Longomontanus’ *De maculis in Luna* and the determination of terrestrial longitudes

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Abstract

In *Astronomia Danica*, Longomontanus provides a method for calculating the terrestrial longitude for a given location on Earth. To do so, he relies on precise calculations of the lunar position so that he can know when he is observing it without any parallax in longitude. As I will show, this method has a fatal flaw that renders it unusable. However, Longomontanus also provides a simple observational method that indicates, via the disposition of the lunar spot and/or horns, the time when the Moon shows no parallax in longitude. This last method, though it does not need any use of tables, also has some problems. In this paper I will explain in detail these methods provided by Longomontanus, together with the problems they carry.

Keywords

Longomontanus, Galileo Galilei, longitude problem, Astronomia Danica, Renaissance astronomy

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Introduction

In *Almagest* V, 3 Ptolemy gives a fairly detailed account of his study on the second lunar anomaly.[1] In order to do so, he uses some observations of the lunar elongation throughout the synodic month. For those observations to be useful though, they had to fulfill a set of conditions. Even if Ptolemy did not always follow his own advice, the criteria listed in that section are indeed necessary to carry out the investigation Ptolemy had in mind. It is of particular interest to us to look at the third condition given by Ptolemy: he says that at the times of observation, "[...] the moon had no longitudinal parallax".[2] At this stage of the *Almagest* Ptolemy still had no theory of lunar parallax, so it makes sense for him to look for moments when he knew that the Moon was not affected by it, at least regarding its position in longitude. To show that the first of the observations he gives is indeed free of parallax problems, Ptolemy tells us that at the time "The apparent position of the moon was ♏ 9 °, and that was its true position too, since when it is near the beginning of Scorpius, about 1 ½ hours to the west of the meridian at Alexandria, it has no noticeable parallax in longitude".[3] But why is that?

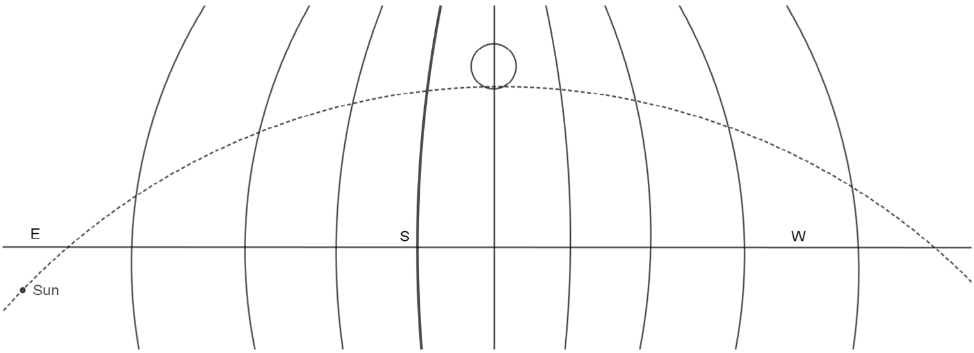

Figure 1. Stereographic projection of Alexandria's southern horizon during Ptolemy's observation (Feb 9, 139 in the morning). The horizontal line ESW is the horizon. All the lines perpendicular to the horizon are altitude circles, with the meridian represented by a thicker line. The west is to the right of the image, and the east is to the left. The dashed curve that intersects the horizon twice is the ecliptic. The Sun is on the east and has yet to rise. The Moon is some degrees to the west of the meridian, just as Ptolemy says. It can be seen that the altitude circle of the Moon intersects the ecliptic at a 90° angle.

1 Toomer, *Ptolemy's Almagest*, 222.
2 *Ibid.*, 223.
3 *Ibid.*

Refer to Figure 1. The Moon is to the west of the local meridian, which crosses the horizon to the south. As the figure shows, the altitude circle of the Moon is at that time intersecting the ecliptic at a right angle. It can be proven that, with respect to the longitudinal component of the lunar position, this configuration allows the observer to see the Moon as if he was at the center of the Earth, thus rendering the apparent longitude equal to the true longitude. The figure also shows that at that time – and only at that time – the point of the ecliptic determined by the altitude circle is 90° apart from the ecliptic's rising and setting points.[4]

In *Almagest* II, 13 Ptolemy provides[5] a table that gives the value of this angle for 0° in each sign and various latitudes. Ptolemy must have used it to determine the best available moments for his lunar observations, even if not always in the strictest of manners.[6]

Some fifteen centuries after Ptolemy worked on his tables of ecliptic angles and used them to test the Hipparchian lunar model by looking for moments where the Moon did not show parallax in longitude, Longomontanus explained in his *Astronomia Danica*[7] a method to take advantage of these configurations in order to determine the terrestrial longitude of any given location. In the section appropriately titled *De maculis in Luna, & ipsarum usu (On the spots on the Moon, and about their use)*, Longomontanus discusses the nature of the lunar features – and indeed of the Moon itself – , and how the lunar position and appearance can be used to solve one of the problems which was beginning to become extremely relevant in the wider European context: the problem of determining terrestrial longitudes.

The *De maculis* can be divided in two thematic parts: first, a discussion about the nature of the Moon and the causes of its spots and varying luminosity. Second, a method for determining the terrestrial longitude of the observer via carefully chosen lunar observations. In the first part Longomontanus deals with the typical discussions of the time: is the Moon opaque? What is the cause of its luminosity? What is the ultimate reason of its irregular appearance? In the course of his exposition Longomontanus shows his acquaintance with the recent works on these subjects, and in particular with Galileo's telescopic investigations.[8] Also, he seizes the opportunity to mention one of the hot theological topics of the day: the question of the merit of human deeds towards salvation. Just as there are people who say that the Moon produces its own light, so are people that profess that our meritorious works come from our own nature. This is labelled as the heresy of *synergism*, a somewhat diplomatic way of alluding to the Catholic position on the matter.[9] The true

4 See the Appendix for a geometrical demonstration of this relation.

5 Toomer, *Ptolemy's Almagest*, 123.

6 Neugebauer, "A History…", 92.

7 Longomontanus, *Astronomia Danica*, 191-197.

8 *Ibid.*, 192.

