Alae seu Scalae Mathematicae…* in which he presented new methods for measuring small parallax angles like those expected for planets and other objects at similar distances. On the basis of our new translation into English of some sections of this book, we point out several relevant innovations developed by Digges. These concern the accuracy of astronomical measurements and a discussion on the origin of observational and computational errors. Furthermore, Digges introduced a simple but effective method for the falsification of observational results, which was useful for disproving some incorrect claims reported by other observers. It is also clear that Digges is completely disinterested in the astrological implications of this unexpected celestial phenomenon.

Keywords

history of astronomy, Copernican system, individual astronomer, Thomas Digges

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> "Who, therefore, wants to participate in the Olympic Astronomical competition, must convince himself […] that the Senses can never precisely aim to satisfy the demands of Reason".
>
> Thomas Digges

1. Introduction

The sudden and unexpected appearance of a very bright New Star in November 1572 prompted the European astronomical community to resume study of the nature of celestial objects. In 1543 in *De Revolutionibus Orbium Coelestium*, Nicholas Copernicus had published the heliocentric structure of the known universe which in 1540 in *Narratio prima de libris revolutionum Copernici* Georg Joachim Rheticus (1514-1574) had described in summary form. M. Boas Hall describes the impact that the new system of the cosmic world and the New Star of 1572 had on the culture of the time:[1]

> By the last quarter of the sixteenth century, the Copernican system, though it had gained few adherents, was widely known; after thirty years of discussion and debate, non-scientists were familiar with the fundamental problem […] Indeed, events in the heavens – a new star (nova) in Cassiopeia in 1572, […] – naturally called everyone's attention to astronomy and to the heated discussion ranging among astronomers.

Johnson describes the reception of the Copernican system in England, which was mainly outside of Universities.[2] In 1556 John Field (Felde or Field; c. 1520-1587) published the *Ephemeris anni 1557 currentis iuxta Copernici* in which he revised for the London meridian the Prutenic tables calculated on the assumption of heliocentrism. The Preface of Field's book was written by John Dee (1527-1609), a leading advocate of Copernican theory, who declared that he asked Field to prepare these new ephemerides because the old ones were not sufficiently precise. In the same year the mathematician Robert Recorde (c. 1510-1558) published the *Castle of Knowledge* in which a teacher describes the Copernican heliocentric system to a young pupil.

Thomas Digges (c. 1546-1595) attempted to measure the diurnal parallax of the New Star, but it was undetectable and Digges reached the conclusion that this phenomenon was at least at the same distance as stars and was not a comet or similar nearby body. (Diurnal parallax is the apparent angular displacement as seen by an observer resulting from the Earth's rotation. It uses the Earth's radius as a baseline, and is not to be confused with annual parallax whose baseline is the Earth's orbital radius.) Digges reported his measure-

1 Boas Hall, *The Scientific Renaissance 1450-1630*, 101.

2 Johnson, *Astronomical Thought in Renaissance England*, 13, 137-138, 196-197.

ments of the angular distances between the New Star and other known stars in the book *Alae seu Scalae Mathematicae…* (hereafter simply *Alae*) printed in London by Thomas Marsh in February 1573 (Fig. 1), while the *Parallaticae Commentationis…* by Dee (1573) followed about a month later and was printed by John Day, also in London. These works appeared when the New Star was still shining brightly in the night sky. Digges wrote that the problem of measuring such small parallaxes was discussed with Dee and both developed observational and computational methods based on new theorems of spherical trigonometry and modes of data collection, which they published independently.

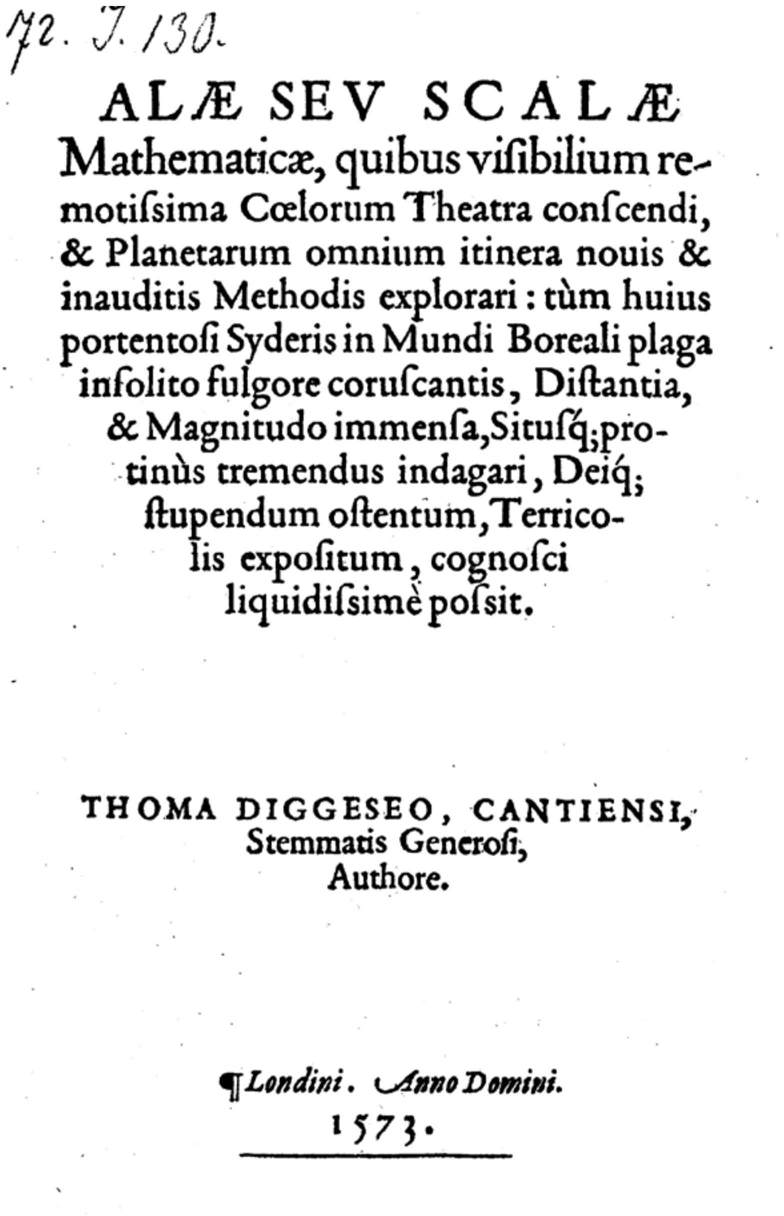
In *Alae*, Digges proposed new and robust mathematical methods to derive diurnal parallaxes of planets, from which he hoped to decide between the geocentric and the heliocentric models of the Universe. Digges proposed new methods that differed from that of Regiomontanus (1436-1476) which at the time was the most accurate way to determine the diurnal parallax of nearby comets. The greatest deficiency in the old approach was inaccurate measurements of time that elapsed between measurements owing either to the practical difficulty of performing nearly simultaneous observations or to the inaccuracy of mechanical clocks which introduce errors so large as to provide completely erroneous results. Digges considered his own effort as the most important for the astronomy at that time. He likened this challenge to an 'Olympic competition', and he used this term at the end of *Proemium* and several times in the Conclusion.

Digges asserted his Copernican conviction, but he published only numerical examples of his methods. Nevertheless, he added sections on the accuracy of measurement and the propagation of uncertainty without which the determination of small parallax angles provided incorrect values. The considerations by Digges reveal a modern approach. For instance, he discussed repeating observations to improve accuracy, but did not express using any averaging procedure. We stress that at that time statistical approaches were completely lacking and that the methods of algebraic calculations were still in their infancy.

Curiously, only some relatively short pieces of Digges's book have been translated into English and none deal with error analysis. We therefore decided to make available the translation of some passages selected for their historical and philosophical content. In the following we first describe the structure of the book, which resembles a compilation of several papers some of which very likely were written before the appearance of the supernova. The main section is written in a traditional style and its organization is similar to that of Dee's booklet.

2. The structure of Alae

Digges's book *Alae* appears as a collection of different works with different typefaces. We distinguish the following sections, with the number of pages and page identifications given in parentheses:



ALÆ SEV SCALÆ
Mathematicæ, quibus viſibilium remotiſsima Cœlorum Theatra conſcendi, & Planetarum omnium itinera nouis & inauditis Methodis explorari: tùm huius portentoſi Syderis in Mundi Boreali plaga inſolito fulgore coruſcantis, Diſtantia, & Magnitudo immenſa, Situſq; protinùs tremendus indagari, Deiq; ſtupendum oſtentum, Terricolis expoſitum, cognoſci liquidiſsimè poſsit.

THOMA DIGGESEO, CANTIENSI, Stemmatis Generoſi, Authore.

¶ *Londini. Anno Domini.*
1573.

Fig. 1 – The title page of the Digges's book *Alae* printed by Thomas Marsh in London in 1573.

Introductory section:

Title (1p.; no sig. A.i^{r}).
Astronomical map of the Cassiopeia constellation with SN 1572 (1p.; no sig. A.i^{v}).
Coordinates of Cassiopeia stars and angular distances to SN 1572 (1p.; sig. A.ii^{r}).
Emblem of William Cecil, Lord Burghley (1p.; sig. A.ii^{v}).
Dedication letter to William Cecil (4pp.; sig. A.iii^{r}-A.iiii^{v}).

