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GALILÆANA

Studies in Renaissance and Early Modern Science

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GALILÆANA XXIII, 1 (2026)

– FOCUS –



MATHEMATICAL THOUGHT IN THE RENAISSANCE
AND THE GENESIS OF MODERN MATHEMATICS

edited by Riccardo Bellé and Beatrice Sisana



Mathematical thought in the Renaissance and the genesis of modern mathematics

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Abstract

This article argues that the Renaissance rediscovery of Greek mathematics functioned both as a stimulus to renewal and as a powerful constraint. The classical paradigm was not simply revived: it had to be reorganized and reinterpreted, yet its internal grammar – marked by tensions between form and extension, number and magnitude – continued to shape what could count as an admissible object and a legitimate proof. Francesco Maurolico provides the central case study, through his effort to construct a renewed framework for Archimedean geometry of measure – an effort that did not culminate in a stable synthesis, but instead exposed internal tensions within the classical framework. Galileo is used as a stress test: his attempt to mobilize the Euclidean theory of proportions for the description of motion reveals both the productivity and the limits of that inherited structure.

Keywords

Renaissance mathematics, Maurolico, Euclidean–Archimedean tradition, New quantities and the mathematization of nature

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1. Introduction

Which pathways led to the mathematical revolutions of the seventeenth century – to Descartes and Leibniz? This question can now be posed on firmer grounds than was possible a few decades ago, after a sustained wave of studies and critical editions has reshaped our understanding of fifteenth- and sixteenth-century mathematics.

The Renaissance, long described either as a mere interlude between medieval calculatores and early modern science or as a simple revival of ancient models, increasingly appears as a field of tensions shaped by the encounter of two major strands. On the one hand, the humanist recovery of Greek mathematical texts fostered practices of commentary, rewriting, and rigorous philological control. On the other hand, algebraic and abacus-based traditions cultivated symbolic procedures, practical geometry, and techniques of measurement tied to the needs of commerce, engineering, perspective, and mechanics.

These lineages were not separated by a rigid divide. They met in courts and in civic life, where mathematicians and practitioners confronted shared problems and negotiated standards of acceptability across languages, genres, and institutional settings. In such contact zones, classical geometry could be mobilized, adapted, or resisted within technical contexts, and abacus-based techniques could be reformulated, translated, or warranted through classical forms.

A fundamental contribution to the consolidation of these exchanges was made by the advent of printing with movable type. Beyond multiplying circulation, print reshaped the conditions of comparison and stabilization of mathematical texts, helping to sustain communities of reference beyond local settings – a development of special importance for mathematics, whose transmission depends vitally on stable written forms.¹ Over the course of the sixteenth century, this infrastructure of comparison helped to weave the two strands more tightly together: Euclid, Archimedes, and Apollonius were recovered, corrected, and subjected to increasingly demanding demonstrative scrutiny, while algebraic and practical traditions circulated alongside them and pressed on their boundaries. The resulting process was neither peaceful nor linear: it generated productive exchanges, but also revealed limits within the inherited framework.

Against this background, the present special issue approaches Renaissance mathematics not as a juxtaposition of local developments, but as a historically structured field in which practices, texts, institutions, and demonstrative constraints evolved under changing conditions of transmission and authority. The guiding questions of the call – the paths leading toward Descartes and Leibniz, including possible blind alleys as well as durable innovations, the interplay of Greek recovery and abacus-based algebra, and the role of pro-

¹ On the role of print in the formation of scholarly communities, see Elizabeth L. Eisenstein, *The Printing Revolution in Early Modern Europe* (New York: Cambridge University Press, 2005), especially 209–285.

portion in the ‘geometrisation’ of nature – are thus taken up here as elements of a broader reassessment.

The present article adopts a deliberately more circumscribed perspective within this broader framework.² Rather than surveying the entire Renaissance mathematical landscape, it focuses on a structural problem that becomes especially visible within the classical geometric tradition: how the Euclidean-Archimedean framework was reworked, strained, and in some cases pushed to its limits in the course of the sixteenth century.

Within this perspective, Francesco Maurolico occupies a particularly significant position. Deeply committed to the recovery of ancient texts, he does not confine himself to philology, but treats classical sources as material to be reorganized and, where needed, extended in the service of a more coherent mathematical whole. He departs from a strictly philological posture whenever system-building and demonstrative standards pull in different directions. Following Maurolico will allow us to bring into view a set of problems that arise when one attempts to restructure the geometry of measure while remaining within the Euclidean-Archimedean standards of proof.