9 The metaphor is somewhat flawed, though, since synergists support a lesser claim, i.e., that salvation at least requires some kind of free cooperation on the part of man.

Christian doctrine, though, is given to us not only in the letter of the Sacred Scriptures, but also metaphorically in the skies: in the same way that the Moon shines inasmuch as it reflects the solar light which is freely given to it, so does man acts meritoriously inasmuch as he receives "Christ's spirit and the total good illumination".[10]

Whatever the theological and astronomical connotations of this first part of the section, this paper will be focused on the second one, where he develops his method for determining the terrestrial longitudes. While it contains some interesting, original aspects, Longomontanus' method is one of many examples that make use of the Moon to find the east-west position on the surface of the Earth. So, the first section of the paper is a brief historical introduction to the problem of finding terrestrial longitudes via the use of astronomical observations. The following section explains in detail the method Longomontanus proposes to make this calculation, and the problems involved in it. This method relies on the use of ephemerides and, as we will see, is plagued with various kinds of problems. Longomontanus provides, however, an additional idea which correctly indicates how an observer might use the orientation of the lunar face in order to determine the best moment for the lunar observations involved in the method. The explanation of this idea is the subject the next section. Finally, I will discuss some of the implication of Longomontanus' text in a Conclusion, which is followed by a technical appendix which discusses some geometrical properties of celestial configurations that are relevant to Longomontanus' argument.

Lunar observations and determination of terrestrial longitudes

Ptolemy describes, in *Almagest* II, 1, the only method that we know of which was used in antiquity to determine distances in terrestrial longitude via astronomical observations. According to him, the inhabited part of the world was comprised within one half of the northern hemisphere. To support this claim, he points to the fact that: "[…] observations of the same eclipse (especially a lunar eclipse) by those at the extreme western and extreme eastern regions of our part of the inhabited world (which occur at the same time), never differ by more than twelve equinoctial hours; and the quarter [of the earth] contains a twelve-hour interval in longitude […]".[11] So, given that a lunar eclipse is a phenomenon which takes place at the same absolute time for all observers, but not at the same local time, then it can be used to determine the distance in terrestrial longitude between two observers. Observer A would see the eclipse happening at 1 PM local time, and observer B will see it happening at 2 PM local time. This means that there is a time difference of one equinoctial hour between them, or 15° in terrestrial longitude. The text indicates that this can be used to determine that the extremes of "our part of the inhabited world" cannot

[10] Longomontanus, *Astronomia Danica*, 192.
[11] Toomer, *Ptolemy's Almagest*, 75.

be separated by more than 180°. In his *Geography* Ptolemy actually provides an example – maybe the only one he knew of – of this kind of calculation, when he refers to a lunar eclipse which took place on 20 Sept. -330, "[…] the one that was seen at Arbela at the fifth hour and at Carthage at the second hour […]",[12] which would indicate that Carthage was about 45° to the west of Arbela (it is actually just about 34° to the west). This method continued to be used right to the end the middle-ages: Columbus tried to determine the positions of two American islands with lunar eclipse observations.[13]

While the lunar eclipse method took advantage of the fact that during those events observers could ignore the effects of parallax, because the phenomenon only depended on the relative positions of the Earth, the Sun and the Moon, during the middle-ages other methods that involved the Moon began to be used, ones that did not need for a lunar eclipse to take place. In the XIIIth century *Theorica planetarum* attributed to Gerard of Sabbioneta we find the following procedure: "If the Moon is in the middle of the heavens and you equate it by means of a table for a certain region, you will know the longitude between the two regions by the difference between the places of the Moon without having to wait for an eclipse".[14] When the Moon is on the local meridian, it shows no parallax in RA. So, the procedure first asks for observer A to observe the right ascension of the Moon when it is on the local meridian via the determination of an angular distance to a star of known position. Then he has to compare that RA to the one predicted for the lunar meridian passage by tables computed for a specific location B, and thus to a specific, different, local time. The difference in RA will indicate, if the lunar speed for that day is known, the interval between the culmination at A and at B. If the intermediate lunar motion is accounted for, this allows for the determination of the difference in terrestrial longitude between the two locations. With infinite variations, and a continuous sophistication, this method continued to be used up until the first half of the 19th century.[15] In Longomontanus' times the problem was still being attacked by his fellow astronomers. As we will see later, the Danish astronomer had fruitfully read Galileo's *Sidereus Nuntius*, where the great Pisan first presented his discoveries of the Jovian moons to the wider public. Galileo had, in 1616, approached the Spanish government with a proposal to provide a method for finding longitudes which relied on the use of the Jovian moons (whose periods he was studying in detail) as a universal clock. After a second try in 1630, the negotiations fell through.

12 Berggren & Jones, *Ptolemy's Geography…*, 63.

13 West & Kling, *The Libro de las Profecías…*, 226-227. The eclipses he used were the one on 15 Sept 1494, and the one on 1 Mar. 1504.

14 Cremonensis, 1478 , pág. NP. In the printed edition I consulted, the folia are not numbered. The quoted text is in the next to last chapter. The translation is from Pedersen, *A Survey…*, 463.

15 Even today tables for "clearing" the lunar observed position of the effects of parallax and refraction in order to calculate the observer's terrestrial longitude are still published. Cf. Stark, *Stark Tables…*.

Several years later, in 1635, he approached the Dutch, who finally accepted it. However, the method was never practical at sea.[16]

As we will see in the following section, while it presents some interesting original aspects, Longomontanus' method is part of the "lunar tradition".

