



Descartes and the law of refraction

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Abstract

Historical research on refraction in the modern era has often focused on the question of attributing the law of refraction to Descartes, Snel and Harriott. The discovery of Ibn Sahl's work has put an end to the dispute over priority between these authors, leaving another problem largely unnoticed: most of the literature refers to this discovery without distinguishing the native form of the statements on refraction from the current conception of the law of refraction. This anachronism is problematic. Ancient texts do not contain a single statement of the law of refraction as we know it today. Descartes' contribution is in line with Maurolico's and Kepler's work on refraction. Like them, Descartes makes no mention of sines or refractive indices, nor does he formulate a physical law of refraction. Since these ideas developed gradually between the late 17th century and the 19th century, the recognition of a "law of sines" or a "law of refraction" is not an idea native to the 16th and 17th centuries. We suggest that the work of Descartes does not fit into the modern quest for the laws of nature, but rather into the framework of the theory of proportions.

Keywords

refraction, law, proportion, Descartes, Maurolico, Kepler

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Introduction

The “law of refraction” has long been regarded as the great optical discovery of the 17th century, and its invention is still one of the most hotly debated issues in the history of optics. While the many studies devoted to the law of refraction have clarified the relationships between certain contributions and authors, they have sometimes had a detrimental effect on the history of science, due to a presupposition inherent in any search for anteriority.

Historically, research into the law of refraction began with Descartes’ *Dioptrique* (1637). It then turned to Willebrord Snel’s *Libellus de refractione* (1621/6), then to Harriot’s *Optics and caustics* notebook (circa 1605). The discovery of an early expression of the law of refraction in Ibn Sahl’s *Book of burning <instruments>*, ca. 984,¹ struck a definitive blow to the search for priority among 17th-century scholars, but left the original presupposition intact. In order to demonstrate that one author preceded another, any claim to priority must assume that the scientific content of the competing authors is identical. Authors can only be aligned in the same narrative if they are pursuing the same intellectual quest and achieve comparable results.² This requires a minimum degree of alignment between the concepts and research goals.

As for the law of refraction, it is by no means certain that these conditions are met. Since this identity is assumed by those who wish to identify precursors or predecessors, it is possible that certain differences in concept or purpose – deemed minor – were downplayed to align the authors within the same series.

Ibn Sahl came close to discovering the law of refraction by studying the properties of burning lenses;³ Grosseteste approached the law of refraction while seeking to explain the rainbow;⁴ Kepler was motivated in his research by astronomical concerns, then by his discovery of the Galilean telescope;⁵ Snel, a specialist in geodesy, wanted to explain the strange lifting of objects when viewed from a certain angle through water;⁶ Descartes

¹ Roshdi Rashed, “A pioneer in anaclastics: Ibn Sahl on burning mirrors and lenses,” *Isis* 81 (1990): 464–491.

² Remember here Georges Canguilhem’s criticism of the precursors, *Études d’histoire et de philosophie des sciences* (Paris: J. Vrin, 1968), 21.

³ Rashed, “A pioneer in anaclastics.”

⁴ Carl B. Boyer, “Robert Grosseteste on the rainbow,” *Osiris* 11 (1954): 247–258.

⁵ Johannes Kepler, *Ad Vitellionem Paralipomena, quibus Astronomiae pars optica traditur...* (Frankfurt: C. Marnius, 1604); *Optics. Paralipomena to Witelo & Optical Part of Astronomy*, trans. W. H. Donahue (Santa Fe: Green Lion Press, 2000). On these aspects: Carlos Alberto Cardona, “Kepler: analogies in the search for the law of refraction,” *Studies in History and Philosophy of Science* 59 (2016): 22–35, 34.

⁶ Cornelis De Waard, “Le manuscrit perdu de Snellius sur la réfraction,” *Janus* 39 (1935): 51–73; Johan Adriaan Vollgraff, “Snellius’ notes on the reflection and refraction of rays,” *Osiris* 1 (1936): 718–725; See in particular the emphasis on motivations by Klaus Hentschel, “Das

studied refraction in connection with the anaclastic problem,⁷ etc. Why should we assume that authors who do not have the same mathematical skills and do not pursue the same scientific goals would have exactly the same understanding of the “law of refraction”?

All those involved in this search for priority – setting aside their disagreements over attribution – have jointly defended the same view: namely that Ibn Sahl, Harriot, Snel or Descartes were all seeking the same law of refraction. And yet, as soon as we delve into material from any historical source, we find that the “law of refraction” does not appear as we understand it today – that is, as a physical law involving sines and refractive indices. The aim of this article is to characterize, through a careful study of texts, the exact status of statements concerning refraction.

Since there are so many texts, a selection is needed. The question of the contribution of 15th- and 16th-century mathematics to modern science defines the general time frame. After careful consideration, I chose Descartes as focal point – even though he falls slightly outside this period – because he is one of the key figures of modernity and one of those credited with discovering (or at least understanding) the law of refraction. To provide a contrast with how refraction was previously understood, the study of Descartes’ work is preceded by a review of some earlier works on refraction: Maurolico, *Diaphana* (1554) and Kepler, *Paralipomena* (1604) and *Dioptrice* (1611).⁸

1. Maurolico, *Diaphana*, 1554

Francesco Maurolico wrote the *Diaphana* in January 1523: *Diaphaneon... libellus completus est... anno 1523... die 30 Ianuarii* (Paris, BnF, latin 7249, fol. 11v). The text was revised in Catania in 1554, before being passed on to the publisher on August 26, 1555. The text was finally edited in Naples the next century (1611). Bellè and Takahashi have recently published a critical edition of Maurolico’s *Optics*.⁹

The *Diaphana* are written *more geometrico*. They open with a set of definitions and postulates (*supposita*) whose status oscillates between rules and observational statements.

Brechungsgesetz in der Fassung von Snellius,” *Archive for History of Exact Sciences* 55 (2001): 297–344.

⁷ René Descartes, *Règles utiles et claires pour la direction de l’esprit en la recherche de la vérité*, trad. J. L. Marion and P. Costabel (La Haye: M. Nijhoff, 1977), 27.

⁸ I have omitted Willebrord Snel’s *Libellus de refractione* so as not to reopen the debate over priority between Snel and Descartes. This work is known only from an abridgment (Amsterdam, Universiteitsbibliotheek, Bc 1), which may not accurately reflect Snel’s ideas on refraction.

⁹ Francesco Maurolico, *Diaphana* (1554), *Photismi de lumine & umbra... Diaphanorum partes, seu libri tres...* (Naples: T. Longo, 1611), 31–80; *Francisci Maurolyci Optica*, ed. by Riccardo Bellè et Ken’ichi Takahashi (Pisa: F. Serra, 2017), text 95–133, commentary 83–93.

The most important of these postulates is P3: “If the angle of incidence¹⁰ is multiplied, the angle of deviation¹¹ must also be multiplied equally” *Multiplicato angulo inclinationis, angulum quoque fractionis aequaliter multiplicari*. This postulate echoes Euclid’s theory of equimultiples (*Elements* V, defs. 4–7).¹² The importance of postulate P3 lies in the fact that it leads to a false statement about refraction, which is set out in Theorem 10.

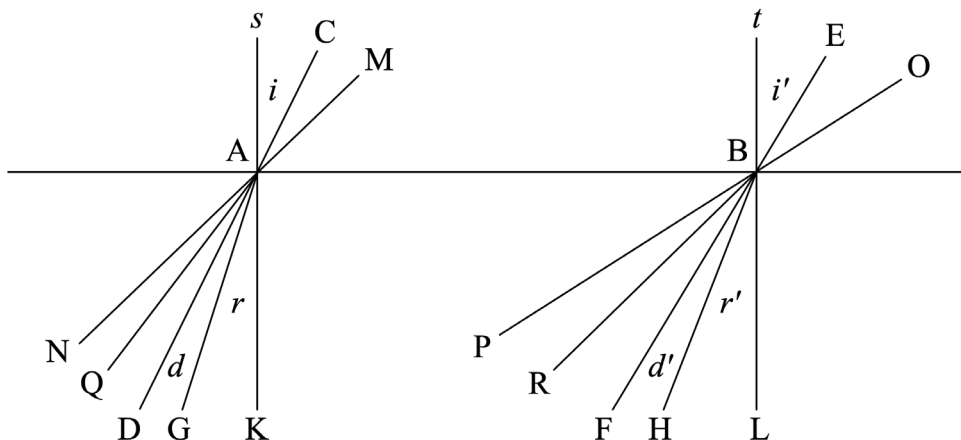


Fig. 1 – Maurolico’s diagram of refraction¹³.

Theorem 10: Angles of incidence¹⁴ are proportional to angles of deviation¹⁵
*Anguli inclinationum sunt fractionum angulis proportionales*¹⁶ (Figure 1).

Let two rays MAN and OBP fall obliquely on the plane interface AB. Theorem 10 is demonstrated using the theory of proportions (postulates P1 to P3):

[Assume that medium ABKL is denser than ABCE]

Thus incident rays are refracted towards AK and BL normals (postulate P1)

Let the ray CAD refracted along AG

¹⁰ inclinatio] lit. inclination, i.e. incidence

¹¹ fractio] deviation $d = i - r$

¹² Euclide, *Les Eléments*, trad. B. Vitrac (Paris, PUF, 1994), vol. 2: Livres V–VI, 35–47.

¹³ Inadvertently, the figure in the MS BnF, Latin 7249, fol. 2v, represents MA, OB rays with the same obliquity on the interface. I take angles such that $\angle DAK < \angle FBL$. I add the letters s t and the angles of incidence i , deviation d and refraction r for discussion.

¹⁴ inclinatio] incidence

¹⁵ fractio] deviation $d = i - r$

¹⁶ Maurolico, *Diaphana*, 35; *Maurolyci Optica*, 99.

And EBF refracted along BH

[We want to show that the angles of incidence are like the deviations]

That is $\angle DAK : \angle FBL :: \angle DAG : \angle FBH$

Take two other rays: MAN refracted towards Q, OBP towards R

$\angle DAK = k \angle NAK$ and $\angle FBL = k \angle PBL$; with any k

Therefore $\angle DAG = k \angle NAQ$, $\angle FBH = k \angle PBR$ (postulate P3)*

If $\angle NAK > \angle PBL$

Then $\angle NAQ > \angle PBR$ (postulate P2)

If $\angle NAK = \angle PBL$

Then $\angle NAQ = \angle PBR$

If $\angle NAK < \angle PBL$

Then $\angle NAQ < \angle PBR$

Thus $\angle DAK : \angle FBL :: \angle DAG : \angle FBH$.

Corollary: The angle of incidence and the angle of deviation are always in the same ratio. As we proved by experiment in the transparent sphere, it is double and $\frac{2}{3}$ superpartiens [8 : 3]

*Ergo et angulus inclinationis ad angulum suae fractionis semper unam servat rationem. Sicut experimento in crystallina sphaera probavimus, estque dupla et $\frac{2}{3}$ superpartiens.*¹⁷

Since $\angle CAs = \angle KAG = i$ and $\angle EBt = \angle LBF = i'$ (angles opposite by the vertex), Maurolico's relationship can be written:

$$i : d :: i' : d' :: 8 : 3$$

$$d = i - r$$

$$r = i - d$$

$$i : r :: i' : r' :: 8 : 5.**$$

Commentary

* Theorem 10 is based on postulate P3, which is false. If we note the angle of incidence i and the deviation $d = i - r$, postulate P3 states that if $i' = ki$, then $d' = i' - r' = k(i - r)$. Relying on the theory of equimultiples, Maurolico establishes a proportion that incorrectly describes the relationship between the angle of incidence and the angle of deviation. Fermat was one of the first to notice the error. In a letter to Mersenne dated April-May 1637, he wrote:

Maurolic Abbé de Messine, en son traité posthume *De lumine & umbrâ*,¹⁸ a soutenu que les angles qu'il appelle d'incidence, sont proportionaux à ceux qu'il nomme de refraction. Si cette

¹⁷ Maurolico, *Diaphana*, 36; *Maurolyci Optica*, 100.