Main section:

Author preface (9pp.; sig. A1^{r}-B1^{v}).
Introduction / Proemium (4pp.; sig. B1^{v}-B3^{r}). Definitions (3pp.; sig. B3^{v}-B4^{v}).
Basic theorems / Protheoremata (4pp.; sig. C1^{r}-C2^{v}).
Problems, Canons, revision of the Regiomontanus method (42pp.; sig. C2^{v}-H3^{v}).
Short Practical Preface (2pp.; sig. H4^{r}-H4^{v}).
Supplement on the Astronomical Radius, in 10 chapters (15pp.; sig. I1^{r}-K4^{v}).

Closing section:

Conclusion (6pp.; sig. K4^{v}-L3^{v}).
Erratum (1p.; sig. L3^{v}).
Emblem of Digges family (1p.; sig. L4^{r}).

The Appendix gives translations of sections marked in different font in which some general astronomical and technical topics are summarized and discussed. These help to understand the methodological and philosophical concepts that Digges adopts.

3. The precision of astronomical measurements and the nature of observational uncertainties.

The first modern study of *Alae* is likely that of Johnson which includes translations into English of a few short passages.[3] This author focuses mainly on Digges's support of the Copernican system and on his concept of an infinite universe. He states clearly the problem of accurate measurements of small parallaxes like those expected for the New Star, but does not discuss the new methods developed by Digges and the practical problems involved. A new and more detailed analysis of Digges's work is that of Goulding[4] who presents an accurate analysis of the new methods that Digges developed to avoid some practical flaws in the approach of Regiomontanus. Goulding points out that Digges's algorithms were generally not

3 *Ibid.*, 158-159.

4 Goulding, "Wings (or Stairs) to the Heavens, The Parallactic Treatises of John Dee and Thomas Digges".

accepted by astronomers because of their complexity and in particular not by Tycho Brahe (1546-1601) (herein simply 'Tycho') who studied the parallactic problem extensively.

The historical treatment of observational data and of their accuracy in astronomy can be traced back to the Hellenistic epoch,[5] and in the Renaissance it became a relevant subject. According to the picture given by Chapman,[6] up to the middle of the sixteenth century the typical accuracy of astronomical measurements was between 6 and 10 arcminutes, and in the second half it improved to about 1 arcminute and continued to increase thereafter. A widely accepted opinion is that Tycho, who had exceptional financial support in building the greatest observatory of the time, was the first astronomer to develop new precision tools for increasing the accuracy of the measurements of astronomical coordinates and for understanding the occurrence of instrumental errors.[7] We will show that Tycho was not the only one to follow this approach and that in *Alae*, Digges independently had a clear idea of errors incurred in making astronomical observations.

Digges had two main objectives: The determination of the distances of planets and the verification of the heliocentric system, as he wrote in the dedication letter to William Cecil and in the last part of the Conclusion, and the detection of the parallax of the New Star of 1572 now known to have been a Type 1a supernova (SN1572).

In seeking the parallax of SN1572, Digges measured the angular distances between this star and five other stars in the Cassiopeia constellation. Digges reported ecliptic Longitudes and Latitudes for some of these stars with the poor precision of 10 arcminutes (or 5 arcminutes in only a couple of values); these values are clearly included not for their astrometric value but just as an aid to identify stars in the associated map with those reported in other work. One of these coordinates is wrong by 1 degree and it is likely a typographical error that was probably corrected by Tycho.[8] A handwritten corrigendum can be seen in the page A.ij of the copy considered by us, here reported in Figure 2. Note that the angular separations of SN1572 from the five stars are reported with a much higher precision. Fractions of a degree are given in 'scruples' which correspond to the smallest amounts of a considered quantity, and it is easy to verify that in this context they are the same as arcminutes. For five of the six stars, distances are reported in degrees and arcminutes, and for the sixth star, the accuracy is 1/2 arcminute = 30 arcseconds. In the Conclusion (see the attached translation in [*App* vii-4]) it is clearly stated that an accuracy of 1/2 arcminute is Digges's goal. This is the highest resolution normally expected for human vision[9] and could have been reached either by

5 Sheynin, "The treatment of observation in early astronomy".
6 Chapman, "The accuracy of angular measuring instruments used in astronomy…".
7 Christianson, "Tycho Brahe's Earliest Instruments", 131-144.
8 Green, *Assessment of early-modern observations of comets and supernovae…*, 122.
9 Usher and Massaro, "The Sixteenth-Century Empirical Disproof of Ptolemaic Geocentrism: Paper II", 68-69.

increasing the size and quality of the instrumental scales or by reducing the uncertainty due to observational errors.

The analysis of Digges of the origin of observational errors is surprisingly modern and indicates that he studied this subject deeply. In the Conclusion, Digges considers three main sources (see the attached translation in [*App* vii-2]): (*i*) When a datum is obtained by means of several measures, the accumulation of errors during data reduction increases the uncertainty of the final value which could be much different than the true value; (*ii*) if some observations must be made in a short time, a small error in time can affect subsequent readings and produce a large deviation; (*iii*) errors in reading instrumental scales, even when accurately calibrated, owing to limitations in visual acuity. Digges added a fourth cause of error originating in mathematical calculations [*App* vii-5], particularly of trigonometric functions and of square roots, which generally are irrational numbers, which cannot be evaluated with a very high precision. These unavoidable mathematical approximations may be a further source of uncertainty, particularly for very small parallaxes.

Another topic that Digges discusses briefly is the possibility of repeating a measure several times to obtain reliable observational data. As noted above, Digges did not report any averaging method, but it is possible that he computed a mean value, for instance, when he gave in the first page of the book the angular separation between the New Star and star numbered 11 (κ Cas) as 1° 28.5′ which has an accuracy of 1/2 arcminute (see Fig. 2) that may not have been achieved visually. Apparently, Digges did not know the statistical nature of errors, but it appears that he realized that owing to the propagation of errors, the final uncertainty can be greater than the value itself.

The use of an arithmetic mean for computing the best estimate of a measure was already recognized in the work of Hellenistic scholars, and in particular of Hipparchus (190-120 BCE),[10] despite a few other ancient historical and literary indications.[11]

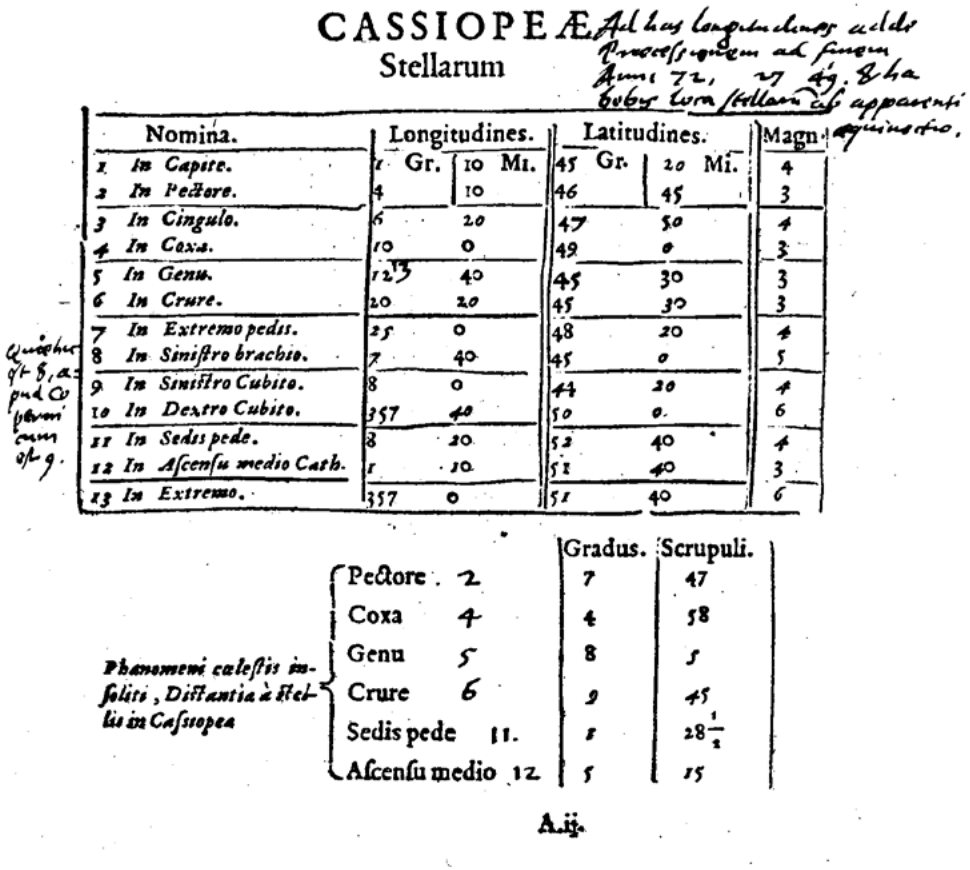
According to Plackett, the first use in an astronomical context is reported in Tycho's *Astronomiae Instauratae* completed in 1588 where Tycho appeared to have applied arithmetic means to a couple of stellar data to eliminate systematic errors, but Buchwald has shown that actually Tycho applied a recursive averaging mean between couples of observational data ordered chronologically.[12]

Digges placed a high value on technical innovation in the fabrication of instruments to increase their sensitivity. He considered the matter so important that he added a Supplement to *Alae* in which he describes the use of the Astronomical Radius, the term he uses for the cross-staff. This instrument is described by Petrus Apianus (1495-1552) in *Instru-*