Longomontanus' method

a) Determination of the time for observation

The method presented by Longomontanus requires for the Moon to show no parallax in longitude, that is, the same requirement Ptolemy had asked for the lunar observations needed in his evaluation of Hipparchus' model. Unlike Ptolemy, Longomontanus does not use a table of ecliptic angles. Instead, he will look for the moment when the Moon is 90° away from the ecliptic's rising and setting points. As I indicated earlier, this is equivalent to the Ptolemaic requirement. To predict when this will happen for a given day is not a trivial calculation.

As an example, Longomontanus calculates this moment for 4 October 1617 (Julian).[17] This was the day of full Moon. First, he obtains the solar longitude at noon. For this he consults David Origanus' *Ephemerides Brandenburgicae*, which were calculated for Frankfurt an der Oder ("horizonti Francofurtano ad Viadrum").[18] He gives a solar longitude of 202;6°. In fact, this corresponds to 5 Oct. in Origanus' tables.[19] Then he also obtains the longitude

[16] Heilbron, *Galileo*, 235-236, 346-348.

[17] Longomontanus, *Astronomia Danica*, 194.

[18] Cf. Origanus, *Ephemerides Brandenburgicae*… . Origanus had already calculated a set of ephemerides for the years 1595-1630, also for Frankfurt an der Oder: the *Ephemerides Novae* (Origanus, 1599). These were derived from the Prutenic tables, although regarding the true nature of the cosmos he kept his geocentric preferences. During the intervening years, Origanus went through a sort of Tychonic conversion (Omodeo, "David Origanus'…", 440), and adopted not only the new cosmological framework, but also the parameters of the Tychonic model. For example, in the 1599 work he assumes the Prutenic solar eccentricity of 32,222 for a radius of 1,000,000, while in 1609 he adopts the Tychonic 35,840. These variations of course translated into different predictions for the same days. For 4 Oct. (Julian), for example, the 1599 ephemerides give a solar longitude of 200;48,45°, while the 1609 ephemerides give 201;5,30°. Although Longomontanus does not specify which ephemerides he is using, it is only natural that, being Tycho's disciple, he would use the Tychonic version from 1609. Also, the values he gives in *Astronomia Danica* fit much better with the 1609 work than with the 1599 one.

[19] Origanus gives 201;5,30° for 4 Oct. 1617 (Julian). No reduction for Longomontanus' location will account for the difference. The longitude given for 5 Oct. 1617 (Julian) is 202;5,13°. In the front page of the book he gives Frankfurt an der Oder's terrestrial longitude as 36°. Longomontanus gives 36;40° as Copenhagen's terrestrial longitude. (Longomontanus, *Astronomia Danica*, 195). This means a time difference of just 2.6 minutes. This is not nearly enough to account for

of the Moon for that moment: 20;20°[20] and adds 90° to it, obtaining 110;20°. This last step means that when the point of the ecliptic with longitude 110;20° is rising, then the point with longitude 20;20° will be 90° away from the rising and setting points of the ecliptic.

At this point he uses Origanus' *tabulae domorum*, which are part of the *Novae Motuum Coelestium*, a work that accompanied his 1609 ephemerides.[21] The tables rely on an astrological division of the local sky, the subject of which is beyond the scope of this paper.[22] Here it is enough to know that the *tenth house* or *10 domicilius* is the point of the ecliptic that is at the local meridian at a given time. The *horoscopus* (as Origanus labels it in the tables) or *prima domus, first house* (as Longomontanus references it) is the point of the ecliptic that is rising in the east at a given time. The tables are calculated for different latitudes, from an equatorial one (*sphaera recta*) up to 60°, with special section for the precise latitude of Frankfurt an der Oder, 52;20°. They have a first column labelled "Tempus a meridie" (from now on *time column*) which indicates the distance in time between the culmination of Aries 0° and the local noon for different solar longitudes,[23] or between the culmination of Aries 0° and the moment when a given point in the ecliptic is in a given position for the local observer.[24] So, first Longomontanus chooses the table with the cor-

the difference between both solar longitudes: it barely yields a solar motion of 7''. The correct difference in terrestrial longitude between those locations, though, is about 2°. This yields a time difference of about 8 minutes, and a solar motion of about 20''. This reduction would give a solar longitude at noon, for Copenhagen, of 202;5,33°, and Longomontanus could be giving a rounded value. This, of course, would mean that Longomontanus had a better value for the difference in terrestrial longitude between both cities than the one derived from Origanus' work.

20 In this case he correctly gives the value for Origanus' 4 Oct. (Julian). The tables give 20;13°, and a lunar motion for that day of 15;14°. A reduction for Copenhagen assuming modern terrestrial longitudes for both cities gives a lunar longitude of 20;18°, while a reduction using the difference derived from Origanus' terrestrial longitude for Frankfurt an der Oder gives a lunar longitude of less than 20;15°. This supports the idea that Longomontanus had a better estimation of the distance between both places.

21 Origanus, *Ephemerides Brandenburgicae…*, 299-370.

22 Cf. North, *Horoscopes and History*, 1-6) for a more detailed introduction to the astrological house system and its history.

23 The tables indicate at the top of each page that they are calculated "☉ existente in *x*", where *x* stands for any given sign. From 0° to 30°, that given sign is listed in the *10* column, the one that stands for the position at the local meridian. So, if the Sun is at a given longitude, then we enter the table through the *10* column and find that longitude, and that gives us, in the first column, the time between local noon and the culmination of Aries 0°.