¹⁸ *Diaphana* are included with *Photismi de lumine et umbra* in the 1611 edition.

proposition estoit vraye, elle suffiroit pour nous marquer les vrayes figures que doivent avoir les corps diaphanes qui produisent tant de merveilles. Mais pource qu'elle n'a pas esté bien démontrée par Maurolic, & que l'expérience mesme semble la convaincre de faux, il en est resté assez à Monsieur Descartes pour exercer son esprit [a discussion of *Dioptrique* follows].¹⁹

** Maurolico studies a crystal sphere (*crystallina sphaera*). A modern interpretation would say that the ratio of the refractive indices of air and crystal is 8 : 5, and since $n_{\text{air}} = 1$, it follows that $n_{\text{crystal}} = 1.6$. However, Maurolico stops short of characterizing the double and $\frac{2}{3}$ superpartiens ratio without using the notion of refractive index, introduced much later, without distinguishing the index of crystal and air, and without involving other refractive media.²⁰

In 1554, the very year of the *Diaphana* revision, Maurolico also wrote *De proportione sermo noster*, in which he sought to simplify the theory of proportions. His aim was to establish the notion of proportion independently of commensurability: "It was therefore necessary to find a condition common to these and those [commensurable and incommensurable magnitudes] which is proper to proportional quantities" *Cogitandum igitur fuit de aliqua conditione, quae his et illis communis esset, solisque proportionalibus propria quantitativibus* (1554, fol. 22r).²¹ At this time, Maurolico tried to transcend the difference between discrete and continuous magnitudes by means of the theory of equimultiples: the four terms of a proportion are not necessarily homogeneous with each other; it is sufficient that they are homogeneous two by two. Maurolico adhered to this theory before abandoning it in his 1567 *Compendium of the Elements* (Paris, BnF, Latin 7463).²² The same treatment of the two types of magnitude enables him to write: "This is why proportion is the equality or identity between two ratios" *unde proportio est rationum aequalitas sive identitas* (1554, fol. 21r). Maurolico thus reduces the gap between the concepts of identity and equality and, as a result, promotes the assimilation of proportion (as a comparison of qualitative ratios $a : b :: c : d$) and equality of ratios (as numerical equality of quantitative ratios $a / b = c / d$).

Maurolico's application of proportions to refraction was not an isolated gesture. The same theory is applied to mechanics. In *De momentis aequalibus*, Maurolico similarly shows that "gravities suspended at equal distances are proportional to their moments."²³

¹⁹ *Correspondance de Descartes*, ed. Charles Adam et Paul Tannery (Paris, Cerf, 1897), vol. 1, 356.

²⁰ Bellè and Takahashi, *Maurolyci Optica*, 178.

²¹ Maurolico, *De proportione sermo noster* (1554), MS: Molfetta, Biblioteca del Seminario Minore Vescovile 5–7 H 15, fol. 22r; ed. Pier D. Napolitani, *Francisci Maurolyci Opera Mathematica*, <http://www.dm.unipi.it/pages/maurolic/intro.htm>

²² Maurolico, *Compendium* (1567), MS: Paris, BnF, latin 7463, ed. Jean-Pierre Sutto, "Le compendium du 5e livre des Éléments d'Euclide de Francesco Maurolico," *Revue d'histoire des mathématiques* 6 (2000): 59–94.

²³ Sutto, "Le compendium," 66.

The confidence with which proportions were applied to various subjects is probably an effect of the immense credit it enjoyed during the Renaissance. The explanatory power of this theory has, so to speak, suspended its critical verification. Maurolico, who occasionally resorts to experimental data, *Sicut experimento... probavimus*, did not test the relationship between the angle of incidence and the angle of deviation. P3's error was therefore transmitted to Theorem 10 and its corollary.

2. Kepler, *Ad Vitellionem Paralipomena*, 1604

Kepler's *Ad Vitellionem Paralipomena* is a massive 487-page work published by Marnius in Franckfort (1604). Its modern, logical plan evades the conventions of ancient and medieval optics. Kepler first examines the physics of light (chapter 1) and the formation of images (chapter 2), revisits the theory of reflection (chapter 3) and refraction (chapter 4) and then applies these principles to vision (chapter 5). Refraction is the subject of chapter 4. After introductory considerations, Kepler formulates Propositions IV, 1 and IV, 2, dealing with incidence and density, and then presents their synthesis in Proposition IV, 3, which can be seen as a precursor to the modern law of refraction.

Proposition IV, 3: The angles of deviation²⁴ increase by greater increments²⁵ than the angles²⁶ of incidence

*Refractionum anguli crescunt maioribus rationum incrementis, quam obliquitas incidentiae (Paralipomena IV, 3).*²⁷

Kepler then writes:

The angle of deviation²⁸ is therefore composed of a part proportional to the incidence and a part proportional to the BM lines. The BM lines increase very little at first, but a lot at low incidence,²⁹ as shown by the secant table, where equal degrees correspond to larger and larger secants. Thus, part of the deviation angle is proportional to incidence, and part increases with increasing increments. Thus, the whole angle increases with larger increments

²⁴ refractionum anguli] angles of deviation, *corr.* refraction! *Donahue*

²⁵ Competing translations: "Les angles de réfraction augmentent dans des proportions plus grandes que l'obliquité de l'incidence" (Simon) or "Le rapport sous lequel croissent les angles de déviation est supérieur à celui sous lequel croissent les angles d'incidence" (Simon) lose the idea of increment, i.e. the difference between two consecutive values.

²⁶ obliquitas] lit. angle of incidence, sg.

²⁷ Kepler, *Ad Vitellionem Paralipomena*, 111; *Paralipomena to Witelo*, 124.

²⁸ angulus refractionis] angle of deviation

²⁹ in humili incidentia] with low incidence

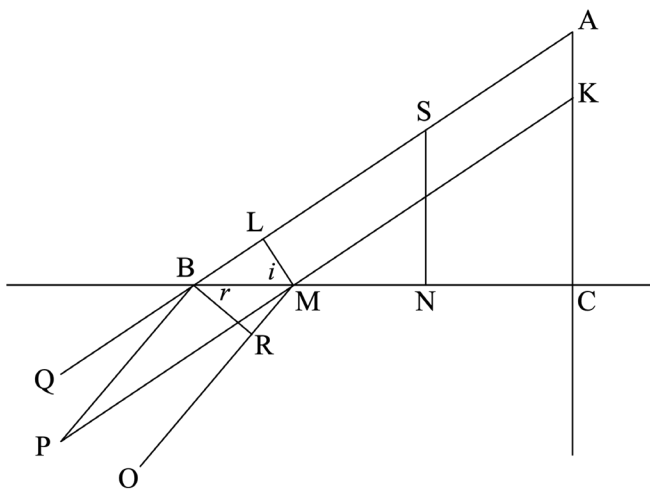


Fig. 2 – Kepler's diagram of refraction³⁰.

Componitur ergò angulus refractionis ex aliquo, quod est proportionale incidentiis, et aliquo, quod est proportionale lineis BM. At lineae BM crescunt initio parum, in humili incidentia crescunt multum, ut ostendit tabula secantium, ubi aequalibus gradibus semper maiores atque maiores secantes respondent. Ergò pars anguli refractionum proportionatur incidentiis, pars maioribus rationis incrementis crescit. Totus igitur angulus maioribus incrementis crescit. (Paralipomena IV, 3; Figure 2)³¹

This passage has been variously interpreted.

Gérard Simon's thesis that Kepler did not use the angle of incidence, as is our custom, but the complementary angle $90^\circ - i$, can be quickly dismissed.³² This interpretation is refuted by the very title of the column in the refraction table: "distance [angular] from source to vertex," *distantia radiantis à vertice*,³³ which refers to the angle of incidence. Kepler was therefore trying to express the deviation as a function of the angle of incidence.

Kepler's conjecture involves a combination of factors: the deviation $i - r$ is proportional to both the angle of incidence i and the secant of the angle of refraction $\sec r$. Several historians have attempted to mathematize this relationship:

³⁰ I adjust the figure to ensure the properties $KP \parallel AB$, $MO \parallel BP$, $LM \perp KM$ and $BR \perp MO$. I add the angle of incidence i and the angle of refraction r .

³¹ Kepler, *Ad Vitellionem Paralipomena*, 111.

³² Gérard Simon, *Kepler, rénovateur de l'optique* (Paris: Garnier, 2019), 100–101, note 28.

³³ Kepler, *Ad Vitellionem Paralipomena*, 115.

Cornelis de Waard proposed to recognize the relation $a = m_1 \gamma + m_2 \sec \gamma$, in which a notes the angle of incidence, γ the deviation, $a - \gamma$ the angle of refraction.³⁴ Applying usual notations, this relationship can be rewritten $i = k_1 (i - r) + k_2 \sec (i - r)$. Accordingly, Kepler would have imagined an additive combination of factors.

Jean Itard³⁵ refutes the idea that Kepler's idea can be written as $i - r = mi + n \sec r$ and shows that it is in fact $i = r + ki / \cos r$.

Buchdahl, Simon, Dijksterhuis and Cardona argue that the formula is of the type $i - r = ki \sec r$. Kepler would have used a multiplicative combination of factors.³⁶

Cardona also claims that Itard is mistaken: "M. J. Itard summarizes in this way the Kepler conjecture: $d = mi + n \sec(r)$, with m and n constants of proportionality (1957: 65). I intend to show that the composition that Kepler has in mind is not additive as Itard suggests, but that is of the form: $d = ki \sec(r)$, with k a constant of proportionality."³⁷

However, in his article on refraction, Itard cites the additive formula only to refute it, and also defends the idea that Kepler had in mind a multiplicative composition of factors. The confusion arises from the fact that Itard expresses this relationship by isolating i . All you have to do is pass r into the other member of the equation to recover the multiplicative form: if $i - r = ki / \cos r$ and $\sec a = 1 / \cos a$, then $i - r = ki / \cos r$ is equivalent to $i - r = ki \sec r$. So, despite criticism, Cardona uses the same formula as Itard.

In the end, only two interpretations remain: De Waard's (1933) and Itard's (1959), expressed in different notations. These interpretations can be decided by comparing the angular values appearing in Kepler's refraction table.³⁸

Let us note i the angle of incidence (*distantia radiantis à vertice*), i' the complementary angle, r the angle of refraction, $d = i - r$ the angle of deviation (*angulus refractionis*) and k some constant. The use of secants³⁹ is justified by the fact that $BM = RM \sec r$.

³⁴ *Correspondance du P. Marin Mersenne*, ed. Cornelis de Waard et al. (Paris: Beauchesne, 1933), vol. 1, 357, 432.

³⁵ Jean Itard, "Les lois de la réfraction de la lumière chez Kepler," *Revue d'histoire des sciences* 10 (1957): 59–68, 65.

³⁶ Gerd Buchdahl, "Methodological aspects of Kepler's theory of refraction," *Studies in History and Philosophy of Science* 3 (1972): 265–298, 291; Simon, *Kepler*, 122; Fokko Jan Dijksterhuis, *Lenses and Waves. Christiaan Huygens and the Mathematical Science of Optics in the Seventeenth Century* (Dordrecht: Kluwer, 2004), 121; Cardona, "Kepler," 34.

³⁷ Cardona, "Kepler," 32.

³⁸ Kepler, *Ad Vitellionem Paralipomena*, 115. Remember that Kepler's goal was to correct Witelo's inaccurate refraction tables.

³⁹ The use of secants is again recommended in IV, 6: "It must therefore be known that the secants of the angles of incidence which are established at the surface of the densest medium contribute to the measurement of refractions" *sciendum igitur, eorum angulorum incidentiae secantes concurrere ad mensuram refractionum, qui constituuntur ad superficiem in medio densiori*, Kepler, *Ad Vitellionem Paralipomena*, 113.

We know that:

$$\angle LMB = i$$

$$\angle MBR = r$$

Here is the air-water refraction table given in Proposition IV, 8:

<i>Distance from source to zenith in rare medium i</i>	<i>Deviation proportional to incidences d_1</i>	<i>Complement to secants d_2</i>	<i>Total deviation demonstrated $d = d_1 + d_2$</i>	<i>Results of Witelo experiments d_w</i>	<i>Difference $d - d_w$</i>
10°	2;25	0;01	2;26	2;15	– 0;11
20°	4;49	0;10	4;59	4;30	– 0;29
30°	7;14	0;35	7;49	7;30	– 0;19
40°	9;39 ⁴⁰	1;23	11;02	11;00	– 0;02
50°	12;04	2;42	14;46	15;00	+ 0;14
60°	14;28	4;40	19;08	19;30	+ 0;22
70°	16;52	7;19	24;11	24;30	+ 0;19
80°	19;17	10;43	30;00	30;00	0;00
90°	21;43	14;47	36;30		

KEPLER'S REFRACTION TABLE.

These angular values can be tested by calculating the ki and $\sec r$ terms separately. The calculated values and the Kepler values are adjusted by the least-squares method. The best fit is given by $k = 0.2411$ (column 2).