Are these problems merely Maurolico’s, or do they point to a more general limitation of the classical framework as it was reworked in the sixteenth century? Galileo enters here as a stress test – not as a general chapter in the history of his physics, but as an illustrative case from a different field and a different scholarly tradition. The same underlying constraints reappear when proportional reasoning, grounded in the Euclidean theory of proportions, is carried into natural philosophy, where Aristotelian ‘qualities’ must be stabilized as magnitudes.

2. Tensions in the Classical Mathematical Tradition of the Sixteenth Century

Vitality and tension

During the sixteenth century, mathematical activity displays a level of intensity and diversity that resists any simple characterization in terms of continuity or rupture. On the

² Interpretations of Renaissance mathematics have often moved along two broad lines: accounts that explain transformations primarily through social settings, institutions, and regimes of circulation, and approaches that foreground practices, epistemic styles, and the material and textual conditions under which knowledge is produced and stabilized. The present contribution does not engage these frameworks at their own scale. Its premise is that such debates may remain under-determined unless they are constrained by an analysis of what, in a given tradition, can count as an admissible mathematical operation. We therefore adopt an analytical perspective centered on demonstrative constraints within the Euclidean-Archimedean framework – a dimension that has received comparatively limited systematic treatment, despite its centrality to sixteenth-century reworkings of classical geometry.

one hand, the period is marked by an increasingly systematic recovery, translation, and assimilation of the *disiecta membra* of Greek mathematics, a process that accelerates and deepens over the course of the century and leads to a renewed and critical engagement with classical texts.³

On the other hand, within the culture of the abacus, the manipulation of algebraic techniques reaches a level of sophistication that goes beyond the limits encountered by earlier traditions, including Arabic algebra. Italian algebraists succeed in formulating general procedures for the solution of third-degree equations, while algorithms of commercial arithmetic spread widely across Europe, reshaping practices of calculation and problem-solving in a variety of applied contexts.⁴

The mathematics of this period is not a linear transition toward a new scientific order but a space of sustained tension, in which inherited conceptual frameworks coexist with increasingly ambitious demands for generalization, rigor, and applicability. This tension does not arise from a lack of technical sophistication; on the contrary, it emerges precisely when classical mathematical texts are widely available, actively studied, and critically reworked, and when established procedures are pushed to their limits across a broad range of problems.

A common constraint

Despite the variety of sixteenth-century mathematical contexts, problems, and methods, many major renewal efforts remained constrained by the ways in which mathematicians of the period received and made operative the classical tradition, especially the Euclidean theory of proportions, which served as the main framework for relating magnitudes. Within this Renaissance reading of the classical tradition, quantitative reasoning remained tied to proportional comparison between homogeneous magnitudes; this requirement shaped both the admissible objects of mathematical investigation and the forms of reasoning through which they could be treated.

³ Rose remains to date the most comprehensive overview of the humanist rediscovery and assimilation of Greek mathematics and science from Petrarch to Galileo, even if its general thesis is shaped, at points rather explicitly, by the anti-Duhemean and broadly culturalist controversies of the 1970s; cf. Paul Lawrence Rose, *The Italian Renaissance of Mathematics: Studies on Humanists and Mathematicians from Petrarch to Galileo* (Genève: Droz, 1975). For a different reading of Renaissance mathematics, centered on the internal organization of problems and traditions, see Pier Daniele Napolitani, “Il Rinascimento italiano,” in *La Matematica. Vol. 1: I luoghi e i tempi*, ed. by Claudio Bartocci and Piergiorgio Odifreddi (Torino: Einaudi, 2007), 237–281.

⁴ On the development of algebra in sixteenth-century Europe, see Sabine Rommevaux, Maryvonne Spiesser, and Maria Rosa Massa Esteve, eds., *Pluralité de l’algèbre à la Renaissance* (Paris: Honoré Champion, 2012). On the impact and circulation of abacus-based arithmetic and commercial calculation, see Raffaele Danna, *The Craft of Indo-Arabic Numerals: How Practical Arithmetic Shaped Commerce and Mathematics in Western Europe, 1200–1600* (Cambridge, MA: Harvard University Press, 2026).