24 The rest of the columns, labelled *11*, *12*, *Horosc.*, *2*, and *3*, reference other positions with respect to the horizon. For example, as we said, the *horoscopus* is the ecliptic point that is rising. So, the tables allow for one to know how much time there is between the moment when a given ecliptic point is rising, and the culmination of Aries 0°. *Mutatis mutandi*, the same can be said about the other *houses* in the tables.

responding latitude considering the 55;43° he indicates for Copenhagen.[25] Then he enters the table through the *10* column and looks for the longitude 202;6° he found for the Sun. This is because Origanus' ephemerides had given him the location of the Sun at noon, that is, when it is on the local meridian, or at the *tenth house*. The value in the time column is 13 h 18 m.[26] In modern terms, he found the RA of the Sun for 5 Oct. 1617 at noon (Julian) (although he thought he was doing it for 4 Oct.).

Next, he enters the table through the *horoscopus* column, and looks for 110;20°. As we said earlier, when that point of the ecliptic is rising, then the point with longitude 20;20° (where the Moon was at noon) will be at the appropriate position, and no parallax in longitude will be observed. The time column indicates 23 h 1 m.[27] In modern terms, he has found the RA of the local meridian when the ecliptic point 110;20° is rising. After that step, he calculates 23 h 1 m – 13 h 18 m = 9 h 43 m. This is the amount of time after noon when the point of the ecliptic with longitude 110;20° will rise that day, and therefore when the point with longitude 20;20° will be 90° away from the rising and setting points of the ecliptic.

However, by the time the ecliptic point 110;20° rises, almost 10 hours after noon, the Moon will no longer be at 20;20°. Instead, it will have increased its longitude, so that by then the ecliptic distance between it and 110;20° will be less than 90°. So, Longomontanus has to iterate the calculation to account for the difference. The procedure he carries out, though, is flawed. What Longomontanus needed to do was to calculate the advance in longitude during the intervening 9 h 43 m, and add that to the original 110;20°. Because that is the amount by which the position of the Moon will have moved, then it is also the amount by which the point 90° away from the Moon will have moved. Then, using the same *tabulae*, he could calculate as before the time when that resulting value is at the *horoscopus*.

[25] Although Longomontanus explicitly indicates this step, there is no need for it when dealing with the *tenth house*, since for any latitude the time value for the meridian position will be the same. In fact, the *tabulae domorum* only give it once per page, whatever the latitude they provide first. However, because Longomontanus is also talking about obtaining the *horoscopus* related to the Moon, and because in that latter step the latitude is relevant, then his indication is not without utility.

[26] In fact, Origanus gives 13 h 21 m 18 s for 202°, and an interpolation that accounts for the 6 minutes would only bring it higher. My opinion is that he simply took the value for 202° and that, similarly to the case in note 28, this is probably a slip of the pen, and he simply mixed the minutes with the seconds.

[27] The preciseness of the value given by Longomontanus suggests some kind of interpolation. The tables provide values for latitudes 54° and 57°. Also, the longitudes in the *horoscopus* column are, in no case, exactly 20;20°. There is no obvious method of interpolation that provides exactly the value given by Longomontanus. The results I obtained by trying some combinations differ from 23 h 1 m by amounts that range from 2 min. to 9 min.

To do that, Longomontanus should have used the daily lunar motion, which according to Origanus' ephemerides was 15;14° on that day, and calculated the proportional for 9 h 43 m, which is 6;10°. That would give an ecliptic point in 110;20° + 6;10° = 116;30°, with a time value of about 23 h 40 m, and a rising time after noon of about 23 h 40 m – 13 h 18 m = 10 h 22 m.

Instead, Longomontanus explicitly points not to the daily lunar motion in longitude, but to the daily motion in elongation, which according to Origanus was 14;14°. This yields a proportional of 5;46°,[28] and an ecliptic point in 110;20° + 5;46° = 116;6°. Once he has the – incorrect – new longitude of the Moon, he repeats the previous step with the *tabulae domorum*, and finds that the time column for 116;6° at the *horoscopus* is 23 h 34 m,[29] with a rising time of 23 h 34 m – 13 h 18 m = 10 h 16 m. after noon.

Now, during the extra 33 minutes he has,[30] the Moon will have moved an extra distance: given the daily motion for that date, about 0;21°. The method thus requires an iterative procedure to be carried out, until a satisfactory value is reached. Longomontanus' explanation does not go further than this. So, in this way Longomontanus determines, for a given date, the time when the Moon is 90° away from the rising and setting points of the ecliptic, thus showing no parallax in longitude.

b) Determination of the terrestrial longitude difference between two places

The procedure Longomontanus proposes is part of the tradition I mentioned earlier. The path he proposes, though, is extremely flawed. The example he gives is for 6 Oct. 1617 (Julian).[31] On that date, he says, the Moon was 90° away from the rising and setting points at 11 h (noontime). According to Longomontanus, the tables indicate for that time a lunar longitude of 57;20°.[32] The known longitude of Aldebaran for that year was 64;26°.[33] So the calculated distance in longitude between the two was 7;6°. Having provided before

28 In the text, he gives 5;0;46° ("5 gr. 46 sec."), clearly a misspelling.

29 In this case the result is a close match to a double interpolation, one for the exact ecliptic longitude, and a second one for the exact geographical latitude, just as the one described in note 35.

30 That is, 10 h 16 m – 9 h 43 m = 33 m.

31 Longomontanus, *Astronomia Danica*, 195.

32 From from Origanus' ephemerides – with the proper reduction for Copenhagen and linear interpolation for 11 PM – we get 57;18°, so Longomontanus is here giving a rounded value. This might point to the fact that here Longomontanus *first* found the time, and *then* he found the lunar longitude. Such a procedure would go against the calculation method he explained, and suggests that he was simply working using the observational method described later in the paper.