Similarly, the values of the refraction angles are calculated using the law of sines, in order to minimize the resulting total deviation. If we apply the least-squares method again, we see that the values show significant deviations at 80° and 90° of incidence (difficult to measure). If we adjust the other values, we find the best fit for $n_1 / n_2 = 3 : 4$, a common value for air-water refraction. We deduce from the table $d = i - r = ki \sec r$ (column 4), then $d_2 = d - d_1$ which is the value of the “complement” produced by the secants (column 3).

If we now compare columns 2, 3 and 4 from the two tables, we can see that the reconstructed values correspond fairly well to Kepler's values, except at high incidences. Kepler seems to have used a multiplicative combination of factors, before obtaining the complement by subtraction – hence the table's additive appearance, which deceived De Waard. We can now decide between De Waard's and Itard's interpretations: there is little doubt that Kepler used Itard's formula.

⁴⁰ 9;39] 3;39! misprint.

Incidence relative to zenith i	First part of the deviation $d_1 = ki$	Complement to secants $d_2 = d - d_1$	Kepler total deviation $d = ki \sec r d_1$	Actual total deviation d	Refraction for ratio 3:4 r
10°	2;25	0;01	2;26	2;31	7;29
20°	4;49	0;10	4;59	5;08	14;52
30°	7;14	0;34	7;48	7;59	22;01
40°	9;39	1;22	11;00	11;11	28;49
50°	12;03	2;40	14;44	14;56	35;04
60°	14;28	4;34	19;02	19;30	40;30
70°	16;53	6;55	23;47	25;11	44;49
80°	19;17	9;19	28;37	32;23	47;37
90°	21;42	11;06	32;48	41;25	48;35

RECONSTRUCTED REFRACTION TABLE.

A comparison of columns 4 and 5 shows that Kepler's values for total deviations deviate progressively from the actual values calculated from the law of refraction. The deviation is greater than 1° at 70°, 4° at 80° and 8° at 90°. It follows that, even neglecting calculation errors, the deviation values proposed by Kepler are not those actually observed. The formula $i - r = ki \sec r$ does not correspond to reality.

From proposition IV, 3, Kepler derived the *refractio simplex*, or Kepler's law of small angles, described in *Dioptrice* (1611).

Science historians have attempted to elucidate the historical significance of this proposition by returning to the inaccuracy of refraction angles. For Lohne, the absence of experimental research on the subject would have led Kepler to record erroneous angular values, from which it was impossible for him to discover the law of refraction.⁴¹ It can be argued that Kepler's corrected values were sufficient to formulate a relationship between angle of incidence and angle of refraction, since they diverge from reality at large angles. Heffer considers that Kepler was in possession of an adequate representation of the refraction problem.⁴² He would have been the first to conceive of the existence of a relationship between certain geometric lines and the index of refraction, but would have missed the right relationship. This interpretation also raises a difficulty: as the concept of refractive index appeared at the beginning of the 19th century, it cannot be used to explain a text from 1604.

One thing is certain: Kepler used trigonometry and was intellectually prepared to express the statement on refraction in sine form. In fact, there is direct evidence of this

⁴¹ Johannes Lohne, "Zur Geschichte des Brechungsgesetzes," *Sudhoffs Archiv* 47 (1963): 152–172.

⁴² Albert Heffer, "Data-driven induction in scientific discovery: a critical assessment based on Kepler's discoveries," E. Weber et al., eds., *Logic, Reasoning and Rationality* (Dordrecht: Springer, 2014), 59–76, 70.

possibility. In a passage from the *Paralipomena*, Kepler calculates the position of the stars on the horizon using a sine ratio identical to that on which the modern law of refraction is based:

$$\sin \angle FCE : \sin \angle FE :: \sin \angle CFE : \sin \angle CE$$

*Ut sinus CFE ad sinum CE sic sinus FCE ad sinum FE*⁴³

The obstacle that prevented Kepler from formulating the law of refraction was not intellectual. He probably went astray in thinking that refraction had to be explained by a combination of factors.

Because of its independence of view from ancient optics, *Ad Vitellionem Paralipomena* is Kepler's great book on optics. His later research in optics was more dependent on his exchanges with Galileo.⁴⁴

3. Kepler, *Dioptrice*, 1611

Kepler was promptly informed of Galileo's discoveries with the telescope. Galileo sent him a copy of *Sidereus Nuncius* (1610) as soon as it was published. Immediately realizing that the study of refraction could improve the telescope's lenses, Kepler set to work hard, and *Dioptrice* was published less than a year later by Franck in Augsburg (1611).

Dioptrice comes in an *more geometrico* altered form. The propositions do refer to definitions, axioms and postulates, but these are not grouped together in a preamble, as tradition dictates. Kepler distributes them according to need, so that the text includes a definition, two axioms, two problems, six axioms, four propositions, a sensible axiom, two propositions, then an axiom again, etc.⁴⁵ This organization reflects both the influence of the classical model and the relaxation of the Euclidean norm. The most important for the theoretical study of refraction are axioms 7 and 8 and proposition 12:

Axiom 7: Up to an incidence⁴⁶ of 30°, deviations⁴⁷ in the glass are roughly proportional to the inclinations

*Crystalli refractiones usque ad tricesimum inclinationis, sunt ad sensum proportionales inclinationibus.*⁴⁸

⁴³ Kepler, *Ad Vitellionem Paralipomena*, 131.

⁴⁴ M. Walter Burke-Gaffney, *Kepler and the Jesuits* (Milwaukee: Bruce Publishing Co, 1944), 60–70; Simon, *Kepler*, 163–164.

⁴⁵ Kepler, *Dioptrice* (Augsburg: D. Franck, 1611), 1–9.

⁴⁶ inclinatio] incidence

⁴⁷ refractio] deviation

⁴⁸ Kepler, *Dioptrice*, 3.

For $i \leq 30^\circ$, $d = k i$.

Axiom 8: Up to this limit, the deviation in the glass is about one third of the [angle] of incidence in air

*Angulus refractionis in Crystallo est usque ad dictum terminum, quàm proximè tertia pars inclinationis in aëre.*⁴⁹

For $i \leq 30^\circ$, $d = \frac{1}{3} i$ ($r = \frac{2}{3} i$).

Proposition 12: Carefully evaluated, the deviations are not proportional to the incidences in the air

*Refractiones exquisitè pensitatae non sunt proportionales inclinationibus in aëre.*⁵⁰

Kepler justifies this assertion by noting the inconsistency: according to axiom 8, an incidence of 90° should correspond to a deviation of 30° ($d = \frac{1}{3} i$), yet experience gives a deviation of 48° (i.e. a refraction angle of 42°).

The main purpose of Proposition 12 is to draw attention on the approximate nature of axioms 7 and 8. These axioms constitute Kepler's law for small angles, or *refractio simplex*. The simplification $i \leq 30^\circ$, $r = k i$ will be much used in later literature. It is worth noting that, whatever Kepler's solution, the three fundamental statements of *Dioptrice* do not seek a law, but a proportion linking incidences and refractions.

If we now turn to Kepler's use of proportion, we see that his conception of proportion in general is not strictly mathematical, as was the case with Maurolico. According to Kepler, proportional relationships do not describe the physical world; they carry a physical and cosmological meaning in themselves. This conception is clearly apparent in the epigraph to the *Harmonices Mundi* of 1619, where Kepler quotes a passage from Proclus' *Commentary on Euclid Book I*:

Mathematics also makes contributions of the very greatest value to physical science [i.e. the study of Nature]. It reveals the orderliness of the ratios according to which the Universe is constructed and the proportion that binds things together in the cosmos."⁵¹

Kepler was not interested in the strictly mathematical significance of proportions, but in the fact that they enable us to unravel the mysteries of the universe, being themselves the principles on which the world is built. This concept underpins all Keplerian astronomy,

⁴⁹ Ibid., 4.

⁵⁰ Ibid., 4.

⁵¹ Kepler, *Harmonice mundi libri V* (Linz: G. Tamach, 1619), 1; Proclus, *A Commentary in the First Book of Euclid's Elements*, trad. Glenn R. Morrow (Princeton: Princeton University Press, 1970), 19; Judith V. Field, *Kepler's Geometrical Cosmology* (London: Bloomsbury, 1988), 99.

in particular the idea that planetary orbs follow the series of regular polyhedra inscribed in the sphere.⁵²

4. *Descartes, La Dioptrique, 1637*

Another milestone in refraction theory can be found in the *Dioptrique*, one of the three treatises illustrating Descartes' *Discours de la Méthode*, published by Maire in Leiden (1637). In the First Discourse, the behavior of light is described by a mechanical analogy: light would behave like a ball thrown against a wall (reflection) or against a piece of water (refraction). As has been noted, Descartes' constant use of analogies raises more difficulties than it resolves, as it vitiates the meaning of physical theory.⁵³ The mechanical analogies of the ball and the stick introduce an inconsistency between the thesis of instantaneous propagation and that of temporal propagation of light, while at the same time it leads to the counterintuitive thesis that the speed of light is greater in dense media than in sparse media, an idea that many of Descartes' predecessors and contemporaries disputed.

The purpose of the Second Discourse is to determine the measure of refractions.⁵⁴ In it we find the formulation that resembles the modern law of refraction. Descartes starts from the mechanical analogy of the First Discourse: "Si une balle recontre la surface d'un corps liquide, elle se détourne et change son cours en y entrant..." and retraces the same figure in a refined form. He then shows that "la force ou la facilité" with which the ray moves in the second medium relative to the first is like the line AH to GI, and then generalizes, showing that this relationship is the same, whatever the incident angle:

Comme, par exemple, s'il passe un rayon dans l'air d'A, vers B, qui rencontrant au point B la superficie du verre CBR, se détourne vers I dans ce verre; & qu'il en viene un autre de K vers B, qui se détourne vers L; & un autre de P vers R, qui se détourne vers S; il doit y avoir mesme proportion entre les lignes KM & LN, ou PQ & ST, qu'entre AH & IG... Si bien que vous voyés maintenant en quelle sorte se doivent mesurer les refractions; & encores que pour determiner leur quantité, en tant qu'elle dépend de la Nature particuliere des cors où elles se font, il soit besoin d'en venir à l'experience, on ne laisse pas de le pouvoir faire assés certainement & aysement, depuis qu'elles sont ainsi toutes reduites sous une mesme mesure⁵⁵ (Figure 3).

⁵² Bruce Stephenson, *The Music of the Heavens. Kepler's Harmonic Astronomy* (Princeton, NJ: Princeton University Press, 1994), 4.

⁵³ Peter Galison, "Descartes's comparisons: from the visible to the invisible," *Isis* 75 (1984): 311–326; Gideon Manning, "Analogy and falsification in Descartes' physics," *Studies in History and Philosophy of Science* 43 (2012): 402–411.

⁵⁴ René Descartes, *Dioptrique* (Leiden: I. Maire, 1637), 13–25.

⁵⁵ *Ibid.*, 21–22.

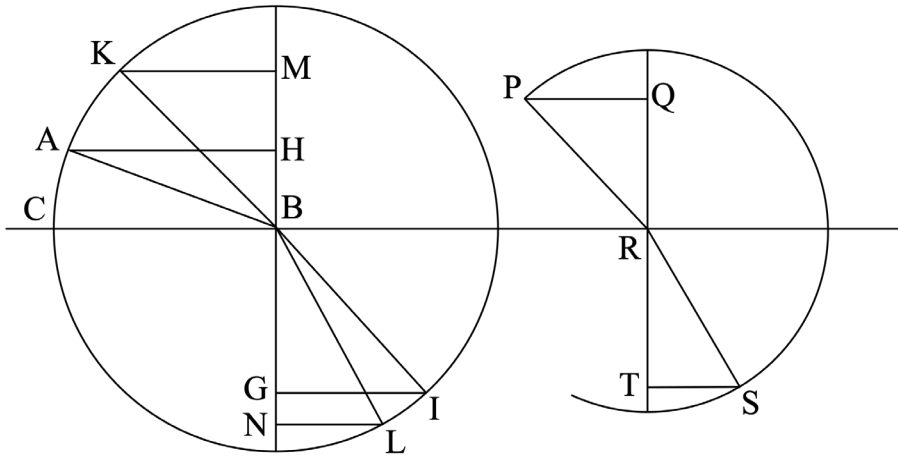


Fig. 3 – Descartes' diagram of refraction.