10 Plackett, "The principle of the arithmetic mean", 130-135.

11 See for istance Bakker and Gravemeijer, "An Historical Phenomenology of Mean and Median", 149-168.

12 Buchwald, "Discrepant measurements and experimental knowledge in the early modern era", 565-649.



CASSIOPEÆ Stellarum

Nomina.	Longitudines. Gr.	Mi.	Latitudines. Gr.	Mi.	Magn.
1 In Capite.	1	10	45	20	4
2 In Pectore.	4	10	46	45	3
3 In Cingulo.	6	20	47	50	4
4 In Coxa.	10	0	49	0	3
5 In Genu.	12 (13)	40	45	30	3
6 In Crure.	20	20	45	30	3
7 In Extremo pedis.	25	0	48	20	4
8 In Sinistro brachio.	7	40	45	0	5
9 In Sinistro Cubito.	8	0	44	20	4
10 In Dextro Cubito.	357	40	50	0	6
11 In Sedis pede.	8	20	52	40	4
12 In Ascensu medio Cath.	1	10	51	40	3
13 In Extremo.	357	0	51	40	6

Phaenomeni caelestis insoliti, Distantia à Stellis in Cassiopea

	Gradus.	Scrupuli.
Pectore 2	7	47
Coxa 4	4	58
Genu 5	8	5
Crure 6	9	45
Sedis pede 11.	1	28 1/2
Ascensu medio 12	5	15

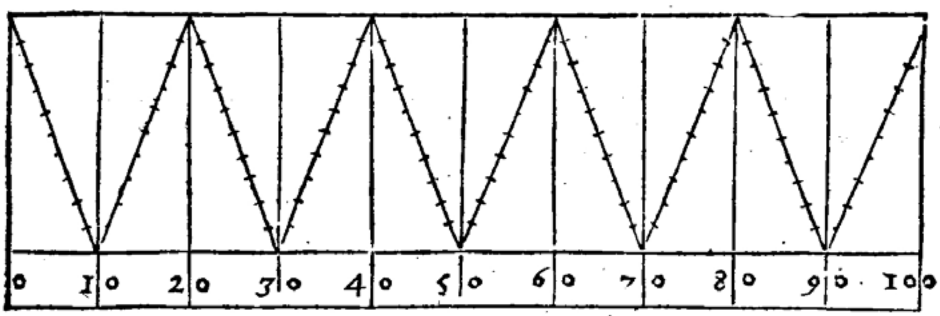
A.ij.

Fig. 2 – The lower part of page A.ij of the copy of *Alae* with some notes written by a contemporary astronomer. Coordinates of Cassiopeia stars and their angular distances from SN1572 as measured by Thomas Digges are reported in the two tables. See the correction in the longitude of a star (13 degrees instead of 12) that can be originally attributed to Tycho, but who was not the author of these notes because the calligraphy looks different from that in Tycho's notebook.

ment Buch (1533), Gemma Frisius (1508-1555) in *De radio astronomico…* (1545), and Leonard Digges the Elder (c. 1520-1571?) in his book *Tectonicon* of 1556 on land surveying methods, and its use was well-known in England.[13] In *Alae* (*Supplementa,* Chapter IX, K3^{r}) Digges writes words of admiration for Richard Chancellor (Richardus Chancelerus; c. 1520-1556), to whom he attributed the adoption of the transverse scale (Fig. 3) for increasing the precision of measurements. The history of this useful device is ancient and pos-

[13] Roche, "The radius astronomicus in England", 2-3.

sibly it was independently developed by various people. It was known and likely invented about a couple of centuries before by Levi ben Gerson (1288-1344) (also known as Gersonides).[14] The books of ben Gerson are generally written in Hebrew and were not largely known in the astronomical community.[15] Christianson reported that transverse scales were also adopted by the young Tycho in 1564,[16] when he was aided by Bartholmæus Scultetus (1540-1614) in constructing a cross-staff. Scultetus had studied with Johannes Hommel or Homelius (1518-1562), professor of mathematics at the University of Leipzig and instrument maker, also known by Tycho who assisted at his lectures.[17] In turn, this idea seem to be suggested to Homelius by the poorly known Richard Cantzlar in 1552 or 1553.[18] Other astronomers who used this tool were Paul Wittich (c 1546-1586), considered however to be a poor observer,[19] and Christoph Rothman (c. 1550/1560-later than 1590) from the town of Kassel, both visitors to Tycho's Uraniborg. Furthermore, in the *Supplement,* Digges analyzed in detail sources of error owing to misalignments of the eye during observation.



Qui verò Radio liberè sine Machina vti affectauerit, pinnulam illam extremitati oculo propinquiori ad motam, integram habeat, quadratæ formæ, et in eius medio (æqualis altitudinis cum Transuersarij acie superiori) foramen exile fiat: cui oculum applicare licebit, et solito more ad libitum baculo vti, absque hallucinatione aut visus parallaxi vlla. Mirum in modum hæc baculi rectificatio Nautis proderit, qui maximè ex oculi eccentricitate in Po-

Fig. 3. The transverse scale described by Thomas Digges in the Chapter V of the *Radii Astronomici Supplementa* (page I4v) of *Alae.* Note the use of decimal divisions.

14 Goldstein, "Levi ben Gerson: On Instrumental Errors and the Transversal Scale", 102.
15 Rudavsky, "Gersonides: Levi ben Gerson".
16 Christianson, "Tycho Brahe's Earliest Instruments", 135-136.
17 Helfricht, *Astronomiegeschichte Dresdens,* 29.
18 q.v. Krisciunas, "Observatories", 5n1. Note added in press: The name Richard Cantzlar could refer to the German "Kanzler", meaning "Chancellor", suggesting the name Richard Chancellor.
19 Mosley, *The Biographical Encyclopedia of Astronomers,* 1234.

4. The scientific approach of Thomas Digges

Digges's approach to SN1572 is free of astrological conviction. He believed that the event is due to the action of God, like everything in the Universe, and that it belongs to the natural world in the remote region of stars, but it would not affect human events at all. By contrast, Tycho devoted about half of his book *De Nova stella* to a discussion of possible connections between SN1572 and astrology.[20] Next, Digges excluded the possibility that this unusual and rare Phenomenon - which is the term that he used frequently to describe the new star possibly to underline its material nature - was a comet. This is clearly affirmed in the Dedicatory Letter to William Cecil [*App* ii-1] where he wrote about the starlike appearance and the absence of any coma or tail, and in the closing sentence of the Conclusion that it does not resemble the star that guided the Three Wise Kings [*App* vii-7]. Furthermore, the changes in brightness are not explained by any miraculous action but by means of a variation of its distance, without consequences for humankind [*App* iii-1].

A possible consequence of this hypothesis is Digges's change of the size of the Universe. *Alae* was written about three months after the appearance of SN1572 when it continued to be very bright, and as a firm believer in heliocentrism, Digges expected to see some modulation in its brightness owing to variations of distance due to the motion of the Earth around the Sun. But SN1572 started to decline in brightness steadily and disappeared from view in March 1574. If this fading is explained by a receding motion of the new star [*App* iii-2], then this very far object could not be fixed on a solid sphere but it could move in space. A possible consequence is that this motion could happen only if a solid sphere of fixed stars does not exist. Consequently, stars could be dispersed within an infinite space and their number could be extremely large, or potentially 'infinite' which according to the Oxford English Dictionary means 'having no limit or end (real or assignable); boundless, unlimited, endless; immeasurably great in extent, duration, or other respect', and with an apparent brightness of distant ones too faint to be detectable visually. Digges presented this model of the Universe three years later.[21] William Gilbert (1544?-1603) later accepted this revolutionary view.[22]

Thus, in *Alae* in 1573, Nature and the Universe are considered topics for research and exploration to be carried out with the aid of more refined instruments [*App* vi-3], together with the support of Mathematics to ensure results by means of correct demonstrations. It is interesting that in the same context Digges suggested, according to the Copernican system, the possibility of searching for parallax due to the orbital motion of the Earth. Furthermore, Digges adopts an innovative approach for investigating celestial objects and

20 Håkansson, "Tycho the Apocalyptic: History, Prophecy and the Meaning of Natural Phenomena", 211-236.

21 Digges, *A Perfit Description of the Caelestiall Orbes*, f. 43.

22 Gilbert, *De Mundo Nostro Sublunari Philosophia Nova*, 202.

refuses to consider any interpretation in ancient and Middle-Ages books that is founded on philosophy or religion. He is free from any mystical approach to knowledge, unlike his mentor John Dee.

According to Digges, a scientist must observe and perform measurements, and the procedures used must be correctly planned and executed [*App* v-1]. An observational result can be wrong and can lead to misinterpretations if the errors are not properly considered. Theory should be based on mathematics and geometry and the results verified by observations. The fundamental role of mathematics is frequently mentioned by Digges as in the letter to Cecil, where he considered it "the most worthy of all sciences for any intellect" [*App* ii-2]. Digges is arguably one of the first "modern" scientists.

The research methods applied by Digges distinguish clearly between the verification of a theory (or a mathematical model) and the falsification of a wrong result. A clear example can be found in Chapter X of the Supplement on the Astronomical Radius, where Digges writes that some astronomers (without mentioning names) reported for SN1572 an erroneous value for the diurnal parallax of about 1 degree and so they concluded that it should be located in the sublunar world. Digges then proposed a simple test for disproving this large value based on finding an alignment of SN1572 with a couple of stars and on the searching for a displacement of the new star from this line in the course of the night [*App* vi-1]. This effort was aided by the fact that SN1572 is circumpolar at typical European latitudes. A deviation of 1 degree would be clearly detectable even to an inexpert observer and thus the claim for the sublunar origin of the Phenomenon could be safely disregarded since its parallax was too small to measure reliably.