33 In order to have a proper list of reference stars, Longomontanus gives the 1620 coordinates of 15 stars (Longomontanus, *Astronomia Danica*, 196): in Aldebaran's case, it is 64;29°. It is a rounded value, as it is the precession rate of 1' per year instead of Tycho's 51" per year (Brahe, *Opera Omnia vol. II*, 280).

an example of how to compute these values, here Longomontanus is just offering the final result of 11 h as the time, and 57;20° as the lunar longitude. We have no indication as to how he actually calculated the values.

Let us reconstruct this initial calculation by following the method provided in the previous section, using the same sources. For the given date, Origanus gives a solar longitude at noon of 203;4,58°, and a lunar longitude of 50;29°. If the times are reduced to Copenhagen using linear interpolation,[34] and we round them as Longomontanus does, we get 203;5° for the Sun (with the corresponding RA 13 h 25 m in Origanus' *tabulae domorum*), and 50;34° for the Moon. Now, 50;34° + 90° = 140;34°. The RA of the meridian when the point of the ecliptic with longitude 140;34° is rising is about 2 h 1 m.[35] This gives a time difference of 24 h – 13 h 25 m + 2 h 1 m = 12 h 36 m. The lunar motion for that day, according to Origanus, was 14;41°. The proportional motion is thus 7;43°, which puts the Moon at 50;34° + 7;43° = 58;17°. Again, 58;17° + 90° = 148;17°. According to the *tabulae* the RA of the meridian when this last ecliptic point is rising is 2 h 48 m.[36] So, we now get that 24 h − 13 h 25 m + 2 h 48 m = 13 h 23 m is the amount of time after noon when the Moon was 90° away from the rising point of the ecliptic. As before, more iterations are necessary to get better results: after all, during the extra 47 minutes of motion the Moon will have moved an extra 29' in longitude, resulting in 58;46°, with an RA of the meridian when the ecliptic point 58;46° + 90° = 148;46° is rising equal to 2h 51 m. This would mean that the correct time for observation on 6 Oct. is 24 h − 13 h 25 m + 2 h 51 m = 13 h 26. Longomontanus' error for the proper observation time is at least 13 h 26 m – 11 h = 2 h 26 m.

As I mentioned earlier, Longomontanus makes a crucial mistake when he uses the daily motion in elongation instead of the daily motion in longitude. So, if we retrace the path we just followed, but instead of using the 14;41° of the daily longitude for 6 Oct. we use the 13;41° Origanus gives for the daily elongation, we get a final time for proper observation, after three iterations, of 13 h 22 m after noon. Limiting the iterations to two, as in Longomotanus' previous example, only slightly changes the result. This shows that the 11 h given by Longomontanus is not a consequence of the conceptual error of using the daily motion in elongation instead of daily motion in longitude, but instead a product of a gross computational error. As I said in note 32, there are reasons to think that Longomontanus found this time not by going through the computational method he described, but instead

34 All the following reductions are calculated assuming a difference in terrestrial longitude of 2° between Frankfurt an der Oder and Copenhagen. This is the modern value for that distance, and as I said before, it seems to fit much better with the reduced values that Longomontanus gives.

35 For the interpolation, I first interpolated between two close longitudes in the *horoscopus* column for latitude 54°. Then I did the same for the *horoscopus* for latitude 57°. The I used those two values to interpolate for latitude 55;43°.

36 Same method as in note 35.

via the observational method described in the next section of this paper. In fact, such an error is completely within the margin of that method, as I will explain later.

So, up to now we know the following data: a) the time *x* for a given date when the Moon shows no parallax in longitude as seen from Copenhagen; b) the true distance in longitude between the Moon and Aldebaran for time *x*. Both a) and b) can be calculated from information readily available in tables. So, Longomontanus tells us, an observer at a place of unknown terrestrial longitude can calculate, using that information and with an additional lunar observation, the distance in terrestrial longitude from Copenhagen. All he has to do is observe the distance between the Moon and Aldebaran at his local time *x*. Because his local terrestrial longitude is different from Copenhagen's, then he will be looking at the Moon at a different absolute time than that calculated in a). So, the Moon will have moved from the position assumed in b). Because the tables provide the motion in longitude for the given date, then it is possible to calculate the elapsed interval between time *x* at Copenhagen, and time *x* at the place of unknown terrestrial longitude. As I said, this is a variation of the meridian method previously described by Gerard. In our case, Longomontanus assumes that at the second location, the observed distance in longitude between the Moon and Aldebaran is 10°, 2;54° more than the distance calculated for Copenhagen.[37] Given the lunar motion for that day of 14;41°, this means that between both times an interval of 4 h 45 m elapsed. Because 1 h is equal to 15° in terrestrial longitude, then we have that the second location is 71° 15' to the west of Copenhagen.

As it was usual with these methods that relied on the lunar distances to reference stars, one of the problems was that they needed very accurate lunar observations in order to give reasonable results. An error of just 5' in the observed distance amounts to a mean error[38] of 2° 17' in the resulting terrestrial longitude. Also, the method Longomontanus uses needs several iterations to determine the appropriate moment of observation in order to be below that value. But these are all practical problems which could, ideally, be managed. The fundamental problem with the method is the following: the ephemerides used for calculating the moment when the Moon showed no parallax in longitude – i.e., time *x* – are ephemerides composed for Copenhagen or, as in our case, reduced to the terrestrial longitude of Copenhagen. This means that while time *x* is the local time in Copenhagen when the Moon shows no longitudinal parallax, it is not the appropriate local time for other locations with other terrestrial longitudes. So, when the observer at the second location observes the Moon at his local time *x*, he will not see it free of the effect of parallax. Thus, the difference in the calculated and observed distances to the reference star will be a product of both the difference in terrestrial longitude between the two places and the effect of parallax in longitude in the second observation.