By considering several rays AB , KB falling on the point of incidence B of the dioptré CBR , Descartes finds a relationship between the projections of the points of the circle on the normal:

$$AH : GI :: KM : LN :: PQ : ST (= \text{constant})$$

This relationship does not involve sines, a fact that few authors have noticed apart from Eduard Dijksterhuis: "It is typical of the geometrical orientation still dominating mathematical thought that neither Snellius nor Descartes formulated the law as a constant ratio of sines."⁵⁶

Consequently, when we invoke the "law of sines" in *Dioptrique*, it is a translation; and this translation needs to be made explicit.

If we equate the $KACLI$ circle with the trigonometric circle:

$$\begin{aligned} AH &= \sin \angle ABH = \sin i_1 \\ GI &= \sin \angle IBG = \sin r_1 \\ KM &= \sin \angle KBM = \sin i_2 \\ LN &= \sin \angle LBN = \sin r_2 \end{aligned}$$

⁵⁶ Eduard Jan Dijksterhuis, *The Mechanization of the World Picture* (Oxford: Clarendon Press, 1961), 390.

Proportion $KM : LN :: AH : GI$ can then be rewritten as $\sin i_1 / \sin r_1 = \sin i_2 / \sin r_2$ (= constant). The equivalence of the two formulations can be invoked. However, the rewriting exercise underlines the existence of a translation process and, in doing so, reveals a gap between Descartes' original formulation: "Il doit y avoir mesme proportion entre les lignes KM & LN , ou PQ & ST , qu'entre AH & IG ,"⁵⁷ and the law of refraction expressed in terms of sines.

Since historians of science and Cartesians have attributed the law of sines to Descartes, and have multiplied evidence, we can only shed full light on the subject once these witnesses have been analyzed. For the time being, we must suspend all judgment. Let us say from the outset that the problem is not about attribution, but rather about the nature of knowledge and the administration of proof. Ten documents mark this history, and none of them suggests that Descartes formulated a true law of refraction.

Descartes and Mydorge, 1626

Among Father Mersenne's papers is a treatise by the mathematician Claude Mydorge, *L'inclination et la refraction de quelconque rayon* (Paris, Bibliothèque nationale de France, naf 5176, fols. 15r-21r). Since the text was published in Mersenne's correspondence,⁵⁸ it has often been mistakenly assumed to be a letter from Mydorge to Mersenne, whereas it is in fact a treatise of its own.

Mydorge's treatise consists of two parts. Propositions 1 and 2, deal with refraction in general and date from the collaboration between Mydorge and Descartes (1626). Propositions 3 to 5, devoted to the study of anaclastic curves (hyperbola, ellipse), belong to Mydorge alone (1626/9).⁵⁹ Proposition 1 states the fundamental relationship:

⁵⁷ Ibid., 21–22.

⁵⁸ *Correspondance de Mersenne*, vol. 1, 404–415.

⁵⁹ Although this treatise bears no direct trace of Descartes' intervention, the argument of an interrupted collaboration between Mydorge and Descartes suggests a two-phase redaction: propositions 1–2 by Mydorge and Descartes; propositions 3–5 by Mydorge alone, Robert Goulding, "The harvest of optics: Descartes, Mydorge, and their paths to a theory of refraction," *Annals of Science* 79 (2022): 164–214, 178. This division respects historical attributions. In the *Examen du livre des recreations mathematiques*, Mydorge admits Descartes' contribution: "Ce noble subject de refractions dont la nature n'a point esté cogneuë ny aux anciens, ny aux modernes Philosophes & Mathematiciens iusques à present, doit maintenant l'honneur de sa decouverte à un brave Gentilhomme de nos amis," Claude Mydorge, *Examen du livre des recreations mathematiques* (Paris: R. Boutonné, 1630), 139. At the same time Ferrier credits Mydorge with the discovery of anaclastic solutions: "Monsieur Mydorge propose un moyen qu'il a de tracer la ligne necessaire pour brûler à un point qu'il determinera à tout verre donné..." (October 26, 1629), *Correspondance de Descartes*, vol. 1, 68.

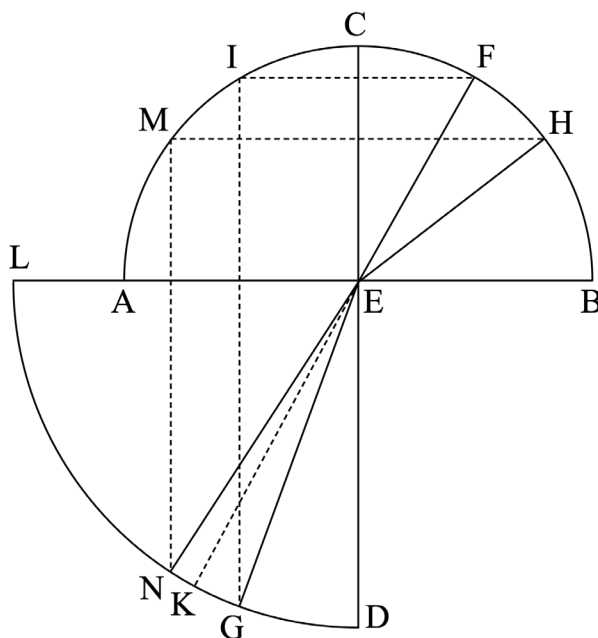


Fig. 4 – Mydorge's diagram of refraction⁶⁰.

Proposition 1: “L’inclination et la refraction de quelconque rayon sur la surface de quelque corps diaphane que l’on voudra estant donnée, donner la refraction de tous les autres rayons inclinés sur la mesme surface”⁶¹ (Figure 4).

AEG is the plane interface separating the rare medium ACB from the dense medium ADB; CED is the normal at the point of incidence E. The incident ray FE is refracted along EG. In that time, “l’angle de l’inclination” represents the angle of incidence $\angle CEF$, “l’angle de la refraction” the deviation $\angle KEG$, “l’angle rompu,” the angle of refraction $\angle DEG$. The angle of refraction of another incident ray HE is found by geometric construction:

We describe circle ACB with center E and aperture EB

Thus circle ACB cuts EF at point F

And circle ACB cuts EH at H

We draw FI // AB

⁶⁰ I reproduce the figure on the sticky note in manuscript naf 5176, fol. 13r.

⁶¹ Claude Mydorge, *L’inclination et la refraction de quelconque rayon* (1626), MS: Paris, BnF, naf 5176, fol. 15r, *Correspondance de Mersenne*, vol. 1, 404–415.

We draw IG // CE

Thus IG meets EG at point G

We describe circle LDG with center E and aperture EG

Thus LDG cuts the extended diameter CE in D

And LDG cuts the extended diameter BE in L

We draw HM // BA

We draw MN // CD

Thus MN intersects circle LDG at point N

Thus HE is refracted into EN, forming an angle \angle DEN.

Mydorge and Descartes conclude: “On trouvera de la mesme sorte la refraction de tous les autres rayons.”⁶² This geometric construction involves neither law nor sine. The authors speak in terms of “la raison de la droite GE à la droite EF” or “la raison de la refraction”⁶³ – raison being the ancient term for the ratio involved in a proportion.

Descartes, 1628

A reference to the law of refraction has also been recognized in *Règles pour la direction de l'esprit (Regulae ad directionem ingenii)*. This uncompleted work was written in the winter of 1628, before Descartes' final departure for the United Provinces in the spring of 1629. The manuscript, identified by the letter F in the inventory drawn up in Stockholm on February 14, 1650, following Descartes' death, was passed on to Clerselier. The text was published for the first time in 1701.

Rule VIII (according to which the mind must never examine the consequences of a difficulty without having elucidated the difficulty itself) is illustrated by an example taken from optics:

If, in a word,⁶⁴ someone who has only studied Mathematics asks⁶⁵ about this line, which in Dioptrics is called Anaclastic, i.e. the one where parallel rays refract, so that after refraction they all intersect at a single point, he will no doubt easily notice, following the fifth and sixth rules, that the determination of this line depends on the proportion observed between the angles of refraction and the angles of incidence; but because he will not be able to pursue the search, since it does concern Mathesis, but Physics, he will be forced to stop at the threshold ...

Si, verbi gratia, quaerat aliquis solius Mathematicae studiosus lineam illam, quam in Dioptrica

⁶² Paris, BnF, naf 5176, fol. 15r, *Correspondance de Mersenne*, vol. 1, 405.

⁶³ *Ibid.*, 407.

⁶⁴ Verbi gratia] lit. for example

⁶⁵ quaerat] asks, i.e. wonders about the nature of

*Anaclasticam vocant, in qua scilicet radij paralleli ita refringantur, ut omnes post refractionem se in uno puncto intersecant, facile quidem animadvertet, juxta Regulas Sam & 6am, hujus lineae determinationem pendere a proportione, quam servant anguli refractionis ad angulos incidentiae; sed quia hujus indagandae non erit capax, cum non ad Mathesin pertineat, sed ad Physicam, hic sistere cogetur in limine.*⁶⁶

Because of the philosophical issues it raises, this passage has given rise to a number of interpretations – some of which are properly apologetic.

Costabel writes about the proportion between the angles of refraction and incidence: “Il s’agit évidemment de la loi des sinus.”⁶⁷ This interpretation goes far beyond the text. Descartes states the proportion observed between the angles of refraction and incidence, without employing any trigonometric concept. Rule VIII therefore does not state a “law of sines.” Descartes is merely formulating a proportion between geometric magnitudes.

Costabel adds: “Descartes entend déjà ici par angle de réfraction un angle compté comme l’angle d’incidence par rapport à la normale à la surface de séparation des deux milieux en présence” and: “Descartes considère les angles d’incidence et de réfraction comme nous les entendons aujourd’hui.”⁶⁸ This interpretation is also questionable, as Descartes employs three different sets of definitions of angles in *Dioptrique*, see below: *Definition of angles*.

Anaclastic is often taken as a marker of Descartes’ advanced research in optics. Only recently, it has been written that “Rule 8 is arguably the most important document detailing Descartes’s reflections on how to discover the law of refraction and the shape of the anaclastic lens.”⁶⁹ This judgment ignores all the work of Descartes’ predecessors on the subject.⁷⁰ Furthermore, in this passage, Descartes does not state the law of refraction in ex-

⁶⁶ refringantur] refringuntur ; admavertet] admavertat ; etiam] enim *MS Cambridge*. Descartes, *Règles utiles et claires pour la direction de l’esprit en la recherche de la vérité*, trad. J.L. Marion, notes mathématiques de P. Costabel (La Haye: M. Nijhoff, 1977), 27; Descartes, *Regulae ad directionem ingenii, An Early Manuscript Version*, ed. Richard Serjeantson and Michael Edwards (Oxford: Oxford University Press, 2023), 196–197.

⁶⁷ Descartes, *Règles*, 198.

⁶⁸ *Ibid.*, 196, 198.

⁶⁹ Tarek R. Dika, *Descartes’s Method. The Formation of the Subject of Science* (Oxford: Oxford University Press, 2023), 276.

⁷⁰ Around 984, Ibn Sahl showed that the hyperbolic plano-convex lens is anaclastic for a light source at infinity, and that the hyperbolic bi-convex lens is anaclastic for a source at finite distance (Rashed, *Geometry and Dioptrics in Classical Islam* (London: al-Furqān, 2005), 108–109, 138–141). In 1554, Maurolico stated the anaclastic problem in clear terms (*Diaphana*, 80). In 1604, Kepler took up the anaclastic problem and wondered whether the solution was a parabola, a hyperbola or an ellipsis (*Ad Vitellionem Paralipomena* V, 19, 194). In 1611, Kepler wrote that the hyperbolic lens is anaclastic, justifying this with arguments, but no mathematical demonstration

PLICIT terms; he does not say how the anaclastic line is deduced from the law of refraction; he mentions “this line” in the singular, whereas for a bundle of parallel rays, two solutions already exist (hyperbola, ellipse). Ultimately, the passage is purely allusive, and the formulation so vague as to be compatible with any state of knowledge of the anaclastic problem prior to the 17th century.

Beeckman, 1628/9

Descartes’ work on refraction is also mentioned in Beeckman’s Journal, or *Loci communes*, the manuscript of which is preserved in Middelburg, ZB Bibliotheek van Zeeland, MS 6471, parts I-IV. Beeckman’s Journal was edited by Cornelis de Waard in four volumes between 1931 and 1953.

The refraction research is attached to the date October 8, 1628, and the solutions to the anaclastic problem to February 1, 1629.

Beeckman writes in his Journal: “[Descartes], I say, came to see me in Dordrecht on October 8, 1628” *Is, inquam, die 8^o mensis octobris 1628, ad me visendum venit Dordrechtum*. Refraction is one of the subjects covered. Beeckman begins by reporting the measurement of refraction angles on a right prism, a passage similar to Proposition 2 of Mydorge’s treatise, *L’inclination et la refraction de quelconque rayon* (1626).