The central role of proportional reasoning also helps explain why repeated attempts were made throughout the sixteenth century to reform, extend, or reinterpret the classical theory of proportions itself. Such efforts – pursued in different ways by figures such as Maurolico, Guidobaldo dal Monte, Benedetti, Valerio, Galileo, and others – testify to the pressure exerted on it by new classes of problems and magnitudes.

The Archimedean geometry of measure, based on so-called ‘exhaustion’ procedures and double *reductio ad absurdum*, is connected to the proportional framework. Archimedean methods provide a powerful, flexible toolkit for extending results to broader classes of figures while maintaining strict control over the objects involved. Mathematical entities are generated and justified through specific procedures of construction, comparison, and proof.⁵

A further, closely related resource was provided by the recovery of Archimedes’ so-called ‘mechanical’ works. Treatises such as *On the Equilibrium of Planes* and *On Floating Bodies* show, with exemplary clarity, how a strictly geometric mode of demonstration can be mobilized to articulate conditions of balance, buoyancy, and the determination of centers of gravity. What Archimedes handles in these works are magnitudes that can be directly placed under proportional comparison within a classical setting – weights and distances in statics, weights together with volumes or surfaces in hydrostatics, and, in the case of the *Spirals*, times together with traversed spaces. However, this still does not constitute a universal calculus for physical quantities. By contrast, quantities that later readers would be tempted to reconstruct as derived magnitudes – moment, pressure, specific gravity, velocity – do not figure in Archimedes’ analysis at all.

Archimedean mechanics exemplifies the tight link between admissible magnitudes and demonstrative procedures that defines the classical paradigm. This coupling is its strength: it enables major extensions of classical results and the systematic unification of previously separate problems. But it also imposes rigid limits: the demand for homogeneous magnitudes, the primacy of proportional relations, and the procedural generation of objects.

As a result, many sixteenth-century innovations take the form of an intensification of classical techniques rather than a reconfiguration of their conceptual foundations. Procedures are refined, generalized, and applied to new domains, but they remain embedded within a framework that is, in crucial respects, still Greek.

Extension of procedures and ambitions of generality

One of the most striking features of late sixteenth-century mathematics is the growing ambition to extend classical procedures by extracting from Archimedean proofs those fea-

⁵ On the structural role of Archimedean procedures of measure and their limits, see Ken Saito, “Archimedes and Double Contradiction Proof,” *Lettera Matematica International* 1 (2013): 97–104, <https://doi.org/10.1007/s40329-013-0017-x>.

tures that can be made portable, and then redeploying them as general results for whole classes of objects.⁶

A clear and particularly accomplished example is offered by Luca Valerio's *De centro gravitatis solidorum* (1604). Instead of treating centers of gravity and quadrature case by case, Valerio builds an organized corpus of theorems intended to apply to whole classes of figures, defined by explicit structural properties.

In this setting, a condition that originally serves to make an Archimedean-style argument go through – for instance, the monotone decrease of sections – can be promoted into a definition, so that the existence of suitable inscribed and circumscribed approximations is secured *by construction* and general theorems can replace ad hoc constructions.

The gain is methodological: an approximation theorem can be proved once and for all and then reused across an open-ended range of cases. Valerio's project aims to transform a set of powerful, though heterogeneous, classical arguments into a systematic, reproducible procedure – a true *via regia*, to use Valerio's own expression.

This drive toward generality comes at a conceptual cost. First, the push for procedural generalization increasingly outpaces the availability of genuinely new mathematical objects to which such procedures can be applied. Once the Archimedean corpus has been systematically reworked and extended, further progress often consists in reiteration, variation, or virtuoso recombination, rather than in opening fundamentally new domains of application. The very success of general methods thus reveals a structural limitation: procedures gain autonomy without a corresponding expansion of admissible mathematical objects. It is therefore not surprising that, in the following decades, Bonaventura Cavalieri will further drive the instrumental use of figures and the 'extraction' of quantitative relations from them.