37 Longomontanus, *Astronomia Danica*, 195.
38 The true error depends on the corresponding daily lunar motion.

In order to avoid that problem, one possibility is to have a way of determining the appropriate moment of observation without having to consult any table. In the next section we will look at how Longomontanus explains how to do this in his *De maculis*.

Lunar spots and horns and the determination of terrestrial longitudes

As it was indicated above, Longomontanus' method needs for the lunar observation to be void of parallax effect in longitude. While the method he proposes ignores that the lunar observation made from the place of unknown terrestrial longitude will be affected by parallax, in this section Longomontanus provides a supplementary method to determine the appropriate moment for observation that does not make use of tables.

As I pointed out above, the situation with no parallax in longitude can be described in several ways: a) the moment when the altitude circle of the Moon intersects the ecliptic at a right angle, b) the moment when the Moon is 90° away from the ecliptic's rising and setting points. The first one is the way Ptolemy describes it, and the second one the way Longomontanus does. But it can also be described in a third way c) when the plane determined by the observer, his zenith and the center of the Moon coincide with the plane determined by the center of the Earth, the ecliptic pole, and the center of the Moon. This last situation only takes place when a) or b) – which are equivalent – take place, and viceversa.[39] Longomontanus refers to this configuration as that when the Moon shows the observer its *erecta dispositio*,[40] its erect or upright position. By this he means that the observable features on the near side of the Moon are positioned with respect to the observer's horizon in the same manner as they would be to an observer on the ecliptic pole (Refer to Illustration 1). So, if the observer knows beforehand how the lunar spots look in the upright position, then he only has to wait until they look that way for him, and he will know that the Moon is in the correct configuration to determine its position with respect to the reference star. However, this is not always simple. An easier method is to observe the Moon when it is on a phase, showing its horns – the days before and after new Moon – , and to wait until the moment when the line determined by the tips of the horns is perpendicular to the observer's horizon. Because the Moon is roughly in the same plane as the ecliptic, then the Sun's rays will be cast on it from a direction parallel to the ecliptic. Therefore, the line determined by the tips of the horns will always be perpendicular to the ecliptic. This means that if that line is perpendicular to the observer's horizon, then he is looking at the Moon as if he was on the ecliptic pole, i.e., on the center of the Earth.

39 Again, see the Appendix for a proof of these relations.
40 Longomontanus, *Astronomia Danica*, 194.

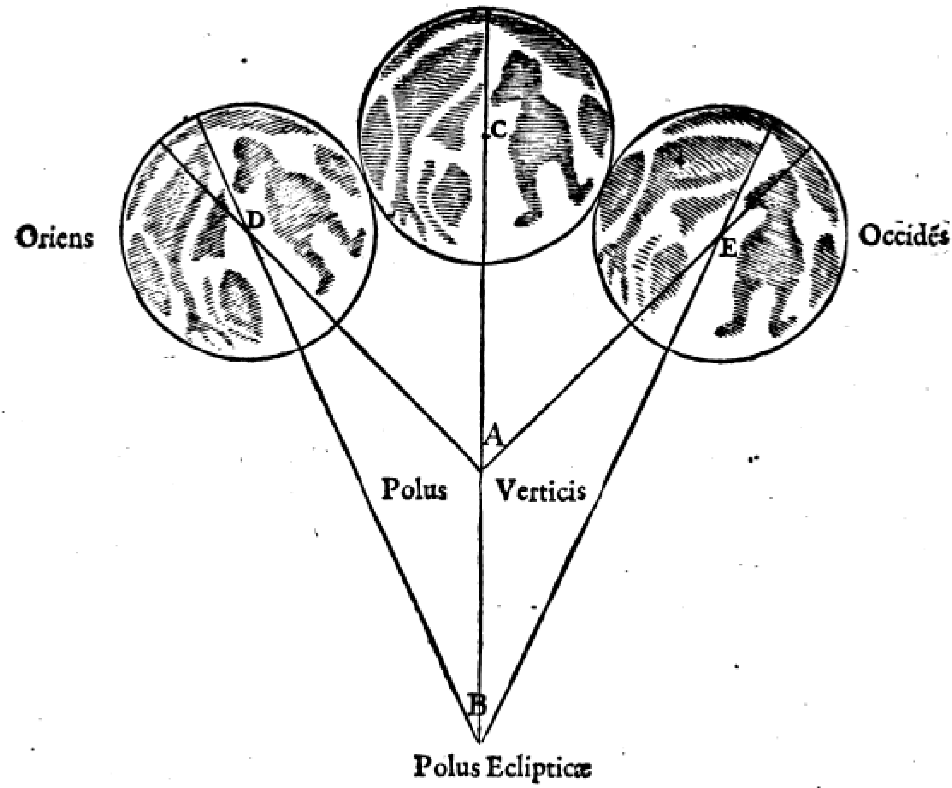

Illustration 1. Longomontanus' illustration for explaining description c) of the correct configuration. Point A is the observer's zenith, and B is the ecliptic pole. When the Moon is in the correct position C, then the plane determined by the center of the Earth, the center of the Moon, the observer's zenith, and the ecliptic pole, will coincide. In every other position (D and E, for example) the lunar disposition will be different as seen from A and B.

So, if an observer determines via tables the time when this configuration takes place for a place of known terrestrial longitude, and he then determines it via this observational method for the place of unknown terrestrial longitude he is at, then he has the two appropriate moments to determine the lunar distance to the reference star. By using the lunar daily motion, he can then calculate the amount of time between both positions, and therefore the difference in terrestrial longitude. As I said, while Longomontanus himself explains this alternative way of determining the appropriate moment of observation, he does not use it to avoid the problems his previous method has.