He then presents a figure derived from proposition 1 of the same treatise:

One angle of refraction being known, we deduce the others from the sine of the angles because, it is said, CD is to IF as AB is to HG. We consider that there is water under ST, that the rays are AEG and CEF, and that they seem to undergo the same [cause] as the equal arms of a balance, at the ends of which weights are suspended: the one in the water is lighter, and raises the arm. We then look for numerous points such as R, and around them we trace a hyperbola through which all the incident parallel rays converge at a single point. This would be the best glass for making telescopes... He says it worked perfectly

Cognito uno angulo refractionis, deducit inde reliquos secundum angulorum sinus: ut enim, inquit AB ad HG, ita CD ad IF. Considerat enim sub ST esse aquam, radios esse AEG, CEF, idemque videntur ipsi pati quod brachia aequalia bilancis quo finibus appensa sunt pondera, quorum id quod in aqua est levius est et brachium attollit. Tandem quaerit multa puncta qualia est R ac circa

(Kepler, *Dioptrice*, 21–22). In the same year, his pupil Ambrosius Rhodius disseminated the terms *anaclasticis* and *anaclastica* (Ambrosius Rhodius, *Optica Ambrosii Rhodii, Kembergensis* (Wittenberg: L. Seuberlich, 1611), B4v. In 1623, Mersenne took up the problem again, highlighting the difficulties of creating an anaclastic lens (Marin Mersenne, *Quaestiones celeberrimae in Genesim* (Paris: S. Cramoisy, 1623), 163). In 1626/9, Mydorge demonstrated mathematically that the hyperbola and ellipse are anaclastic curves for a source at infinity, i.e. parallel beam (Mydorge, *L’inclination et la refraction de quelconque rayon, Correspondance de Mersenne*, 404–415).

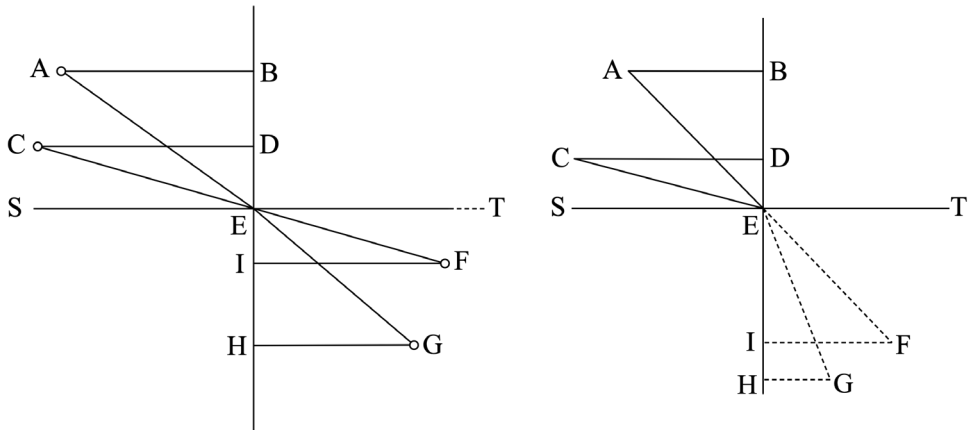


Fig. SAB – Beekman's diagram of refraction⁷¹.

illa hyperbolam ducit per quam omnes radii paralleli incidentes concurrunt in unum punctum. Quod vitrum optimum foret ad faciendos tubos oculares... Ipse dicit sibi perfecte successisse⁷² (Fig. SAB).

This time, the relationship is written:

$$CD : IF :: AB : HG (= \text{constant})$$

The important sentence is obviously: “One angle of refraction being known, we deduce the others from the sine of the angles.” It corresponds to the problem posed in Proposition 1 of Mydorge's treatise.⁷³

Beekman would later return to the anaclastic problem. On February 1, 1629, he found a demonstration for the hyperbola, which Descartes had not found⁷⁴. Descartes thanked him for it: “Tu mérites de grands éloges pour l'hyperbole que tu m'as apprise.”⁷⁵

⁷¹ On the left, I reproduce the figure from MS 6471, fol. 333v. The straight line ST is extended by a third on the side of T. I would like to thank Mrs. Sanne van Splunter, curator of the ZB Bibliotheek van Zeeland, who provided me with a photograph of the Beekman manuscript. The figure on the right is the one that appears in *Correspondance de Descartes*, vol. 10, 336.

⁷² *Correspondance de Descartes*, vol. 10, 336; *Journal tenu par Isaac Beekman de 1604 à 1634*, ed. by C. de Waard (Den Haag: M. Nijhoff, 1939), vol. 1, 97.

⁷³ Mydorge, Paris, BnF, naf 5176, fol. 15r; *Correspondance de Mersenne*, vol. 1, 404–415, 404.

⁷⁴ *Journal tenu par Isaac Beekman*, vol. 3, 109.

⁷⁵ *Correspondance de Descartes*, vol. 1, 163.

Beeckman will further reproduce Descartes' alternative proofs,⁷⁶ which will be adapted and reproduced in *Dioptrique's* Discourse VIII – without calling on the sines.⁷⁷

These fragments of Beeckman's Journal have often been used as evidence that Descartes had discovered the law of refraction and the solution to the anaclastic problem before 1628. Even quite recently, it has been written: "By the time Descartes met with Beeckman in October 1628, he not only already had the sine law in hand, but had also already deduced the shape of the anaclastic in the case of the ellipse."⁷⁸

Here are the limits of this judgment:

1. As most commentators access this text through Adam-Tannery's reference edition *Oeuvres de Descartes*, no one seems to have noticed that Beeckman's text and figure are wrong. Comparison of the handwritten figure (Figure 5A) and the edited figure (Figure 5B) shows that the edited figure has been rectified: the rays AEG and CEF in the manuscript have been intentionally broken at point E and deflected to reveal refractions EF and EG; the origin-points of the incident rays and the end-points of the refracted rays, which have any position in the manuscript, have been placed on a trigonometric circle ACSTFG to justify the relationship of sines. These properties – absent from Beeckman's Journal – are taken from *Dioptrique*, which dates from ten years later. This anachronistic editorial choice forces continuity between the two texts. On the handwritten figure, the segments AB and HG (CD and IF) are derived from the rectilinear rays AEG (CEF) passing through the point of incidence E. Therefore, the sine relationship only appears on the codicologically false figure edited by Adam and Tannery.
2. As the relation of sines, quoted in Beeckman's Journal, does not appear in proposition 1 of the 1626 treatise by Mydorge, nor in later Cartesian texts, we do not know to whom the sines should be attributed: is it Descartes who uses it before later abandoning it, but for what reason would he have discarded a clear trigonometric notion? Was it Beeckman who interpreted in trigonometric terms the figure presented to him by Descartes?
3. The dating is debatable. Beeckman wrote his notes from day to day, before compiling them in his Journal. Several passages show that he did not keep to the chronology of his notes. For example, Gassendi's visit to Beeckman in 1629 precedes a note dated 1627.⁷⁹ The anachronisms are numerous⁸⁰ and appear even in the passage considered here. The date "8 oktober 1628" to which the passage refers is the date of the meeting between Descartes and Beeckman. Later in the text,

⁷⁶ Ibid., 338–340; *Journal tenu par Isaac Beeckman*, vol. 4, 135–136.

⁷⁷ Descartes, *Dioptrique*, 94, 103.

⁷⁸ Dika, *Descartes's Method*, 312.

⁷⁹ *Journal tenu par Isaac Beeckman*, vol. 1, xxxi note 1.

⁸⁰ Ibid., vol. 1, xxviii, xxxviii, 354; 1942: 173; vol. 2, 173; vol. 3, i–ii, 335, 360, 443, etc.

Beeckman writes: “By meditating until the age of 33, he seems to have found more perfectly than the others what he was looking for” *Usque ad 33 aetatis suae annum meditando, eam rem quam quaesivit, perfectius quàm reliqui invenisse videtur*.⁸¹ Descartes’ 33rd birthday provides the *terminus a quo* March 31, 1629. Consequently, the passage on refraction was written well after Descartes’ visit on October 8, 1628. In any case, the content has been altered and reworked.

Mersenne, 1629

Mersenne, who corresponded extensively with Descartes, discusses refraction in his major work *Harmonie universelle*, published by Sébastien Cramoisy in 1636. The printing of this large work was so delayed that it is preferable to refer the text to the ecclesiastical approval of 1629.

Proposition I, 29 reads: “Mais puis que l’un des plus excellens esprits de ce siecle a trouvé la vraye proportion des refractions aux inclinations, ie veux icy en remarquer l’analogie... Il y a mesme raison du sinus GO, de l’angle d’incidence GNO, au sinus PE, de l’angle d’incidence PNE, que du sinus IR, de l’angle rompu NIR, au sinus KQ, de l’angle rompu NKQ, ce qu’il démontrera dans sa Dioptrique, quand il luy plaira.”⁸²

This passage raises two distinct problems: the identification of “l’un des plus excellens esprits de ce siecle” and the form in which the relation of refraction is expressed. Let us leave aside the problem of identification (Descartes) and concentrate on the form of the relation. Mersenne states the “vraye proportion des refractions” $\sin \angle GNO : \sin \angle PNE :: \sin \angle RNI : \sin \angle QNK$ without explanation. Knowing that sines do not appear in Descartes’ texts on refraction, one wonders: who owns the expression of sines in the *Harmonie universelle*? Is Mersenne reproducing a statement by Descartes, but why would Descartes then delete the sines? Or is Mersenne translating the proportion of refractions into his own language? Remember that, like Kepler, the Minime was familiar with trigonometric methods. As early as 1623, he calculated the distance from the center of the solar orb to the center of the world in this way: *Proportio IA ad IK est sinus anguli IAK ad totum sinum... Proportio verò GI ad IK sicut sinus totius ad sinum anguli HGI*.⁸³

Descartes to Ferrier, 1629

There is no autograph of this letter, written from Amsterdam on November 13, 1629. Descartes suggests to Ferrier that he draw the template for the lens according to the proportion set out in Proposition 1 of the treatise on refraction: “Que si vous en voulez tracer une [ligne] qui brûle à une plus grande ou moindre distance, par exemple, à la distance de

⁸¹ Ibid., vol. 3, 95.

⁸² Marin Mersenne, *Harmonie universelle* (Paris: S. Cramoisy, 1636), 65.

⁸³ Mersenne, *Quaestiones in Genesim*, 170.

DE, cherchez EF, qui soit à DE comme BC est à AB... c'est à dire que si vous avez une fois la proportion qui est entre les lignes AB & BC, par le moyen de vostre quadran, elle vous servira pour tous les verres d'un mesme diaphane."⁸⁴ Note again that Descartes expresses a proportion $EF : DE :: BC : AB$, and not the sine relation.

Descartes to Golius, February 2, 1632

The fifth document showing similarities with the modern law of refraction appears in the letter Descartes sent to Golius in Amsterdam on February 2, 1632. It has been claimed that, in this letter, Descartes aimed to inform his correspondent of “sa découverte de la loi des réfractions.”⁸⁵ But Descartes determines the angle of refraction by the proportion $hi : ef :: ab : cd$ (= constant). The text makes no mention of sines. The absence of sines in the *Dioptrique* may well be the result of a pedagogical choice on Descartes' part.⁸⁶ But what can be said about their absence in the letter to Golius? Why would Descartes, a promoter of mathematical clarity, make a clear trigonometric concept disappear in his exchanges with a mathematician?

Descartes to Mersenne, June 1632

This letter has no date. The editors place it in June 1632. In this letter, which deals with the parallels between acoustics and optics, Descartes argues that the refraction of light and sound do not follow the same rules. It is in this context that he recalls: “Pour la façon de mesurer les refractions de la lumiere, j'établis une comparaison entre les sinus des angles d'incidence et des angles de réfraction” *instituo comparationem inter sinus angulorum incidentiae & angulorum refractorum*.⁸⁷ This is Descartes' only document employing the concept of sine. The statement refers to the existence of a proportion between the sines of the angles of incidence and refraction. This letter was included in the reference edition without being checked against the autograph. The text appears only in Clerselier's edition,⁸⁸ whose editorial methods will be examined below.

⁸⁴ *Correspondance de Descartes*, vol. 1, 64–65.

⁸⁵ Gaston Milhaud, “Descartes et la loi des sinus,” *Revue générale des sciences pures et appliquées* 18 (1907): 223–228, 224.