5. Final remarks

Thomas Digges was not a philosopher in the sense that he never wrote a text dealing with a general treatment of the knowledge of Nature and its relation to humanity. However, he had an innovative cultural and methodological approach to the investigation of natural phenomena which was a nascent form of what today we loosely call the scientific method. One has to consider that Digges, as he wrote in the Preface of *Alae*, was educated to the Copernican system of the world first by his father Leonard and later by John Dee,[23] and his belief in this matter was so strong as to cause him to persist in the search for some unquestionable observational proof. For this, *Alae* serves as a preparatory document to his *A Perfit Description…* which appeared three years later. Papers in 2023 and 2024 have discussed the connection between these works.[24]

23 See also Johnson, *Astronomical Thought in Renaissance England*, 157.

24 Usher and Massaro, "The sixteenth-century empirical disproof of Ptolemaic geocentrism"; Id. "The sixteenth-century empirical disproof of Ptolemaic geocentrism: Paper II".

Alae had a rather large diffusion in Europe and copies can be found in several Libraries, including those of relevant Universities of several countries (France, Belgium, Austria, Italy, Ireland, Spain, Czechia, Swiss, Poland and, of course, United Kingdom), in particular a copy is now still preserved in the University Library of Padua that is bound together with the *Parallaticae Commentationes* by Dee and that was transferred from the ancient Library of the Santa Giustina Abbey. Tycho held Digges's data set in high regard and wrote at length about it in *Progymnasmata*.[25]

Johnson pointed out Digges's relevance to the development of modern astronomy in England and its relationship to Giordano Bruno's cosmology which emerged eight years after Digges's paper of 1576 and which the Nolan developed in accordance with his philosophical beliefs.[26] This aspect warrants recondite consideration,[27] but is beyond the purview of this paper. It is possible that Digges's methodology was practiced in other institutions and contributed to the need for precise observations. Digges's exploration of Nature and the sky does not mean that he must be considered a pure empiricist who accumulates data and uses induction to extract their meaning and increase knowledge as does the later philosopher Francis Bacon (1561-1626).

Rogers[28] and the pioneering researcher Taylor[29] defined Digges and his father as Mathematical Practitioners, but this definition, at least for Thomas, may be too reductive. Such a label might apply to activity of the more mature Thomas Digges when he was busy mainly in military and engineering affairs, but not to his astronomical and mathematical work, as Johnston has pointed out.[30]

Digges was mainly intent on testing empirically whether the Copernican system was true and to modify it if required by observations. In *A Perfit Description…*, Digges tried to establish a better physical basis for the rotation of Earth by considering the relative motion observed from moving ships, an argument used also by Copernicus. Digges developed and introduced the invariance of the free fall of a plummet along the mast of the ship and like other scholars, he was concerned with ways to increase the accuracy of measurement. Digges may have acquired this interest from his work on new methods and instruments of

25 Brahe, *Astronomiae Instaurate Progymnasmata…*

26 Johnson, "The influence of Thomas Digges on the progress of modern astronomy in Sixteenth-Century England", 392.

27 See: Granada, "Bruno, Digges, Palingenio: omogeneitá ed eterogeneitá nella concezione dell'universo infinito"; and Id., "Thomas Digges, Giordano Bruno y el desarollo del copernicanismo en Inglaterra".

28 Rogers, "Leonard and Thomas Digges: 16th Century Mathematical Practitioners", unnumbered p. 2.

29 Taylor, *Mathematical Practitioners of Tudor and Stuart England.*

30 Johnston, "Making Mathematical Practice: Gentlemen, practitioners and artisans in Elizabethan England", 103.

surveying, like those described by his father in their jointly authored *Pantometria* of 1571 for which the renovation of Dover harbour would have a need.[31]

Several authors have commented on the role of theological inspiration in Digges interpretation of natural phenomena (e.g. Koyré),[32] but this matter appears to be controversial and in need of further analysis. In the Elizabethan period, episodes of religious intolerance were frequent, and individuals might have a need to mask their thoughts with repeated declarations of faith particularly if they were involved in public affairs. Digges was on good terms with Sir Walter Raleigh (1552-1618), founder of the 'School of Night', known also as the School of Atheism, but beyond that, Thomas Digges adhered to a policy expressed in his address "To the Reader" of *Stratioticos*,[33] where he declared "by the example of my Father, Pythagorically I will content my selfe *Per manus tradere*, and to communicate them only with a few selected friends". He refers to Leonard Digges's emulation of Pythagoras (c. 570-c. 490 BCE) who conversed only with a select few.

Also, in *Stratioticos*, Digges declared that he began to write a treatise on Copernican astronomy entitled *Commentaries upon the Revolutions of Copernicus, by evidente Demonstrations grounded upon late Observations, to ratifye and confirm hys Theorikes and Hypothesis…*,[34] but no evidence for it has been found. Other books dealing with navigation and military subjects met the same fate, indicating that his main interests were already oriented towards engineering and practical works. Digges appears therefore to have characteristics more like those of a modern scientist than an observer or engineer from the Middle-Ages or the Renaissance, and the combined roles of Mathematics and precise observational methods distinguish him as a precursor to Galileo Galilei (1564-1642).[35]

Appendix: English translations of selected pages of Alae

Translations of selected passages of *Alae* presented below are useful for emphasizing the need to establish precise observational methods. This was a priority for Digges, but his literary style is not simple. Translations of the Latin are not literal and in some places we have taken liberties with the original text to facilitate understanding. Some long sentences are broken into parts, but use of italics and parentheses are as in the original text. Sometimes Digges's rhetorical comparisons and eulogies are excessive by modern standards with long and articulated passages and some repetitions, and he uses many superlatives which appear unnecessary in modern writing. We therefore introduced a few limited shortenings of the text, indicated by […], which do not affect meaning and that are not relevant in ex-

31 Margetts *et al.*, "What "incomparable Jewells Havens, and sure harbours are".

32 Koyré, *From the Closed World to the Infinite Universe*, 37.

33 Digges and Digges, *Stratioticos*, Bv^r.

34 *Ibid.*, a.iv^r.

35 On this topic see Wallace (*Prelude to Galileo*).

plaining any topic of precision astronomical observations. Johnson and Goulding reported some translated passages from *Alae* in their works,[36] but the majority of these are from the Praefatio Authoris (Author's Preface) which is not considered here since generally we selected untranslated sections. Translated sections are denoted by a format with italic "*App*" that denotes this Appendix, such as *App* (ii). Segments in the sections are presented in paragraphs that are not present in the original text but are indicated in square brackets with a self-explanatory format, e.g., [*App* ii-1], to allow readers to find relevant sections more easily. Translated parts of *Alae* are:

App (i): Title (sig. Ai^{r});
App (ii): Dedication Letter to William Cecil (1520-1598) (sig. Aiii^{r}-Aiiii^{v}).
App (iii): Introduction (Proemium) to the mathematical treatments (sig. B1^{v}-B3^{r}).
App (iv): Definitions concerning Parallax (sig. B3^{v}).
App (v): Supplement on the Astronomical Radius: Short Practical Preface (sig. H4^{r}-H4^{v}).
App (vi): Supplement on the Astronomical Radius: Chapter X (sig. K3^{r}-K4^{v}).
App (vii): Conclusion of the book (sig. K4^{v}-L3^{v}).

The Latin copy is preserved at the Osterreichische Nationalbibliothek at the location 77.J.130 ALT PRUNK (http://data.onb.ac.at/rec/AC09813307). This copy has several interesting handwritten annotations including a handwritten correction of a typographical error of 1 degree in the Longitude of a Cassiopeia star that was noted by Tycho (see Fig. 2).

App (i): Title

Mathematical Wings or Ladders, with which it is possible to ascend to the very remote Theaters of the visible Heavens and to explore the paths of all the Planets with new and unheard-of methods, in order to ascertain with extreme simplicity, the immense Distance and Magnitude of this portentous Star shining with unusual brightness in the region of the Boreal World, and at the same time to investigate this amazing manifestation of God revealed to the terrestrial inhabitants.

THOMAS DIGGES, man from Kent

36 Goulding, "Wings (or Stairs) to the Heavens, The Parallactic Treatises of John Dee and Thomas Digges"; Johnson, *Astronomical Thought*.

App (ii): Dedication Letter to William Cecil (1520-1598)

To the much honored man WILLIAM CECIL, *most Illustrious Knight of the Order of Gold:*[37] *Baron of Burghley: Lord High Treasurer of England, and personal adviser to Her Royal Majesty.*

[*App* ii-1]

I have been meaning for some time, […] to present you with proof of my gratitude to you, and at last an opportune occasion has arisen when (during your tenure) I have tried to measure position, motion, distance, and brightness [magnitude] of an extremely rare Phenomenon, a bright new star. At first glance, I had not seen it to have a fuzzy appearance in the form of a "mane", "hair", or "tail", and I was further amazed when I observed it for several nights and had not found any movement with respect to the fixed stars. I sought to discover a difference in the position,[38] or Parallax, and found it undetectable so that it was quite evident that [the New Star] lay beyond the Moon. Before stating such a finding and […] after several more observations, I understood more clearly that it was far above the Moon. Then finally I began to recall in my mind the methods of all past and contemporary astronomers that I knew about for measuring distances and magnitudes of comets and celestial bodies, and I could not find anyone who had shown an adequate way of determining such very small parallaxes. Even the methods[39] of Regiomontanus, whose reputation towers above all others, are insufficient in practice as it will be later more fully demonstrated. Alone (as if thrown into the surging ocean of many doubts), and deprived of the aids of all Ancient and Contemporary astronomers, I decided to research the problem myself. In a short amount of time (as if favored by a mathematical breeze) I sailed into the desired port and discovered some very fast routes hitherto unexplored and safe from all peril of error.