Second, the same generalizing moves weaken the traditional role of geometric form in defining mathematical objects. In Valerio and, even more so, in Cavalieri, figures are

⁶ This tendency is already visible in mid-sixteenth-century projects of systematic rewriting and methodological consolidation (Maurolico, Commandino) and continues through a broader constellation of authors, extending into the early seventeenth century – notably Kepler (*Nova stereometria doliorum vinariorum*, 1615) and Bartolomeo Sovero (or Barthélemy Souvey, *Curvæ ac recti proportio*, 1630). It reaches a particularly mature form in Valerio and will later be pushed further, in a different key, by Cavalieri. For Valerio's *De centro gravitatis solidorum* and the generalization of Archimedean procedures of measure, see Pier Daniele Napolitani and Ken Saito, "Royal Road or Labyrinth? Luca Valerio's *De centro gravitatis solidorum*," *Bollettino di Storia delle Scienze Matematiche* 24, no. 1 (2004): 71–134, <https://doi.org/10.1400/18843>. For Cavalieri and the theoretical difficulties of the method of indivisibles, the extended studies by Enrico Giusti, *Bonaventura Cavalieri and the Theory of Indivisibles* (Roma: Cremonese, 1980), and Kirsti Andersen, "Cavalieri's Method of Indivisibles," *Archive for History of Exact Sciences* 31 (1985): 291–367, <https://doi.org/10.1007/BF00348519>, remain fundamental; for a broader reassessment of seventeenth-century debates on indivisibles, see Vincent Jullien, ed., *Seventeenth-Century Indivisibles Revisited* (Basel: Birkhäuser, 2015), <https://doi.org/10.1007/978-3-319-00131-9>.

increasingly treated through processes, sections, or families characterized by shared structural properties rather than fixed configurations. This partial dissolution of form is not accompanied by a fully articulated notion of abstract quantity, necessary for the creation of a new mathematical object.

The extension of procedures, the partial erosion of form, and the unresolved problem of abstract quantity thus bring into view a structural difficulty: the classical pattern can be stretched and recombined, but it resists reform from within.

New magnitudes and the mathematization of nature

If Archimedean mechanics had made it plausible that geometrical proof could articulate physical configurations, sixteenth-century authors soon faced a more demanding question: how to render certain physical *qualities* quantitatively tractable within the Euclidean theory of proportions. The difficulty now shifts from the geometrical description of configurations to the quantitative stabilization of qualities. The issue, in other words, is less a matter of ‘applying geometry to nature’ than of deciding what may count as a magnitude, and under which rules it can be compared and composed.

Within Book V of Euclid’s *Elements*, a ratio is not a numerical object but a relation between magnitudes of the same kind, established and compared through the criterion of equimultiples. This gives the theory its remarkable rigor – allowing, in particular, the comparison of incommensurable magnitudes – while also fixing a strict boundary: quantitative reasoning is limited to relationships among magnitudes of the same kind, and operations on ratios are allowed only by acting on the magnitudes that constitute those ratios.

The consequences are decisive: quantities that, from a modern perspective, are defined through fixed relations between heterogeneous magnitudes – whether by proportionality (space, time, and speed; weight, volume, and specific weight) or by composition (weight and distance in mechanical moment) – cannot be assumed *as such* as autonomous mathematical objects. If they are to enter mathematical reasoning at all, they must be reconstructed indirectly, by reducing them to controlled comparisons between admissible magnitudes (spaces with spaces, times with times, weights with weights). This requirement is not merely a technical limitation, but a structural one: it determines, in advance, the forms under which new physical properties may be stabilized within classical mathematics.⁷

Against this background – that is, against the demand for homogeneous magnitudes imposed by Book V – it is worth recalling that attempts to accommodate physical properties within a proportional framework did not begin with Galileo. Already in the early de-

⁷ On the structural role of the classical theory of proportions in early modern mechanics and its consequences for the treatment of physical quantities, see Enrico Giusti, *Euclides Reformatus. La teoria delle proporzioni nella scuola galileiana* (Torino: Bollati Boringhieri, 1993), especially ch. 2, “La teoria delle proporzioni e il linguaggio della natura,” 33–56.

caes of the sixteenth century, figures working firmly inside the classical tradition explored different ways of regulating new physical situations through proportional schemata.

In his early writings, Maurolico provides a particularly clear example. Problems concerning equilibrium and *mechanical moments* are handled through composed ratios involving weights and distances; in optics, refraction is approached by comparing angles and lines within a Euclidean setting; illumination is discussed by introducing rays and by relating their ‘density’ or concentration to the resulting effect.⁸ In these cases, the relevant quantity is not treated as a numerical quotient, but is introduced (and regulated) as a legitimate magnitude within a classical setting, endowed with explicit definitions and conditions of comparison. Maurolico thus exemplifies an early and systematic attempt to stabilize new physical magnitudes *within* the proportional framework itself.