⁸⁶ Descartes writes in the Avertissement to the *Géométrie*: “Iusques icy i'ay tasché de me rendre intelligible a tout le monde, mais pour ce traité ie crains qu'il ne pourra estre lu que par ceux, qui sçavent desia ce qui est dans les livres de Geometrie” (*Géométrie*, 296). Descartes' rhetorical and pedagogical intentions are analyzed in the literature (John A. Schuster, *Descartes and the Scientific Revolution: 1618–1634*, Doctoral Dissertation (Princeton: Princeton University, 1977), 327; Bruce S. Eastwood, “Descartes on refraction,” *Isis* 75 (1984): 481–502, 498).

⁸⁷ *Correspondance de Descartes*, vol. 1, 255.

⁸⁸ *Lettres de M. Descartes*, ed. Claude Clerselier, 3 vols. (Paris: Angot, 1657–1659), 344–345.

Descartes, *Géométrie* 1637

Refraction, the central subject of *Dioptrique*, reappears in *Géométrie* published alongside *Discours de la méthode* (1637). In Book II, Descartes studies the anaclastic problem, which consists in determining the shape of a lens that focuses rays on a point.⁸⁹

Given that sines are used by Mydorge to study anaclastic curves in the 1626/9 treatise *L'inclination et la réfraction de quelconque rayon*, it would have been logical for this relationship to appear in the 1637 *Géométrie*. Unexpectedly, Descartes' ovals are described only in terms of the proportion stated in proposition 1 of the 1626 treatise, with no mention of sines. The reason for this is probably that the demonstrations of propositions 3 to 5 of the treatise were written after Descartes left for the United Provinces in the spring of 1629.

In his *Géométrie*, Descartes explains the construction of anaclastic curves using the same proportion of refractions as in his *Dioptrique*: “Je tire la ligne droite 56, qui coupe l'autre au point 6, en sorte que A6 soyt moindre que A5 selon telle *proportion* donnée qu'on veut...” and similarly: “en sorte que FL soyt à LG comme A5 à A6, c'est à dire qu'elles ayent la *proportion*, qui mesure les refractions”; “suivant ce qui a esté dit en la *Dioptrique*, elles peuvent toutes estre mesurées par la *proportion* qui est entre les lignes A5 & A6.”⁹⁰

Géométrie makes no mention of a sine ratio that would justify the study of anaclastic curves.

Descartes, *Météores* 1637

A final mention of the relationship of refractions can be found in the *Météores*, the last treatise illustrating Descartes' *Discours de la méthode*, published by Ian Maire (1637). Discourse VIII of *Météores* deals with the rainbow. Here is the central passage, which explains the calculation of refracted angles in the raindrop: “Je connois de ce qui a esté dit en la *Dioptrique*, qu'AE ou HF, & CI, ont entre elles la *proportion* par laquelle la refraction de l'eau se mesure. De façon que si HF contient 8000 parties, telles qu'AB en contient 10000, CI en contiendra environ de 5984...”⁹¹

The proportion of refractions serves as the basis for applying a rule of three. From the relationship $CI : HF :: 187 : 250$, Descartes derives $CI = 187 HF / 250 = 5984$. Two points deserve our attention. Firstly, the proportion of refractions has a calculatory function: “Et calculant en mesme façon tous les autres rayons...” Secondly, contrary to a modern reading that would associate the numerator and denominator each with a refractive medium – say 187 with air, 250 with water – in *Météores*, as in all Descartes' texts, the entire ratio is attached to a single medium: water. Descartes did not use refractive indices.

⁸⁹ Descartes, *Géométrie*, 352.

⁹⁰ *Ibid.*, 353, 356 and 357, italics mine.

⁹¹ Descartes, *Météores*, 262–263.

Discussion

This overview of sources shows that, beyond their particularities, the Cartesian texts written between 1626 and 1637 do not deal with the law of refraction as we understand it today (i.e. sines and refractive indices inserted into a statement with the scope of a physical law), but are oriented by the theory of proportions.

We can now develop a critical analysis of each of these points.

1. Sine relationship. – Descartes is credited with a “law of refraction” based on the sines of the angles of incidence and refraction. Let us leave aside the status of “law,” which we shall come back to, and call the residue: “relation of sines.”

Many philosophers and historians of science credit Descartes with the discovery of the sine relationship⁹² sometimes a relation of cosines⁹³ or a relation of cosecants, which was then transformed into the relation of sines we know today.⁹⁴

A synthesis of the ten documents analyzed above reveals a clear division between two conceptions of refraction.

Seven texts mention a proportion of the type $KM : LN :: AH : GI$, in which the sines (cosines, cosecants) of the angles of incidence and refraction are not involved at all. This conclusion also applies to Proposition 1 of the 1626 treatise *L'inclination et la réfraction de quelconque rayon*; to the 1628 *Règles pour la direction de l'esprit*, to the 1629 letter to Ferrier, to the 1632 letter to Golius, and to the 1637 *Dioptrique*, anaclastic section of the *Géométrie*, and *Météores*.

Only three texts mention sines. None of these is an autograph by Descartes: two are written by his correspondents (Beeckman 1628 and Mersenne 1629); the third is an un-

⁹² For volumetric reasons, I cannot give a complete list of references; See for instance: Montucla, *Histoire des mathématiques*, 1802: 248; Arago, “Descartes,” 1855: 303; Kramer, “Descartes und das Brechungsgesetz des Lichtes,” 1882: 258; Milhaud, “Descartes et la loi des sinus,” 1907: 223; Descartes, *Discours de la méthode*, ed. Gilson 1925: 270; Mach, *Principles of Physical Optics*, 1925: 35; Crombie, *Robert Grosseteste*, 1953: 282; Rodis-Lewis, *L'oeuvre de Descartes*, 1971: 85; Ziggelaar, *Le physicien Ignace Gaston Pardies*, 1971: 174; Mahoney, “Descartes,” *Dictionary of Scientific Biography*, 1971: 59; Costabel, *Démarches originales de Descartes savant*, 1982: 57; Shea, *The Magic of Numbers and Motion*, 1991: 155; Jullien, *Descartes, La Géométrie*, 1996: 106; Horstmann, “Hobbes und das Sinusgesetz der Refraktion,” 2000: 417; Heefer, “Descartes’ Theory of Explanation in Natural Philosophy,” 2002: 11; Antoine-Mahut, *Encyclopedia of Early Modern Philosophy*, 2022: 436; Bellis, *Encyclopedia of Early Modern Philosophy*, 2022: 1116; McDonough, *A Miracle Creed*, 2022: 11; Dika, *Descartes’s Method*, 2023: 279, 283, etc.

⁹³ A. Mark Smith, “Descartes’s theory of light and refraction,” *Transactions of the American Philosophical Society* 77 (1987): 1–92, 64.

⁹⁴ John A. Schuster, “Descartes opticien,” Stephen Gaukroger et al., eds., *Descartes’ Natural Philosophy* (London; New York: Routledge, 2000), 258–312, 265; Schuster, *Descartes–Agonistes: Physico-mathematics, Method and Corpuscular-Mechanism 1618–33* (Dordrecht: Springer, 2013), 186.

dated letter from Descartes to Mersenne, known only from Clerselier's edition. A study of Clerselier's editorial methods shows that he intervened massively in edited texts, to the point of fabricating them from scratch to vindicate Descartes against his opponents. Here is a letter from Descartes to Mersenne dated December 18, 1629, in autograph form (Paris, Bibliothèque de l'Institut, MS 2001, fol. 2r) and as edited by Clerselier.⁹⁵

Descartes

Il ne reste plus que quelque chose touchant la vitesse du mouvemant que vous dite que le Sr Becman vous a mande, ce qui viendra mieus en respondant a vostre derniere

Clerselier

[I 617] Il ne reste plus que quelque chose, touchant la vitesse du mouvemant que vous dites que Monsieur Beecman vous a mandé, mais cela viendra mieus en répondant à vostre derniere. *Pour la proportion de vitesse selon laquelle descendent les pois, ie vous en ay écrit ce que i'en sçavois en la précédente, saltem in vacuo, sed in aëre, ce que vous a mandé Monsieur Beecman est veritable, pourvû que vous suposiez que plus le pois descend viste, plus l'air luy resiste; car si cela est, dequoy ie ne suis pas encore du tout assuré, enfin il arrivera que l'air empeschera iustement autant, que la pesanteur adjouteroit de vitesse au mouvement in vacuo, & cela estant, le mouvement demeurera tou-jours égal; mais cela ne se peut [I 618] déterminer que de la pensée; car en pratique il ne le faut pas esperer. Et pour vos experiences, qu'un pois, descendant de cinquante piés, employe autant de tems à parcourir les vingt-cinq derniers que les premiers, salvâ pace, ie ne me sçaurois persuader qu'elles soient iustes: car in vacuo, ie trouve qu'il ne mettra que le tiers du tems à parcourir les vingt-cing derniers, & ie ne puis croire que l'empeschement de l'air soit si notable, qu'il rende cette difference-là imperceptible. Je suis, &c.*

⁹⁵ Left: Paris, Bibliothèque de l'Institut, MS 2001, fol. 2r. Right: *Lettres de M. Descartes*, ed. C. Clerselier, Paris: Angot, 1657–1667, vol. I: 617–618, vol. II: 482–483.

in qua 1° petis quare dicam celeritatem imprimi ut unum primo momento a gravitate et ut duo 2° momento etc. Respondeo, salva pace me non ita intellexisse sed celeritatem imprimi ut unum primo momento a gravitate et rursus ut unum 2° momento ab eadem gravitate etc. unum autem 1i momenti et unum 2i faciunt duo, et unum 3ii faciunt tria atque ita crescit in arithmetica proportione.

Nam si supponeremus exempli causa plumbi massam deorsum delabentem vi gravitatis et postquam per primum momentum labi coepit,

Deum tollere omnem gravitatem ex plumbo, adeo ut postea massa plumbi non sit magis gravis quam si esset aer aut pluma, perget nihilominus descendere ista massa saltem in vacuo, quoniam et coepit moveri et nulla potest afferri ratio cur desinat,⁹⁶

sed non augebitur eius celeritas atqui si post aliquod tempus restituat Deus gravitatem isti plumbo ad momentum temporis tantum quo elapso rursus eandem subtrahat nunquid secundo isto momento vis gravitatis tantundem impellet plumbum quantum fecerat 1° momento ac proinde duplicabitur celeritas motus? Idem de reliquis momentis dicere licet.

[II 482] Mais permettez-moy de vous répondre que ie ne l'ay pas ainsi entendu; mais bien ay-je dit, que la force de la vitesse s'imprime comme un au premier moment par la pesanteur, & derechef comme un au second moment, & ainsi de suite comme un au troisiemes, &c. mais l'un du premier moment, & l'un du second, font deux momens, & c'est ainsi que croist la proportion Arithmetique.

Ainsi par exemple, si nous supposons qu'une masse de plomb par la force de sa pesanteur tombe en bas, & que si-tost qu'au premier moment elle a commencé à descendre, Dieu luy oste toute sa pesanteur, en sorte que cette masse de plomb ne soit pas plus pesante que l'air, ou qu'une plume. Cette masse ne laissera pas pour cela de continuer à descendre dans le vuide, puis qu'elle a une fois commencé à se mouvoir, & qu'on ne sçauroit donner de raison pourquoy elle dust cesser; (*car il faut se ressouvenir que ie suppose que ce qui a une fois commencé à se mouvoir dans le vuide, continue tousiours à se mouvoir, & i'espere le demonstrier en Physique*) mais sa vitesse ne sera point augmentée: Et si quelque temps après, Dieu vient à rendre pour un moment à cette masse de plomb, [II 483] toute la pesanteur qu'elle avoit auparavant, & qu'un moment après il la luy oste derechef, ne voit-on pas qu'en ce second moment la force de la pesanteur doit pousser autant cette masse de plomb, qu'elle avoit fait au premier moment; & par consequent son mouvement sera augmenté de moitié, & le mesme arrivera aux troisiemes, quatrieme & cinquiemes momens, &c.

⁹⁶ desinat] *add. marg.* a Oportet meminisse nos supponere illud quod semel motū est in vacuo semper moveri et in meo tractatu demonstrare conabor.

dou il suit certainement que si vous laissiés tomber une boule *in spatio plane vacuo* de 50 pieds de hault que de quelle matiere quelle pust estre elle employeroit tousiours iustemant trois fois autant de tans aus 25 premiers pieds quelle feroit aus 25 derniers.