[*App* ii-2]

After gathering these [results] in a little book, I have decided to present it to your Honor as proof of my work (unless such a high regard of myself[40] deceives me). The book is not

[37] Cecil became became chancellor of the Order of the Garter om 1551 and received the title of Knight in 1572: https://www.heraldica.org/topics/orders/garterlist.htm.

[38] Digges uses the Latin word "Aspectus" whose literal meaning is "appearance" or "look". Likely he means "position" because its meaning is clear from the following mention of Parallax (see fn. 47).

[39] Digges uses the word "Demonstrationes" (Demonstrations) likely to underline that a procedure, although based on a rigorous Mathematical reasoning, need not be the best suited one for practical applications.

[40] Digges uses a word of Greek derivation, "Philautia", to denote self-consideration. It was used in a similar context by Erasmus (d. 1536) in the Dedication Letter of Moriae Encomium to Thomas More (1478-1535).

written in the vernacular[41] in order that it will not perish in a short time. Just as in our present age in which your deeds make you absolutely commendable to the hearts of all [...], so this book[42] will endure to Posterity as a tribute to your expertise in Mathematics (the most worthy among all sciences for any intellectual pursuit), and to your benevolence towards scholars of these disciplines. [...] I very humbly offer these first fruits from my previously uncultivated garden, which are the first Astronomical harvests that are rightly due to your Greatness, and which if you will accept them graciously, shall encourage me to greater and greater efforts [...]. Moreover, you will silence ignoble Imposters[43] and Epicureans[44] (who are the most inert and unproductive of all, yet they achieve fame falsely by plundering the works of others). [...]. Let those who deny the truth of this work do so freely: [The book] will not need any patronage, because it is so fortified by the strongest and firmest of geometrical demonstrations that it does not fear any Academician's cunning. These first fruits of my studies and this my first astronomical work were written in a very short time and so contain a series of propositions without any rhetorical embellishments, but if it were to be appreciated by the thorough evaluation of Your Illustrious Dignity, I shall be committed from now on (with the benefit of the favorable Mathematical Muses) to accomplish further and more important works. I shall not stop at this my first effort but I shall progress perhaps to the point where anyone may clearly understand that the mechanism of Celestial Globes and of the Visible World [Sun, Moon, and planets] [...] that was reformed by the divine Copernicus who was provided with more than human talent, might not be fully correct or whether there might be some points remaining to be carefully examined [...]. In the meantime, I beg the Creator of this admirable new star to grant you the longest and most prosperous life on Earth, and the happiest seat in Heaven for your pious soul.

Very much grateful to your Highness
Thomas Digges

41 English.

42 Digges uses the Latin "Monumentum" that is difficult to understand if Digges means his book or Cecil's expertise; the simplest interpretation is that Digges intends the former.

43 Digges uses the word "Sycophantis" of Greek origin having the meaning of informer, liar, or swindler.

44 The word "Epicureis" is likely used by Digges in the broad sense of philosophers who spend their life in leisure and employ specious reasoning for their own advantage. This concept is also present in the widely known *The Colloquies* by Erasmus (1518): "Quia mihi placet otium. Arridet Epicurea vita" (I like idleness. Epicurean life smiles upon me.)

App (iii): Introduction (Proemium) to the mathematical treatments

[*App* iii-1]

An amazing new PHENOMENON is visible in the "seat" of Cassiopeia, which is seen to rotate around the Pole together with the other fixed stars so uniformly that after each revolution it returns to the same position without any detectable difference [in position] as I have verified with the highest accuracy. My observations conform to the perception that either the huge sphere of fixed stars moves in a circle in the interval of 24 hours, or, following Copernicus, only the Earth rotates in the same time while the Sphere of fixed stars remains motionless. Everyone recognizes what is evident from experience, that both motions occur around the axis of the Earth that passes through the center of the Earth, and the same Celestial appearance follows in both cases so that we may assume this model as the basis of our geometrical proofs […].

This PHENOMENON that is [located] in a distant place does not exhibit any evident displacement contrary to the very fast motion either of the Prime Mover or of the Earth in a single Rotation,[45] *and if in aforesaid times any change of position appeared to occur, it would not happen because of any type of motion, nor to Parallax,*[46] *which is a deviation of our line of sight from the centre of the Earth with respect to which it [the Phenomenon] rotates uniformly and without any detectable variation [of its position in the sky].*

[*App* iii-2]

If Regiomontanus (the famous mathematician) did not hesitate to measure the distances of comets whose [parallax] variations are a hundred times greater than those that occur in this [object, i.e. the new star] (if indeed it has any [parallax] at all) […]. I have no doubt, however, that this Phenomenon is now farther from the Earth than it was initially, but its receding motion is so slow that any change [of the apparent position] could not be detected in a single rotation. If one were to observe other phenomena that have a detectable motion in a single rotation [like comets], [their motion] could easily be reduced by means of arithmetical calculations to any fraction of the [daily] rotation. It should be clear that I do not take these [phenomena] into account, because propositions [considerations] on *Parallax* are not necessary to unveil such phenomena that have been discussed extensively by other [astronomers]. Since this subject of astronomical parallax remains […] poorly treated, for a better understanding I will say a few words about Regiomontanus. Even though he excelled above all others in this matter, yet I will show

[45] Here and in the following paragraph Digges uses the Latin word "revolutione" for the Earth's diurnal motion.

[46] Likely, Digges uses a third conditional to stress that any apparent change of position of SN1572 is not physically possible and any report about it originates from errors in the observational procedures.

how the practical uses of his methods are scarcely useful and convenient [for the problem of the new star].

[According to Regiomontanus], a couple of observations of the Phenomenon are required to obtain the *Parallax*, and in both of these observations, the true Altitude above the Horizon and the distance from the Meridian, measured on the Horizon, are necessary; but these are not sufficient because the time between the observations must be known as well, and the method for measuring this time interval is not discussed [by Regiomontanus]. It is very difficult to measure the time in any way with an accuracy (of a minute). Either we use the Altitudes of some fixed [stars], and for this one must know their positions with very high accuracy because if one of their Longitudes or Latitudes is slightly wrong, it [this error] will produce a large deviation in the outcome. Furthermore, if one could know them [those coordinates] it is also necessary to know Altitudes or Azimuths, not at a single time, but twice, that is in both the measurements [for obtaining the parallax]. Moreover, those [astronomers who are] accustomed to observe stars, know how difficult this is […] since in the course of a very fast observation, when one measures any angle, even two, three or four times, even if you consider the same angle, they [measured values] will be hardly in agreement; it [this angle] can only be known exactly by means of simultaneous observations, therefore it is impossible to measure it several times, because time passes rapidly during the observation. If you make an error of only two [time] minutes in the evaluating of the time, such a short a time [interval can] corresponds [up] to 30 [arc]minutes in the graduation along a major circle [Equator or Ecliptic] on the [Celestial] Sphere. […] I do not mention here mechanical clocks, as they are so alien and unfit for use by mathematicians, especially for such fine parallax measurements that they have been rejected for some time with everyone's consent. When, therefore, I observed how inadequate it was to use current clocks to measure small *Parallaxes*, and that no-one had developed other methods, I thought it worthy of a mathematician to remedy the situation. How much I have advanced can be evaluated by others.

[*App* iii-3]

Even though I have not written this short work in vernacular English, and I have done so entirely without any help, yet lest anyone should be offended that I have described my findings on parallaxes devoid of numerical examples, I want publicly to report the following items. I discussed this work with my very learned friend, Mr. John Dee, who showed me a clear demonstration of the method which he had recently discovered for determining the Parallax of the new star Phenomenon that was easy and highly commendable. He told me that he had previously formulated new methods […] for measuring the very small Parallax of this very rare Phenomenon; and to make them really useful, he prepared many new and original instruments. Since the first appearance [of the Phenomenon SN1572], he observed the Position, the Motion, and several Altitudes, with extraordinary energy, diligence and admirable skills, and using a very accurate device, which will allow him to verify all the various *Parallaxes* reported up to now with the highest accuracy, as I will readily testify as eye-witness. However, a complete *account* of the subject cannot be provided in a short amount of time, and because the present booklet was ready to be printed, I decided to publish it right away so that the praise due to [Dee's] innovations would not be overshadowed by a "most" up-to-date edition of my work lest it weaken our inveterate Friendship.

Meanwhile, the reader may, as it were, climb these Stairs to the Sky and be an eye-witness to these mysteries. If you (whose mind is more sublime) choose to exert yourself a little to discover the truth of the subject, you will be more insightful and more qualified to evaluate the truly Herculean efforts in this Olympic competition. To avoid further ambiguities, and since you will soon take into consideration the matters [evaluation of parallax], and to grasp their meaning easily, I will start from the definitions lest the frequent use of Latin words would be unclear or produce doubts in the minds of readers.