The method, however, has a fatal practical flaw, which is that it is extremely difficult even today – and certainly impossible in Longomontanus' times – to determine the precise moment when the line determined by the horns is intersecting the horizon at a right angle. The inclination of that line changes fairly slowly and an error of just 5' in the lunar position will result in errors of more than 2° in the final terrestrial longitude of the place. Assuming the mean lunar speed, this means an error of just 9 min. for the time of observation. The same problem holds, of course, for the version of the method that relies on the disposition of the lunar spots.

Conclusion

In this paper I have analyzed the method proposed by Longomontanus to determine the terrestrial longitude of a given location. The method is one more attempt to solve the this problem via the use of lunar observations, particularly of determinations of angular distances to reference stars. While the method has some basic theoretical and flaws, Longomontanus provides alternative observational paths to determine the correct moments when the Moon should be observed in order to determine its relative position to the chosen star.

It is not clear to what extent was Longomontanus aware of the problems in his method. Although he surely knew that his "lunar horns" method could only give the time for the observation in an approximate manner, he does not indicate to what extent the imprecision inherent in the method would limit the astronomer's ability to obtain a useful time. Even more, in the examples he gives us he does not use the "lunar horns" method, simply assuming that the time which he calculated, for a place of known terrestrial longitude, would also be useful to observe a Moon with no parallax in longitude in a place of unknown terrestrial longitude. This is the most unexpected problem of all, because it is so obvious. Longomontanus, nevertheless, seems to have missed this crucial error, and thought his method to be sound.

Despite all its problems, this section from *Astronomia Danica* is nevertheless not void of importance. In it, Longomontanus pays attention to the features on the Moon as having a significant role to play. In this, he shows openness to the new fields that were rapidly developing, particularly the Galilean application of the telescope to astronomical observations. Although he does not list the telescope as an astronomical instrument in the section he devotes to them,[41] Longomontanus explicitly refers to the *Sidereus Nuntius* in his discussion of the nature of the lunar features,[42] in particular Galileo's hypothesis that the Moon is surrounded by a thick layer of dense vapors, and that this explains why we cannot see large spots reaching the edge of the visible side of the lunar face: the lunar vapors prevent the light from the spots located in those regions to reach

41 See Longomontanus, *Astronomia Danica*, 118-122.
42 *Ibid.*, 192.

us.[43] In this he is with Galileo and *cum praestantissimo Kepplero*. So, despite all the mathematical or practical problems this section has, it reveals an astronomer who is still well within the Ptolemaic tradition of circular, uniform motions, but that at the same time has studied, and accepted, the most striking results of modern telescopic astronomy. As another example we can point his description of the part of the lunar surface visible to us as being comprised of "rough [regions], and earth-like plains".[44] He is no doubt referring to Galileo's lengthy description in *Sidereus Nuntius* of the lunar terrain, and his comparisons to different geographical features extant on Earth. This suggests that he had embraced the anti-Aristotelian side in the debate about the nature of the Moon, and of the celestial bodies in general.

43 *Ibid.*, 192 and Galilei, *Sidereus Nuntius*, 52-53.
44 "[…] in scabros, terreisque similes campos […]" (Longomontanus, *Astronomia Danica*, 192).

Appendix. Demonstration that when the lunar altitude circle is orthogonal to the ecliptic the apparent and true lunar latitudes will be the same. It is also demonstrated that in that situation, and only in that situation, the Moon is 90° away from the rising and setting points of the ecliptic.

Refer to Figure 2. The Earth is the sphere with center C. The solid circle is the celestial equator, with its pole P. The dashed circle is the ecliptic, with its pole E. The solid circle with center on C, and a radius CE and CL will necessarily be orthogonal to the ecliptic, because its plane is determined by the center of the Earth, the pole of the ecliptic, and a point on the ecliptic itself. As all the points in the celestial sphere, the ecliptic pole E will make one revolution around P – the small, dotted circle – every day. This means that the circle orthogonal to the ecliptic plane will sweep the entire celestial sphere once a day.

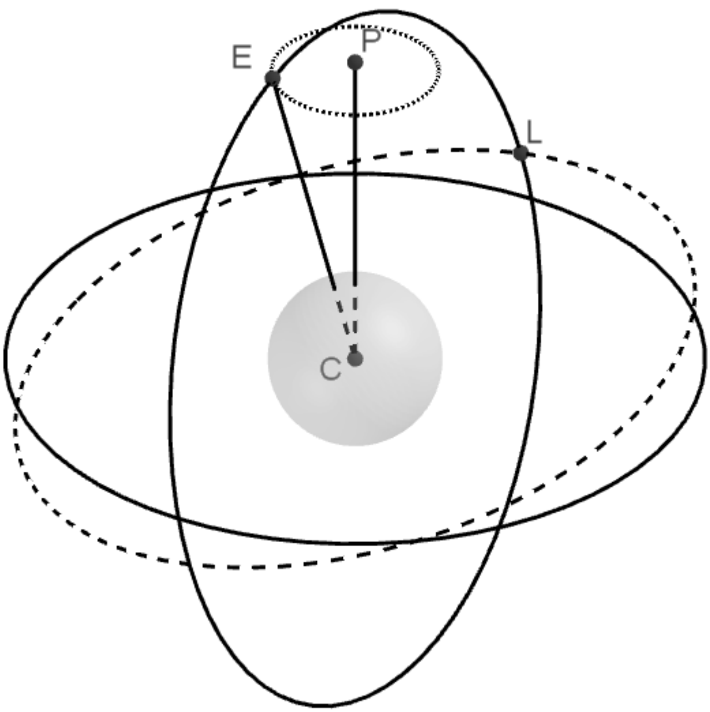

Figure 2. Diagram for the demonstration that a great circle concentric with the Earth, that passes through the ecliptic pole, sweeps the entire celestial sphere once a day.