Mais dedans laer cest tout autre chose et pour revenir au Sr Beecman, encore que ce quil vous a mande soit fauls a scavoir quil y ait un lieu auquel un poids qui descend estant parvenu pour-suit par après tousiours desgale vitesse, toutefois il est vray qu'après certain espace cette vitesse s'augmente de si peu quelle peut estre iugée insensible. Et ie men vois vous expliquer ce quil veult dire, car nous en avons autrefois parle ensemble.

[II 483] D'où il suit certainement, que si vous laissez tomber une boule *dans une espace tout à fait vuide* de cinquante pieds de haut, que de *quelque* matiere qu'elle puisse estre, elle employeroit tousiours iustemant trois fois *autant* de temps à *descendre les vingt-cinq* premiers pieds, que les *vingt-cinq* derniers:

mais dans l'air c'est toute autre chose. Et pour revenir au Sieur N. encore que ce qu'il vous a mandé soit faux, à sçavoir qu'il y ait un lieu auquel *une pierre* qui descend estant parvenuë, elle *descendra* par après d'égale vitesse. Toutefois il est vray que *cette augmentation* de vitesse est si *petite après certain espace*, qu'elle peut estre *estimée* insensible, & ie m'en vais vous expliquer ce qu'il *faut* dire, car nous en avons autrefois parlé ensemble, & ie vous diray *après enquoy il se méprend*.

This excerpt shows that the changes introduced by Clerselier range from simple rephrasing to the dismantling of the text, including significant omissions and additions, and biased rewritings that attribute to Descartes ideas belonging to others.⁹⁷

In such conditions, only Descartes's autographs are reliable, and knowledge of the law of refraction cannot be attributed to Descartes based solely on a single text published by Clerselier ten years after Descartes's death.

This leaves the accounts of Beeckman and Mersenne, two authors who used trigonometry in their own work, and to whom we must leave the authorship of the texts they wrote, in which they set out, perhaps in their own words, what they understood of Descartes' research. In any case, *Descartes never publicly claimed the law of sines*.

The conclusion that Descartes did not formulate a sine-based law of refraction rests primarily on the absence of any autograph documents whatsoever attesting to the use of sines in his writings.

⁹⁷ Giulia Belgioioso, "Un faux de Clerselier," *Bulletin cartésien* 33 (2005): iv (unpaginated); Giulia Belgioioso, Vincent Carraud, *Des éditions du XVIIe siècle aux éditions modernes* (Lecce: Centro dipartimentale di studi su Descartes "Ettore Lojacono," 2024), <http://www.cartesius.net>.

2. *Definition of the angles.* – There is another proof that Descartes was indifferent to writing the refraction relation in sine form, but this evidence is invisibilized by Costabel’s opinion that “Descartes considère les angles d’incidence et de réfraction comme nous les entendons aujourd’hui.”⁹⁸ This statement is false, as can be demonstrated by examining how Descartes defines the angles of incidence and refraction.

Sines can only be used in a mathematical expression if the angles to which they refer are clearly and uniquely defined. However, in *Dioptrique*, the angles of incidence and refraction are subject to varying definitions:

Definition 1. Sometimes angles are measured in relation to the interface, in the manner of the Ancients: “Vous voyés facilement, comment se fait la reflexion, à sçavoir selon un angle toujours esgal à celuy qu’on nomme l’angle d’incidence... en sorte que l’angle de la reflexion FBE, n’est ne plus ne moins grand que celuy de l’incidence ABC.”⁹⁹

Definition 2. Sometimes, the angle of refraction means the deviation $r - i$: “Cette inclination se doit mesurer par la quantité des lignes droites... non par celle des angles... semblables à DBI, qu’on nomme les angles de Refraction.”¹⁰⁰

Definition 3. Sometimes, angles of incidence and refraction are measured in relation to the normal, in the manner of the Moderns: “Il doit avoir même proportion entre les lignes KM et LN... mais non pas la même entre les angles KBM et LBN, ou PRQ et SRT, qu’entre ABH et IBG.”¹⁰¹

Descartes thus uses mutually exclusive definitions of the angles of incidence and refraction just a few pages apart. This refutes Costabel’s opinion.

But the multitude of definitions of angles has implications for the very nature of the law. Indeed, each definition leads to a different “law.” Definition 1 (angles of incidence $i = \pi/2 - i$ and refraction $\rho = \pi/2 - r$ measured relative to the diopter) gives law (1). Definition 2 (angle of deviation $\delta = i - r$) gives law (2). Only definition 3 (angles i and r measured with respect to the normal) gives the modern sine law (3):

$$\cos i_1 / \cos \rho_1 = \cos i_2 / \cos \rho_2 (= \text{constant}) \quad (1)$$

$$\sin i_1 / \sin (i - \delta)_1 = \sin i_2 / \sin (i - \delta)_2 (= \text{constant}) \quad (2)$$

$$\sin i_1 / \sin r_1 = \sin i_2 / \sin r_2 (= \text{constant}) \quad (3)$$

While Descartes’ text is immediately compatible with an approach based on proportions (which requires no definition of the angles of incidence and refraction whatsoever), a law of refraction based on sines does require such definitions. The lack of stable definitions of

⁹⁸ Descartes, *Règles*, 198.

⁹⁹ Descartes, *Dioptrique*, 16.

¹⁰⁰ *Ibid.*, 21.

¹⁰¹ *Ibid.*, 21–22.

the angles has never been pointed out, nor have the implications been drawn: the inconsistent definitions of the angles of incidence and refraction preclude the identification of a sine law, even in implicit form, in Descartes' text.

3. *Refractive indices.* – Another point of contention is the omnipresence of refractive indices in philosophical and historical commentaries on Descartes' optics¹⁰². None of the texts just reviewed – not even the *Dioptrique*, which is the most accomplished – mentions refractive indices. Descartes simply speaks of “air” and “eau” or, to generalize a little, of “corps transparents” that light “pénètre plus ou moins facilement.” The refrangibility of bodies is neither conceptualized, nor quantified.

This absence shifts the focus of the investigation and raises the question of when the refractive indices were incorporated into the law of refraction.

Taking up Newton's idea¹⁰³ of measuring the *refractive power* of different media by the ratio of the sines $n_i : n_r = \sin r : \sin i$, Wollaston (1766–1828) presented a paper to the Royal Society in June 1802, in which he explained how to determine refractive indices using a right-angled prism. His work is compiled in a table of refractive indices for 83 bodies, ordered from the most refractive (diamond 2.44) to the least refractive (atmospheric air 1.00032).¹⁰⁴

1807 saw the publication of the *Lectures* by Thomas Young (1773–1829), a collection of lectures on natural philosophy given at the Royal Institution between 1801 and 1803. The lectures are undated, but Lecture xxxv, *On the theory of optics*, is later than 1802, as it mentions Wollaston's measurement of refractive indices. Although Young's terminology varies,¹⁰⁵ he is credited with establishing the current term *refractive index* (*index of refraction*). He defined it as follows: “The ratio of the angles of refraction and incidence ... in all

¹⁰² I cannot provide full references. See for instance: Rogers, “Genius and Writings of Descartes,” 1852: 12; Kramer, “Descartes und das Brechungsgesetz des Lichtes,” 1882: 239; Peddie, *A Manual of Physics*, 1896: 216; Ball, *A Short Account of the History of Mathematics*, 1901: 286; Preston, *The Theory of Light*, 1901: 128; Ball, *A Short Account of the History of Mathematics*, 1908: 277; Mach, *Principles of Physical Optics*, 1925: 4; Wootton, *The Physical Work of Descartes*, 1927: 473; Scott, *The Scientific Work of Descartes*, 1951: 110; Cohen, *Notes and Records of the Royal Society*, 1964: 135; Sirven, *Les années d'apprentissage de Descartes*, 1987: 328; Rodis-Lewis, *La science chez Descartes*, 1987: 69; Hatfield, “Science, Certainty, and Descartes,” 1988: 258; Thomson, *On Descartes*, 2000: 29; Burnett, *Descartes and the Hyperbolic Quest*, 2005: 62; Smith, *From Sight to Light*, 2014: 395; Watson, *Light*, 2016: 90; etc.

¹⁰³ “Having made these observations, I first computed from them the refractive power of that glass, and found it measured by the ratio of the sines, 20 to 31,” Isaac Newton, “A new theory about light and colors,” *Philosophical Transactions* 80 (1672): 3075–3087, 3077.

¹⁰⁴ William H. Wollaston, “A method of examining refractive and dispersive powers by prismatic reflection,” *Philosophical Transactions of the Royal Society* 92 (1802): 365–380, 570–571, Table 1.

¹⁰⁵ Rarity, density, refractive power, refractive density...

cases, being expressed by the ratio of 1 to a certain number, which is called the index of the refractive density of the medium.” And later on “The index of refraction at the common surface of any two mediums is the quotient of their respective indices.”¹⁰⁶ In his lecture, Young does not attempt, as Wollaston did, to determine the refractive indices of an entire series of refracting substances. He provides only the indices for water (4 : 3), crown glass (3 : 2), and flint glass (8 : 6).

In 1814, Joseph Fraunhofer (1787–1826) resumed the measurement of refractive indices of optical glasses, with the aim of perfecting achromatic lenses.¹⁰⁷ Knowing that chromatic aberration in lenses can be compensated for by combining flint and crown glass (more and less refractive), determining the refractive index of glass more accurately made it possible to better adjust the lenses of an achromatic doublet. Fraunhofer renamed the refractive index “exponent of the refractive ratio” (*Exponent des Brechungsverhältnisses*) and determined the refractive indices of twelve optical lenses to the sixth decimal place.¹⁰⁸

4. *Law.* – Even more numerous are those who credit Descartes with a “law of refraction.”¹⁰⁹ In certain circumstances, this expression may have been used inadvertently. When Arago

¹⁰⁶ Thomas Young, *A Course of Lectures on Natural Philosophy and the Mechanical Arts* (London: J. Johnson, 1807), 413.

¹⁰⁷ Joseph Fraunhofer, “Bestimmung des Brechungs- und Farbenzerstreuung- Vermögens verschiedener Glasarten, in Bezug auf die Vervollkommnung achromatischer Fernröhre,” *Denkschriften der königlichen Akademie der Wissenschaften zu München* 5 (1814): 193–226.

¹⁰⁸ *Ibid.*, 225, Tabelle III.

¹⁰⁹ Again, I cannot provide full references. These are for information only: Montucla, *Histoire des mathématiques*, 1802: 248; Arago, “Descartes,” 1855: 303; Airy, *Geometrical Optics*, 1870: 43; Kramer, “Descartes und das Brechungsgesetz des Lichtes,” 1882: 236; Adam and Tannery, *Correspondance de Descartes*, 1897: 241; Milhaud, “Descartes et la loi des sinus,” 1907: 223; Mach, *Principles of Physical Optics*, 1925: 32; Descartes, *Discours de la méthode*, ed. Gilson 1925: 270; Papanastassiou, *Les théories sur la nature de la lumière*, 1935: 26; Crombie, *Robert Grosseteste*, 1953: 283; Descartes, *Discours de la méthode*, ed. Alquié 1963: 673; Rodis-Lewis, *L’oeuvre de Descartes*, 1971: 85; Ziggelaar, *Le physicien Ignace Gaston Pardies*, 1971: 170; Mahoney, “Descartes,” *Dictionary of Scientific Biography*, 1971: 59; Costabel, *Démarches originales de Descartes savant*, 1982: 66; Lindberg, *Studies in the History of Medieval Optics*, 1983: 23; Smith, *Descartes’s Theory of Light and Refraction*, 1987: 4–5, 29, 81; Shea, *The Magic of Numbers and Motion*, 1991: 241; Jullien, *Descartes, La Géométrie*, 1996: 105; Schuster, “Descartes opticien,” 2000: 258; Blay, *Penser avec l’infini*, 2010: 55; Schuster, *Descartes-Agonistes*, 2013: 187; McDonough, *The Cambridge Descartes Lexicon*, 2016: 552–553; Schuster, *The Cambridge Descartes Lexicon*, 2016: 455; Bellis, *Descartes en dialogue*, 2019: 192; Antoine-Mahut, *Encyclopedia of Early Modern Philosophy*, 2022: 436; Bellis, *Encyclopedia of Early Modern Philosophy*, 2022: 1116; McDonough, *A Miracle Creed*, 2022: 11; Schuster, *Descartes’ mechanichal philosophy*, 2022: 448; Dika, *Descartes’s Method*, 2023: 270, etc.

wrote that Descartes was the first to “communiquer la loi des sinus,”¹¹⁰ he was primarily seeking to establish Descartes’ priority over Snel. As we know, this priority has been contested because Snel’s discovery predates Descartes.¹¹¹ When Gaston Milhaud wrote that Descartes gave a “démonstration théorique de la loi des sinus,”¹¹² his aim was to contrast the contributions of Descartes and Fermat. This interpretation has also been debated: Descartes engages more in an exercise of rhetorical persuasion than in a demonstration.¹¹³ In both cases, the focus is on the relationship between Descartes and his peers. Neither Arago nor Milhaud were interested in the *epistemological status* of the refraction relationship.