App (iv): Definitions concerning Parallax

1. PARALLAX, that others call the change of position[47], is the difference in the angles subtended by two points on the Earth's surface, which comprises the semi-diameter of the Earth with straight lines converging at the center of the comet or of the celestial body. But because the sphere of the fixed stars is so far from the Earth, no *parallax* is measurable for them since the globe of the Earth is a mere point when compared with the immensity of that sphere. In the case of stellar *parallax* therefore another definition can be given after the introduction of some other Definitions. […]

2. As astronomers do in theories of planets and comets, I assume the TRUE LOCUS of the new star to be the [position] located on the sphere of fixed stars by the straight line connecting the center of Earth to the center of the star.

3. The APPARENT LOCUS is that [point] on the sphere of fixed stars established by the straight line connecting our position on the Earth's surface to the center of the star.

4. SIMPLE PARALLAX is the arc of a great circle on the sphere of the fixed stars, passing through the true and apparent positions of the celestial body.[48]

Other Parallaxes are: Total or Partial in Longitude, Latitude, Declination, or Right Ascension. […]

47 In this definition Digges writes "aspectus diversitate" translated with "change of position". He does not mention a celestial body with a detectable proper motion like a comet which was considered by Regiomontanus (see footnote 38).

48 Digges uses the Latin word "Stella" instead of "sydus" to denote the target of the measurements, which we prefer to translate as "celestial body" to avoid confusion with fixed stars.

App (v): Supplement on the Astronomical Radius: Short Practical Preface

[*App* v-1]

Astronomical Practice consists of two chief processes: observational experiments and demonstrations by which the [human] intellect evaluates the truth of things; and Sovereign Reason that generates laws and contributes them to the senses. If these processes are followed adequately, we will never deviate from the TRUTH. However, when we rely solely on the senses with poor and obtuse judgment, and without being able to exactly execute all the rules of the Empress [of Reason], so that we can direct ourselves very close to TRUTH itself, the only aim of Mathematical Arts, two [things] must be mainly observed. First, that the Empress Reason with its absolutely true and infallible demonstrations does not prescribe too difficult work procedures. Secondly, when for improving the senses someone uses mechanical instruments of this kind [for astronomical observations], the smallest things may be detected and measured without error.

[*App* v-2]

Therefore, after I had considered Diopters, Triquetrums, Armillae [or Armillary Spheres], Radii, Astrolabes, Quadrants and many other instruments of ancient and recent [astronomers], by means of which they are accustomed to measure Altitudes, Distances, Longitudes and Latitudes of stars: [I found that] the Astronomical Radius is the most useful of all the others (both for its easy use in each case, and also for the capability in distinguishing the smallest differences of any kind). However, it should be used with caution because it may be subject to some errors (due to the position of the eye and the practice not being sufficiently studied), which must be corrected in the most accurate way before dedicating oneself to this very delicate practice. I noticed that this topic was completely neglected by those who wrote on the structure and use of the Radius and that the visual use of the Radius for parallax [measurements] has not been corrected by anyone until now. Having corrected all the errors in demonstrations and having avoided all the inaccuracies of the visual work, like very true demonstrations the use of the most exact practice will produce unquestionable truths.

App (vi): Supplement on the Astronomical Radius: Chapter X

[*App* vi-1]

Anyone inexpert in astronomical hypotheses but not entirely lacking in common sense and judgment, would be able to understand easily through the following argument the mistakes of those who believe that this unusual star [SN1572] (located in the Elemental sphere below the Moon) has a parallax greater than that of the Moon.

Take a rod [rule] five or six feet long, made of wood or possibly of metal, in such a way that the sides are two straight lines and are perfectly parallel to each other. Its width is not important, it could be one-half foot or wider. Then after sticking a six-foot high stake in the ground, connect its visible extremity to the rod in such a way the other extremity can be turned in a circle to each side, and if you want to verify if the Phenomenon has parallax, proceed as follows: [In the following Digges explains the choice of two stars aligned with the new star between them.] […].

At first, you may find that no [star] lies along the same vertical with the Phenomenon [the New Star], but after various attempts you can find [such stars], because verticals are continuously changing with time and other stars will be aligned with the [Phenomenon]. When you find [stars] so aligned, take note of them carefully. When they move away, you will lower the rod and point your eyes here and there in such a way that the line of the same passes through the center of the fixed [stars] lines as closely as possible. If the Phenomenon had some parallax then it will not be seen on the same line of as the Rule. But to know the truth of the matter as clearly as possible, six or seven hours after the first observation of the vertical you will be able to see (once adapting the lines of the rod to the same fixed [stars]) if the Phenomenon lies perfectly with them or if it deviates from them. If it does not deviate from it at all, but it seems to fit perfectly, it has absolutely no perceptible parallax. If instead it deviates from it, it is certain that it has a parallax.

[*App* vi-2]

The previous problems will show exactly how large it [the Parallax] is. For this reason, having observed for many nights this extraordinary Phenomenon, I noticed that it appeared in a straight line with the little star in the knee of Cassiopeia, and the other which is in the right hip of Cepheus under the belt and also with the one that is in the thigh of Cassiopeia and another that is found in the left arm of Cepheus. I noticed throughout a night of this month of February that the Phenomenon had not deviated from the same straight lines or circumferences of great circles, only in the width/amplitude of its semi-diameter, so you can deduce with extreme accuracy that its largest parallax, in this duration of time, is less than two [arc]minutes. Even only by eye unaided by any instrument, anyone will be able to see the madness of those who believe that it [the Phenomenon] is in the region of Elements, below the lunar sphere where it would have a parallax greater than a whole degree. If one compares its position only with that of the other little star placed in the basis of Cassiopeia's seat, in reality, you would not discover that the Phenomenon is displaced away from it three times the diameter of the Sun. If it had a Parallax greater than a single degree, it would be necessary that the Phenomenon would be above [this star] along the same vertical, closer to it by two thirds of the previous distance and not differing from that by a single diameter of the Sun, or about 30 [arc]minutes, as it is clear from the proofs in the tenth and fifteenth Problems. There is nobody unable to notice such a big difference in such a small distance by the sight only without an instrument. But if a man wishes to examine this matter, not only would he know that such a big difference in the position does not exist, but he would find it so small and undetectable that he would not know whether to believe that it is without Parallax like a Fixed Star above the spheres of all planets. It is almost the case that only someone blind or benighted by a very crass ignorance would claim that the Phenomenon is below the Moon.

[*App* vi-3]

I had decided not to discuss the [exact] determination of the parallax of this wonderful star, or the evaluation of the limits of its parallax, because it properly concerns the history [sequence of events] of the star. However, a short time ago I heard that some people inexpert in Mathematical science had publicly claimed to have observed with demonstrative methods that the parallax of this unusual star was greater than one degree. They then concluded that its place was below the sphere of the Moon and that it was in the usual region for comets. In order that their terribly gross and almost obvious mistake [on the location] from where the star was shining, might be understood, I thought that adding this simple method [the use of a rod] would be useful so that even men who were uncultured in mathematics but were endowed with intellect and common sense could lay blame on professors of mathematics who make gross errors and could show their incompetence. […].

Furthermore, you will be able to demonstrate in the most exact way by means of this method the true place of the Phenomenon in the Heaven. You will be able to point at it with your finger and show those who are inexpert that it rests always exactly at the intersection of the vertical with the straight line or the great circle through the fixed [stars] selected by the method already discussed. In this way its true place will be made known and be self-evident to all. Inexpert shepherds and sailors, informed in this way, can know the value of the parallax and how it changes […]. If we accept that these starting points are right, we will publish with God's consent these and other unpublished matters. These shall be examined later more extensively with the easiest method and with the help of a new instrument, along with other Parallaxes that no one has treated up to this time or that were known and believed by very few people.[49] Certainly, these Parallaxes result from various locations of the observer relative to the center of the Earth.[50] If God is fa-

49 Digges does not clarify the type of new instrument. It seems quite unlikely that he intended simply an Astronomical Radius of a larger size. He mentions explicitly that none had been used before to measure parallax and that only very few people know these facts. One cannot therefore exclude that this sentence may concern the Perspective Glass, an optical device designed by his father for terrestrial use, but that it was useful also for astronomical observations. See: Usher and Massaro, "The sixteenth-century empirical disproof of Ptolemaic geocentrism", 661-664.

50 Again, Digges does not write his thought explicitly, but it is likely that he considered diurnal parallaxes as well as those due to the orbital motion of the Earth around the Sun, as expected in the Copernican system. This interpretation agrees with Goulding. See Goulding, "Wings (or Stairs) to the Heavens", 50n38.

vourable to such great enterprises and the Parcae[51] have regard for future generations, these matters shall be clearly understood.

[*App* vi-4]

However, someone might criticize me for the many methods presented in this book and for supplying few illustrative examples. I admit that in those respects I am also not particularly satisfied, and in fact I would have liked to have added very short and clear computations to the individual Problems. And I would have liked to have explained tools suitable for this work using vivid images and clear explanations if the short time available and my other affairs had not prevented me. But in this period of my life, I was forcibly removed, and almost torn, from these observations of celestial bodies by some recent human affairs in order to provide for the goods of Fortune.[52] These restrictions moved me to quit writing the book and to remove my hands from the desk [to stop the work]. Furthermore, since the specific goal of my work is to enable people everywhere to make very precise observations [of the Phenomenon] (before it and this most opportune occasion passes). […] Since it is uncertain how long this star will continue to shine, I thought that it would be better […] to lay a few traps and nets as soon as possible [instead of] throwing the largest nets too late [by printing a small book instead of a long treatise before the Phenomenon disappeared]. Later, however, having settled and overcome the obstacles posed by situations of life and fate, again with God's favour, we shall resort to our most serene sources of Mathematics. We shall examine how much our Muses are worth in this Country and with the help of the Almighty we shall faithfully share with all scholars the secrets of Nature which the work shall reveal.