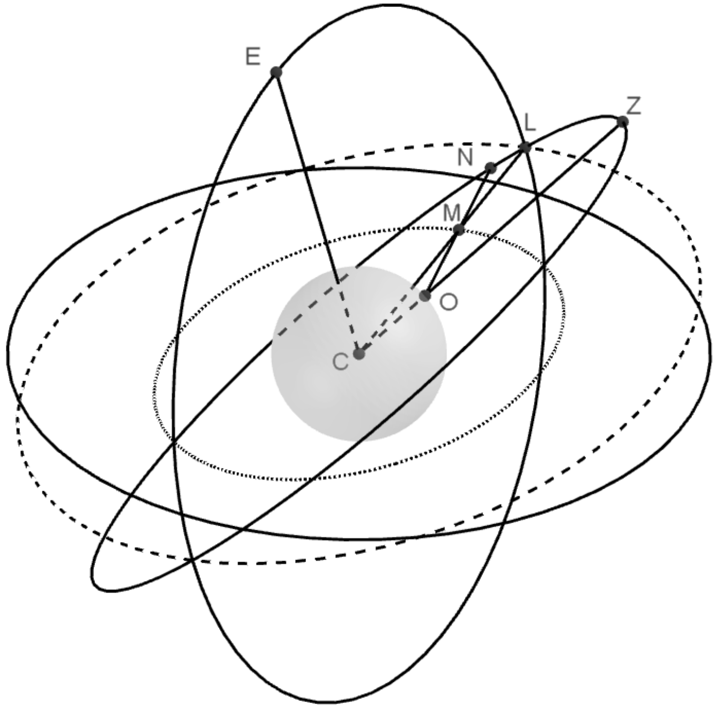

Figure 3. Diagram for the demonstration that an observer located anywhere on the Earth's surface will see the Moon without parallax in longitude once a day.

Refer to Figure 3. Let us add, to the previous figure, a (simplified) lunar model, the dotted circle with center C. For simplicity, we will assume that the Moon M is always on the ecliptic, i.e., the dotted circle has the same inclination to the celestial equator as the ecliptic. Now let us assume that point L is the true longitude of the Moon. This means that M is coplanar with the circle orthogonal to the ecliptic. The plane of this circle is thus determined by points ECM.

Then, we have the observer O at some position of the surface of the Earth, with a zenith at point Z. For him, the apparent position of the Moon will be N. As it can be seen, the Moon shows parallax both in longitude and in latitude. For our problem, we are only interested in the parallax in longitude. It is clear that to have no parallax in longitude we need for line OMN to be on the plane of the circle orthogonal to the ecliptic. When will this happen? Line OMN is on the plane determined by ZCM. But we know that, since Z is a point on the celestial sphere, at some time during the day it will be coplanar with the circle orthogonal to the ecliptic. When this happens, then the planes determined by points ECM and ZCM will be coplanar, and the true and apparent longitudes of the Moon will be the same. Finally, because the circle on plane ZCM will then be orthogonal to the ecliptic, it is the case that both longitudes will be equal when the lunar altitude circle is orthogonal to the ecliptic.

Refer to Figure 4. In the diagram we only have the dashed ecliptic, the circle orthogonal to the ecliptic, and the dotted horizon. Points A and B are the rising and setting points of the ecliptic. At this moment, the zenith Z is coplanar with the circle orthogonal to the ecliptic, so the lunar true and apparent longitudes are the same, as it can be seen in the diagram. Line ZOC is necessarily perpendicular to the plane of the horizon at point C, so it is also perpendicular to line ACB. But line ZOC is on the plane orthogonal to the ecliptic. So every line in that plane that passes through C must also be perpendicular to line ACB. But line LMC is on that plane. So line LMC is perpendicular to line ACB. But at this time L indicates the apparent (as well as the true) lunar longitude. So, when the lunar altitude circle is perpendicular to the ecliptic, it is the case that the point of the ecliptic that indicates the apparent lunar longitude is 90° away from the ecliptic rising and setting points.

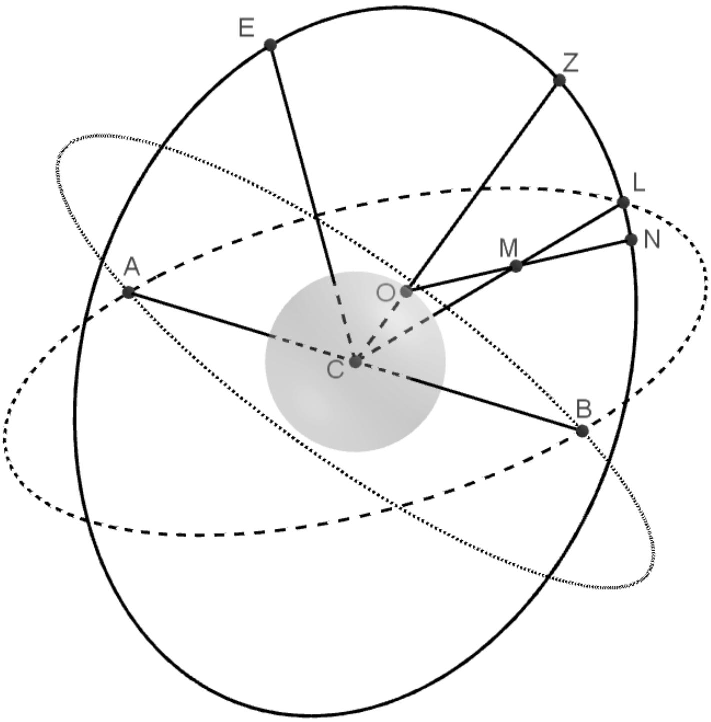

Figure 4. Diagram for the demonstration that when the Moon is seen without parallax in longitude, the point of the ecliptic corresponding to the lunar longitude is 90° away from the ecliptic rising and setting points.

Acknowledgments

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