If we refer to the texts, Descartes does not claim any “law of refraction.” Even when he wrote that “la proportion [AH : GI :: KM : LN :: PQ : ST] demeure la mesme en toutes les refractions qui sont causées par les mesmes cors,”¹¹⁴ his aim is not to state a physical law, but to calculate angles, freeing himself from the need to measure them empirically.

The absence of a law in the strict sense raises secondary questions, and shifts the focus of the investigation.

And first of all, why did Descartes not consider the ratio of refractions to be a law? We know, for one thing, that Descartes conceived of laws of nature. As he wrote in a letter to Mersenne in April 1630: “C’est Dieu qui a établi ces lois en la nature ainsi qu’un Roi établit des lois en son Royaume.”¹¹⁵ We also know that Descartes accepted the existence of “laws of motion” which in fact constitute the foundational hypothesis of *Dioptrique*: “Car il est bien aysé à croire que l’action ou inclination à se mouvoir, que i’ay dit devoir estre prise pour la lumiere, doit suivre en cecy les mesmes loys que le mouvement.”¹¹⁶ Why did these favorable conditions not lead to a law of refraction? As far as I know, Descartes’s attempt to reduce optics to mechanics ended in failure. When comparing the motions of a ball and a ray of light, Descartes had to conclude that they move in different directions relative to the extended incident ray: one toward the surface, the other toward the normal. To explain the opposite trajectories of light and the ball, Descartes assumes that light propagates faster in dense bodies than in rare bodies, which is contrary to experience and to the common opinion that a medium offers resistance proportional to its density.¹¹⁷ Consequently, the

¹¹⁰ François Arago, “Descartes,” *Oeuvres de François Arago*, ed. by Jean-Augustin Barral (Paris: Baudry, 1855), vol. 3, 297–309, 303.

¹¹¹ See Vollgraff, “Snellius’ notes on the reflection and refraction of rays,” 718–725; Hentschel, “Das Brechungsgesetz in der Fassung von Snellius,” 297–344.

¹¹² Gaston Milhaud, “Descartes et la loi des sinus,” 223.

¹¹³ Eastwood, “Descartes on refraction,” 501; Antoni Malet, “Gregorie, Descartes, Kepler, and the law of refraction,” *Archives internationales d’histoire des sciences* 40 (1990): 278–304, 300.

¹¹⁴ Descartes, *Dioptrique*, 21.

¹¹⁵ *Correspondance de Descartes*, vol. 1, 145.

¹¹⁶ Descartes, *Dioptrique*, 8.

¹¹⁷ Descartes’ physics is also self-contradictory: when he claims that light travels faster in a dense body, he is advocating the temporal propagation of light (finite speed); when he says that dense

mechanical analogy could not serve as the basis for the law of refraction. Because of this lack of a strict equivalence between mechanics and optics, Descartes was unable to apply the laws of motion to optics and, consequently, was unable to formulate optical laws such as the law of refraction.

Next, one is also led to wonder when the Snel-Descartes relationship came to be considered a physical law.

It should be noted in advance that ancient texts refer to the law of refraction sometimes in the singular and sometimes in the plural. Since the law of sines is unique, we must first question the use of the plural.

When Mersenne mentions the “laws of refraction” in *Harmonie universelle*, he is not referring to the modern law of sines, but to the ancient rules of refraction set forth by Alhacen, *Optics* VII, 2.77, rewritten by Risner, according to which the ray “s’esloigne” or “s’approche de la perpendiculaire.”¹¹⁸ Let us then exclude occurrences of the laws of refraction in the plural. We can see, by contrast, that the phrase “law of refraction” in the singular only came into common usage from the 1690s onwards, as evidenced by the following expressions:

- loix¹¹⁹ de la refraction (Huygens 1690)
- loi des sinus (Leibniz 1693)
- veritable loy des refractions (Leibniz 1693)
- loi de la refraction (Mariotte 1717)
- loi constante de la refraction (De Montmor 1718)
- loi connuë de la réfraction (Maraldi 1723)
- loi ordinaire des réfractions (Bouguer 1729)
- loi de la refraction (Mairan 1730)
- loi de la réfraction (Clairaut 1741)
- loi de la réfraction (Maupertuis 1748)
- etc.

It is anachronistic to look for a law of sines before the end of the 17th century. A good connoisseur of 17th-century optics might still object that Fermat already speaks of a “law of refractions” in *Synthèse pour les réfractions* (1662). This is an artifact: Tannery forced the translation to include the word law, which does not appear once in the Latin text: *In ipsam tamen Cartesii proportionem incidimus* should be translated as “we nevertheless arrive at the same proportion as Descartes” and not “nous sommes retombés précisément sur la loi

bodies do not resist the passage of light, he is defending the instantaneous propagation of light (infinite speed). Light can have a finite or infinite speed, not both at the same time.

¹¹⁸ Mersenne, *Harmonie universelle*, 64.

¹¹⁹ The plural here refers to all laws of refraction, including the law of sines, Christiaan Huygens, *Oeuvres complètes de Christiaan Huygens*, ed. by Diderik J. Korteweg (La Haye: M. Nijhoff, 1916), 741, 755. These notes, which supplement the *Traité de la lumière*, were written after 1690.

que Descartes a énoncée.”¹²⁰ It is symptomatic that Tannery’s translation error consisted in substituting Descartes’ own word *proportio* with the modern concept of law. But here, as elsewhere, the theory of proportions is the only one historically documented.

5. *Proportions*. – What remains of Descartes’ account of refraction once the modern garb of sines, refractive indices and physical laws has been removed?

What remains is a proportion of the type $AH : GI :: KM : LN$ defined between geometric magnitudes, which requires clarification of Descartes’ conception of proportional relationships.

Unlike Maurolico’s and Kepler’s texts, which are composed *more geometrico*, Descartes’ texts are written freely, making it more difficult to discern the influence of mathematical axioms. And yet, many Cartesian problems boil down to the manipulation of proportions. Here are a few examples. In Book II of *Géométrie*, Descartes introduces an instrument, similar to Eratosthenes’ mesolab, which can be used to find proportional means. The lengths of the instrument’s rules obey the proportion $YC : YB :: YD : YC :: YE : YD :: YF : YE = \mu$.¹²¹ In a letter dated September 12, 1638, he explained the nature of a spiral to Mersenne. The (logarithmic) spiral conforms to the proportion $OB : OA :: OC : OB :: OD : OC :: OE : OD :: OF : OE \dots$.¹²² In another letter dated June 1645, Descartes mentions that Florimond de Beaune proposed a problem to him five or six years ago, which led him to define a (logarithmic) curve by the proportion $BC : CL :: N : BI$.¹²³

The Cartesian idea of proportions is neither *purist* – Descartes often confuses proportion with identity of ratios¹²⁴ – nor truly *new* – many subjects, such as Pappus’ problem, are posed in the terms of the Ancients. The novelty of the Cartesian project lies instead in the role he assigns to proportions in the development of science. Hence their lack of visibility. This role can, however, be documented based on certain passages from *Discours de la méthode* and *Règles pour la direction de l’esprit*.

In *Discours de la méthode*, Descartes stresses the fundamental role of proportions. By questioning the relationships between the mathematical sciences (including optics), he manages to reduce the core of these sciences to proportions. Descartes states: “encore que leurs obiets soient differens, elles ne laissent pas de s’accorder toutes, en ce qu’elles n’y considerent autre chose que les divers rappors ou proportions qui s’y trouvent.”¹²⁵

Further references to the role of proportions appear in Descartes’ *Règles pour la direction*

¹²⁰ Pierre de Fermat, *Synthesis ad refractiones* (1662), *Oeuvres de Fermat*, ed. by Paul Tannery and Charles Henry (Paris: Gauthier-Villars, 1896), vol. 3: 151–152.

¹²¹ Descartes, *Geometrie*, 317.

¹²² *Correspondance de Descartes*, vol. 2, 360–361.

¹²³ *Ibid.*, vol. 4, 229–230.

¹²⁴ Descartes, *Météores*, 262–263.

¹²⁵ Descartes, *Discours de la méthode*, 21.

de l'esprit. In Rule VI, the theory of proportions is the central idea, the “secret” from which numbers can be arranged into continuous proportion. In Rule XIV, Descartes goes further when he aims at the “réduction de ces proportions jusqu'à ce que l'égalité entre la 'chose' demandée et quelque 'chose' connue soit vue clairement” or when he proposes to “réduire les proportions si emmêlées qu'elles soient au point de trouver ce qui est inconnu en égalité avec quelque autre connu”¹²⁶ (Marion 1977: 62). Here, proportions take on a new dimension. They are no longer merely a tool for calculation; they embody an ambitious project of knowledge aimed at unifying the sciences – the *Mathesis universalis*. The reason why it transcends the divisions between arithmetic, geometry, astronomy, mechanics, optics ... is precisely that proportions form the underlying foundation of each of these sciences.

Commenting on the relationship between these texts and geometry, Vuillemin considers that the theory of proportions is more central than analytical geometry in Descartes' work: “Aux yeux du philosophe, l'invention de la géométrie analytique paraît secondaire par rapport à l'invention d'une méthode universelle de pensée, contenue ... dans la théorie générale des proportions.”¹²⁷ Descartes' ambition seems to have been to renew geometry, replacing the consideration of “figures” with that of “lines” between which relationships and proportions can be defined. In this respect, one can even view proportions as a tool for organizing mathematical problems.¹²⁸

Descartes' theory of proportions is the very counterexample to the common idea – which originated with Kuhn – that scientific theories are subject to a law of replacement: as soon as a new paradigm appears, it replaces the previous one.¹²⁹ However, just as Canguilhem was right to denounce the continuous view of the history of science based on the unclear notion of precursor, so too could one challenge the discontinuous view based on the idealization of the notions of theory and paradigm. As Duhem showed with regard to optics, “une théorie qui disparaît ne disparaît jamais tout entière.”¹³⁰ This maxim applies perfectly to Descartes' work, which contains within itself a historical stratification of mathematical principles from different periods. The author of a radical innovation is not entirely new. As the inventor of analytical geometry, Descartes nevertheless made extensive use of the theory of proportions, which guided both his geometric research and his theory of refraction.

Returning now to the theory of refraction, once we set aside the anachronisms represented by the sines, refractive index and physical law, which gradually developed between

¹²⁶ Descartes, *Règles*, 62, 67.

¹²⁷ Jules Vuillemin, *Mathématiques et métaphysique chez Descartes* (Paris: PUF, 1960), 10.

¹²⁸ Vincent Jullien, *Descartes, La Géométrie de 1637* (Paris, PUF, 1996), 27.

¹²⁹ “These men ... are particularly likely to see that those rules no longer define a playable game and to conceive another set that can *replace* them. The resulting transition to a new paradigm is scientific revolution,” Thomas Kuhn, *The Structure of Scientific Revolutions* (Chicago: University of Chicago Press, 1962), 90, and the countless occurrences of the verb *replace* in *Structure*.

¹³⁰ Pierre Duhem, “Les théories de l'optique,” *Revue des deux mondes* 123 (1894): 94–125, 123.

the late 17th century and the 19th century, what remains is an attempt to express a proportion. This observation extends far beyond the authors discussed in this article. This becomes clear when we examine the expressions used at the time:

- sunt proportionales (Maurolico 1567)
- proportion (Harriot av. 1605)
- non sunt proportionales (Kepler 1611)
- ratio (Snel 1621)
- raison de la refraction (Mydorge 1626)
- vraye proportion des refractions (Mersenne 1636)
- proportion (Hérigone 1637)
- mesme proportion (Descartes 1637)
- raison de refraction (Roberval av. 1651)

In the end, even though Maurolico, Kepler, Mydorge, Descartes and others came up with different theories on refraction, they were all firmly rooted in the classical theory of proportions. Incidentally, the influence of the theory of proportions did not disappear in the mid-17th century. Its mark is still visible in the works of Huygens and Newton,¹³¹ but that is another matter, which is beyond the scope of this article.

¹³¹ Niccolò Guicciardini, *Reading the Principia. The debate on Newton's mathematical methods for natural philosophy from 1687 to 1736* (Cambridge: Cambridge University Press, 1999), 118–135.

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