App (vii): Conclusion of the book

[*App* vii-1]

Although the *Problems* and Canons, which have been set forth above, are all shown to be correct by robust proofs, yet I admit that not all are equally useful in *Practice*. Some are more useful than others, and perhaps those unaccustomed to observing will be surprised that even though everything is true, not all are suited to reach the *Truth*. This does not occur because of flaws in the demonstrations of the *Canons*, but, as already noted, it is due to the limitation in the use of the Senses. Therefore, those wanting to participate in the *Olympic Astronomical* competition must convince themselves that the *Senses* can never precisely aim to satisfy the demands of Reason. Like Archers, one must approach the target as close as possible, and those who achieve it are considered more skilled.

[51] "Parcae" is the Latin name for the three Greek Fates represented as old women spinning the threads of human destiny.

[52] "Fortunae bonis" or the "goods of Fortune" are economic benefits for him and his family.

[*App* vii-2]

In this Mechanical[53] controversy I have observed three main sources of errors. *First* and foremost, since an [observer] must measure so many different quantities, a small error in each of them will accumulate in the course of the operation. *Next*, in observations that must be made quickly, if one errs by even a small amount, the resulting errors will be so large that the time used for repeating measurements (necessary for high precision measurements) would be completely wasted. *Last*, when we use [astronomical] *Instruments* of this type,[54] which, although they are made and divided in a very fine and exact way, nevertheless, for small intervals of [arc]minutes or of other fractions, the weakness of the sight does not allow us to measure small differences. Thus, although we are anchored to very robust demonstrations and equipped with correctly-made and finely-divided instruments, in proceeding with this Olympic competition we will receive shame not honor. Instead, […], in order that anyone could directly reach the fortress of TRUTH and obtain the desired prize, let me report a few things on these Methods […].

[*App* vii-3]

Instruments for measuring angles in scruples [arcminutes][55] should have a very large size, and you would be able to obtain the Parallax successfully. However, both the methods [discussed in the 10th and 11th Problems] require measurements of time and if you deviate a little [from the exact time], in the meanwhile fast variations in Altitudes will occur, which will hide a *small Parallax* very easily, or produce a fictitious one, and therefore I do not consider them. […] In those Problems [in which] the positions of the Fixed [Stars], i.e. Longitudes and Latitudes, are clearly not considered at all, one has time enough to examine any [angular] distance[56] twice, three, four times, down to a [minimum] detectable deviation of a semi-scruple [half an arcminute]. Furthermore, if one admits that in mea-

[53] Digges writes "In isto autem Mechanico certamine…"; the word Mechanico referring to certamine (competition) has an uncertain interpretation because its meaning can be related either to the practice of observation or to a scientific context.

[54] Digges does not specify what type of instruments he considered, but on the basis of his text we can interpret them as the Devices for the measurement of celestial coordinates and angular distances like the Astronomical Radius or the Circle.

[55] "Scruple" can be interpreted as the smallest quantity that can be measured. According to the Encyclopedia Britannica, "scruple" (from Latin scrupulus, "small stone" or "pebble") is a unit of Roman commercial weight as well as a unit of coinage weight". In the 1st page of *Alae*, Latitudes and Longitudes of stars in Cassiopeia are given in degrees and arcminutes, whereas angular distances of SN1572 from some of these stars are in Degrees and Scruples. Considering that no written scruple value is higher than 60 (the highest one is 58) it is likely that 1 scruple could correspond to 1 arcminute. The accuracy of coordinates of Cassiopeia stars is 10 or 5 arcminutes.

[56] These Distances are the angular separations between SN1572 and nearby stars.

suring Altitudes (since times are established) we could make an error of a whole degree: nevertheless, in the determination of the Parallax, we should not deviate from the true [value] even by a half-scruple [half arcminute].

[*App* vii-4]

The methods presented are very certain and absolutely free from any defect, and moreover, are well suited for detecting a very small parallax. The evaluation of a Parallax with the method of *Regiomontanus* requires ten or twelve simple data.[57] Changes in *Altitude and Azimuth* are fast, [and when] one misses slightly the correct times as required, who does not know how large in size instruments must be to measure the Azimuths, and to correct the positions of the Fixed [stars with the precision] of [one] scruple? Thus, the Regiomontanus's architecture rests on a very unstable foundation that does not admit of correction, so that if a small [error] occurs in a single measure, then at the end it will produce an [error] of a large amount; and if in the calculation of time one deviates from the truth by only one scruple [time minute], it would correspond to fifteen [arc]minutes[58] on the maximum circle of the Sphere (on which Parallaxes are measured). Who then does not see how difficult it is for those who have adopted these methods, to avoid *Ruin*, particularly, when *very small Parallax* must be measured? For Comets, whose Parallaxes occasionally can reach eight degrees,[59] it may be used quite conveniently. But in the present [case] with so many small [measurements], in which the greatest Parallax does not reach a few [arc]minutes, it is not suited at all. I have dwelled on these matters not to denigrate Regiomontanus who was a really very expert Mathematician, but for fear that others who in our time would not completely miss the palm [trophy, goal] of TRUTH in this Olympic challenge by insisting on useless paths which despite being correct in theory are not [good] in practice. I have no doubt that if Regiomontanus were alive he would have to discard his old method, and he would have liked to find new ones that would be capable of seeking the truth of this Mystery hidden in the pitch-darkness.

[*App* vii-5]

There is another type of error, much less relevant than the others [but] which should not be neglected in the search for very small Parallaxes. Even if measurements had the highest precision, afterwards many Multiplications, Divisions, and extractions of *Roots* of *Sines* must be computed before one can reach the goal, small errors will occur in each of these operations […]. This happens because the precise [true] values of *Sines* cannot be known

57 Simple data are those consisting of only one measurement.

58 Likely here Digges uses the word scruple for 1 minute of time that corresponds to 15 arcminutes in Right Ascension (see footnote 55).

59 In the original book we used there is a handwritten note in Latin by an anonymous reader which makes clear the meaning of the Digges sentence.

because many are irrational [numbers]; furthermore, true values of *Roots* cannot always be known completely as numbers [with a finite number of digits]. In my sixth and seventh Problems, proofs provide the [angular] Distances of the Phenomenon [SN1572] and the [partial] Parallaxes separately.[60] Also, my Friend's[61] Method which he calls Nucleus, [...], can be adapted to all those Problems of mine, which depend on my Sixth and Seventh Problems. Therefore, [...] since both ways of working (i.e., those of John Dee and of my problems, consist in several multiplications, divisions, and extractions of *Roots* of the different *Sines*, [and] excluding the occurrence of large parallaxes, for the very small ones considered here, I would take care that an intelligent Observer of this Miracle [SN1572], would adopt other Methods, which use a simpler, shorter and less error-prone calculation of the *Sines*. This booklet is not lacking in such a rich matter, particularly in the last five Problems, [which are] clearly verified, and not so cryptically complex, that any Mathematician could easily select [among them] according to different needs [...]. In any case, if someone likes to use those complicated and intricate Methods with evaluations of many *Square Roots*, I would suggest he adopt the way of *Algebraic* calculations; i.e., multiplying and dividing the same *Square Sines* (which can be found more precisely) and assigning the proper *Characters* to each result [number][62] so that only one extraction of a *Square Root* should be necessary. This concerns the precautions and checks to be applied in practice.

[*App* vii-6]

I cannot then end here without [...] pointing out once more to all scholars of the *Celestial Science* how the opportunity available to *Terrestrial* beings is great and desirable. The *Monstrous System* of the Celestial orbs[63] constructed in antiquity was perfectly

60 Likely, Digges refers here to the components of parallactic displacement, like the Longitude and Latitude Parallaxes mentioned in the Definition section.

61 The Friend of Digges is clearly John Dee, indicated in the Preface as his mentor and second father who in March 1573 published booklet on the problem of the measurement of parallaxes (Dee. *Parallaticae Commentationis*). The Foreword of Dee's booklet was written by "Thomas Diggseus, Benevolo Lectori".

62 Digges writes: "singulis inventis suos Characteres attribuendo", the meaning of which is not entirely clear. The word "Character" is derived from the ancient abacus terminology and likely was used by the Digges father and son to indicate the exponents of prime factors of a number (Digges and Digges. *Stratioticos*, 32. See also: Cajori, *A history of mathematical notations*, 169-171). Later in 1585, Simon Stevin (1548-1620) used the same word to indicate powers of 10 in his practical book, *La Disme* or *Thiende* on arithmetical calculus: Cajori, 154; Sanford, "La Disme of Simon Stevin".

63 Digges uses the words: "Coelestium globorum Systema" and "Coelestis Systematis". These expressions for indicating the totality of orbs of planets and their spatial order and structure was used by Rheticus, as reported in the extensive historical study of Lerner, who however did not mention Digges.

corrected and improved by *Copernicus*, a divine rather than a human talent, yet there are still some things to be worked out. I realized that this could not have been done except by means of very accurate observations either of this *Very Rare Star*, or of the remaining *Wandering* bodies [comets or planets] and of their variations in appearance [position and brightness], made from different *Regions* of this dark and mysterious *Terrestrial Star* where we spend a troubled life […] wandering like pilgrims across a small region. I really do not see any better reason to spend life (than to contemplate the works of the most good and most great God). In fact, eyes were given to men for this very reason, to appreciate […] how many and invisible things exist by way of the visible things of GOD. You [readers], then, who have a more sublime mind and were not born subjugated by hard chances nor chained by the bonds of avarice, lust, and other vices, have become prisoners in the realm of *Sarcotheus,*[64] may take this province for yourselves. This unexpected opportunity to shake off indolence should not slip out of your hands. [One could hope] that a more fruitful doctrine of Celestial Science would finally emerge from the many considerations about the Celestial spheres ([developed] by very expert Mathematicians), which, in different regions and places on the Earth, are presented and discussed by means of numbers and measures with flawless competence, and then published and collected in public essays.

[*App* vii-7]

If *Copernicus* (who can never be praised enough) were alive at present, this year he would have been a *Centenarian*, and we could hope that on this occasion the true knowledge of the Celestial System [[27]] would be given to Mortals (as far as human weakness can achieve). But it is vain to hope for the return to life of a so great man. Lest this very rare and first desired Occasion should vanish fruitlessly, […] my work should not be considered the close of this Olympic competition, […]. Not to seem myself to be highly indolent and too negligent of my duty to begin this competition, and also to raise the interest of others whom URANIA enriched more with her benefits, I have prepared these STAIRS [SCALAE] by which mathematicians and even Tyrunculi [students] may ascend the Ethereal Towers and measure all Distances in detail, and investigate truthfully the places of the Celestial Globes. They can also examine the syntax [structure] of the World, and the *Magnitude, Distance* and position of this portentous Star presented to *Terrestrial [People]* by the Almighty [Lord]. Finally [they can] explore the remote and terrible region […] of the star (different from the one that announced to the [Three Wise

64 Sarcotheus is a devil whose aim is to rule over the sublunar world in order to have it dominated by passions, illness, and death. Marcellus Palingenius Stellatus (likely Pietro Angelo Manzoli, known as "Palingenius") writes of Sarcotheus in Book VIII (Scorpius, 229, line 28*ff*) of *Zodiacus Vitae (Palingenius, Zodiacus Vitae, Lyon: Jean de Tournes, 1503)*, a poem much admired by Thomas Digges.

Kings] MAGI the advent of the CHRIST GOD [a comet]). Without any doubt they [can] testify the astonishing Miracle of GOD to the others, who cannot raise their faces from the Earth, so that all people may finally know the great things of GOD, to whom alone all PRAISE, HONOR and GLORY must be directed in all Ages.

THE END OF MATHEMATICAL STAIRS

References

Bakker, Arthur, Gravemeijer, Koeno. "An Historical Phenomenology of Mean and Median". *Educational Studies in Mathematics* 62, 2 (2006), 149-168.

Boas Hall, Marie. *The Scientific Renaissance 1450-1630*. New York: Dover, 1962.

Brahe, Tycho. *De Nova stella…*. In *Tychonis Brahe Dani Scripta Astronomica*, edited by John L.E. Dreyer, vol. 1, Hauniae in Libraria Gyldendaliana, 1573.

Brahe, Tycho. *Astronomiae Instaurate Progymnasmata…*. In *Tychonis Brahe Dani Scripta Astronomica*, edited by John L.E. Dreyer, vol. 2, Hauniae in Libraria Gyldendaliana, 1602.

Buchwald, Jed Z. "Discrepant measurements and experimental knowledge in the early modern era". *Archive for History of Exact Sciences* 60, 6 (2006), 565-649.

Cajori, Florian. *A history of mathematical notations* [1928, 1929]. New York: Dover, 1993.

Chapman, Allan. "The accuracy of angular measuring instruments used in astronomy between 1500 and 1850". *Journal for History of Astronomy* 14, 2 (1983), 133-137.

Christianson, John R. "Tycho Brahe's Earliest Instruments". *Nordic Journal of Renaissance Studies* 12 (2017), 131-144.

Dee, John. *Parallaticae Commentationis Praxeosque Nucleus quidam*. London: John Day, 1573.

Digges, Leonard, Thomas Digges. *Pantometria*. London: Henrie Bynneman, 1571.

Digges, Leonard, Thomas Digges. *Stratioticos.* London: Henrie Bynneman, 1579.

Digges, Thomas. *Alae seu Scalae Mathematicae…*.London: Thomas Marsh, 1573.

Digges, Thomas. *A perfit description of the caelestiall orbes*. In *A Prognostication Euerlastinge of Right Good Effecte …. Published by Leonard Digges … Corrected and Augmented by Thomas Digges his Sonne*. London: Thomas Marsh, 1576.

Gilbert, William. *De Mundo Nostro Sublunari Philosophia Nova.* Amsterdam: Apud Ludovicum Elzevirium, 1651.

Goldstein, Bernard R. "Levi ben Gerson: On Instrumental Errors and the Transversal Scale". *Journal for the History of Astronomy* 8, 2 (1977), 102-112.

Goulding, Robert. "Wings (or Stairs) to the Heavens, The Parallactic Treatises of John Dee and Thomas Digges". In *John Dee: Interdisciplinary Studies in English Renaissance Thought,* edited by Stephen Clucas, 41-64. Dordrecht: Springer, 2006.

Granada, Miguel A. "Bruno, Digges, Palingenio: omogeneitá ed eterogeneitá nella concezione dell'universo infinito". *Rivista di storia della filosofia* 47 (1992), 47-73.

Granada, Miguel A. "Thomas Digges, Giordano Bruno y el desarollo del copernicanismo en Inglaterra". Éndoxa: Series Filosóficas, (1994), Madrid: UNED, 7-42. DOI: 10.5944/endoxa.4.1994.4817.

Green, Daniel W. E. *Assessment of early-modern observations of comets and supernovae: Focus on pre-telescopic European astrometric and physical data.*, Durham theses, Durham University, 2004. Available at Durham E-Theses Online: http://etheses.dur.ac.uk/3053/.

Håkansson, Håkan. "Tycho the Apocalyptic: History, Prophecy and the Meaning of Natural Phenomena". In *Science in Contact at the Beginning of the Scientific Revolution,* edited by Jitka Zamrzlová, 211-236, Prag: National Technical Museum, 2004.

Helfricht, Jürgen. *Astronomiegeschichte Dresdens*. Dresden: Hellerau, 2001.

Johnson, Francis R. "The influence of Thomas Digges on the progress of modern astronomy in Sixteenth-Century England". *Osiris* 1 (1936), 390-410.

Johnson, Francis R. *Astronomical Thought in Renaissance England: A Study of English Scientific Writings from 1500 to 1645*. Baltimore: The Johns Hopkins Press, 1937. Reprinted New York: Octagon Books, 1968.

Johnston, Stephen A. "Making Mathematical Practice: Gentlemen, practitioners and artisans in Elizabethan England"- PhD diss. Cambridge University, St John's College, 1994. https://doi.org/10.17863/CAM.19773.

Koyré, Alexandre. *From the Closed World to the Infinite Universe.* Baltimore: Johns Hopkins Press, 1957.

Krisciunas, Kevin. "Observatories". arXiv:astro-ph/9902030 (1999).

Lerner, Michel-Pierre. "The Origin and Meaning of "World System". *Journal for the History of Astronomy* 36 (2005), 407-441.

Marcellus Palingenius Stellatus. *Zodiacus Vitae.* Lyon: Jean de Tournes, 1503.

Margetts, Andrew, Giles Dowkes, Damian Goodburn, *et al.* "What "incomparable Jewells Havens, and sure harbours are": the remains of late 16th century Dover harbour and their wider significance". *Post-Medieval Archaeology* 57 (2023), 177-198. DOI: 10.1080/00794236.2023.2166442.

Mosley, Adam. "Paul Wittich". In Hockey, T., *et al.*, *The Biographical Encyclopedia of Astronomers,* 1234-1235. New York: Springer, 2007. https://doi.org/10.1007/978-0-387-30400-7_1493.

Palingenius: see Marcellus Palingenius Stellatus.

Plackett, Robin L. "The principle of the arithmetic mean". *Biometrika* 45, 1/2 (1958), 130-135.

Roche, John J. "The Radius Astronomicus in England". *Annals of Science* 38, 1 (1981), 1-32.

Rogers, Leo. "Leonard and Thomas Digges: 16th Century Mathematical Practitioners". Oral Presentation, University of Oxford, 669-678, 2004. https://hal.science/hal-01349243.

Rudavsky, Tamar M. "Gersonides: Levi ben Gerson". In Hockey, T., *et al.* Biographical Encyclopedia of Astronomers. Springer, New York, NY, 2014. https://doi.org/10.1007/978-1-4419-9917-7_512.

Sanford, Vera. "La Disme of Simon Stevin - The first Book on Decimals". *The Mathematical Teacher* 14, 6 (1921), 321-333.

Sheynin, Oscar. "The treatment of observation in early astronomy". *Archive for History of Exact Sciences* 46, 2 (1993), 153-192.

Taylor, Eva G.R. *Mathematical Practitioners of Tudor and Stuart England.* Cambridge: University Press, 1954.

Usher, Peter D., Massaro, Enrico. "The sixteenth-century empirical disproof of Ptolemaic geocentrism". *Journal of Astronomical History and Heritage* 26, 3 (2023), 659-666.

Usher, Peter D., Massaro, Enrico. "The sixteenth-century empirical disproof of Ptolemaic geocentrism: Paper II". *Journal of Astronomical History and Heritage* 27, 1 (2024), 63-72.

Wallace, William A. *Prelude to Galileo*. Boston Studies in the Philosophy of Science, vol. 62, Dordrecht: D. Reidel Publishing Company, 1981.