A different, more conservative strategy can be observed around the turn of the seventeenth century in Marino Ghetaldi’s *Archimedes promotus* (1603). Rather than introducing specific weight as an autonomous magnitude, Ghetaldi constructs a proportional model for the loss of weight experienced by a body immersed in a fluid, keeping the analysis within controlled comparisons between homogeneous magnitudes.⁹ Taken together, these cases indicate that ‘new magnitudes’ did not generate a single research program, but a range of local solutions within the same Euclidean constraint.

They also help clarify a second point: the Archimedean revival does not unfold along a single line. As texts are recovered, edited, translated, and re-used, Archimedes is appropriated in different registers – humanist, technical, mathematical, and philosophical – and the ‘mechanical’ works, in particular, often require active clarification and reworking.¹⁰

When the young Galileo engages with Archimedes, he therefore does so within an already articulated landscape. His distinctive profile emerges not from adherence to a sin-

⁸ On Maurolico’s early optical writings (*Photismi*, 1521; *Diaphana*, 1523, reworked in 1553–1554), see Francesco Maurolico, *Optica*, ed. by Riccardo Bellé and Ken’ichi Takahashi, vol. 10, *Edizione Nazionale dell’opera matematica di Francesco Maurolico* (Pisa: Fabrizio Serra, 2017), <https://maurolico.it/Maurolico/sezione.html?path=10>. His treatment of mechanical moment (*momentum*) likewise dates from the 1520s; see Francesco Maurolico, *Archimedeae. Tomus B: De Momentis*, ed. by Riccardo Bellé, Pier Daniele Napolitani, and Beatrice Sisana, vol. 7, *Edizione Nazionale dell’opera matematica di Francesco Maurolico* (Pisa: Fabrizio Serra, 2022), <https://maurolico.it/Maurolico/sezione.html?path=7.B>.

⁹ See Pier Daniele Napolitani, “La geometrizzazione della realtà fisica: il peso specifico in Ghetaldi e in Galileo,” *Bollettino di Storia delle Scienze Matematiche* 8, no. 2 (1988): 139–237.

¹⁰ On the complex transmission of the Archimedean tradition between the Middle Ages and the Renaissance, and on the non-linear character of this process of reception, see Beatrice Sisana, “L’archimedeismo’ negli scritti giovanili di Galileo” (PhD diss., Università degli Studi Roma Tre, 2023) and Pier Daniele Napolitani, “Between Myth and Mathematics: The Vicissitudes of Archimedes and His Work,” *Lettera Matematica International* 1 (2013): 105–112, <https://doi.org/10.1007/s40329-013-0021-1>.

gle model, but from an ability to coordinate different registers into a problem-oriented synthesis – a form of *syncretic Archimedeanism*.¹¹ In Galileo's earliest investigations, and throughout his later work, these constraints remain operative. They are negotiated with success in some domains (notably in treatments of uniform motion), but the same tension will reappear, in different guises, from problems of specific weight and moment to the harder case of accelerated motion.

Alternative strategies and conceptual reorganizations

Alongside the intensification of classical procedures discussed above, the late sixteenth century also witnesses the emergence, across different European contexts, of proposals that aim more explicitly at loosening – or even bypassing – the constraints of the classical paradigm. These attempts do not take the form of incremental reform of the Euclidean-Archimedean framework, but explore paths that move beyond it, often without a clear sense of how such paths might ultimately be stabilized.

A first example is provided by Adriaan van Roomen, whose project of a *mathesis universalis*, articulated in the *Apologia pro Archimede* (1597), aims at grounding a general science of quantity no longer tied either to geometry or to number. The ambition is deliberately foundational; yet it remains largely programmatic, and the lack of shared conceptual resources limits its capacity to generate a stable practice.

A different strategy is pursued by Simon Stevin. By redefining the notion of number and introducing decimal fractions in a systematic way (*La disme*, 1585), Stevin effectively weakens the traditional boundary between number and magnitude, thereby extending the range of quantities that can be treated arithmetically. Here too, however, the conceptual consequences of this move are only partially integrated into the classical demonstrative framework.

At a still different level, François Viète's *In artem analyticem Isagoge* (1593) brings to maturity a range of earlier experiments in algebraic symbolism,¹² and shapes them into a general analytic art explicitly connected to the ancient method of analysis and synthesis transmitted by Pappus, whose *Collectiones mathematicae* appeared in print in 1588. Viète's

¹¹ For the main lines of Archimedean reception in the late sixteenth century and their relevance to Galileo's early writings, see Sisana, "L'archimedeismo' negli scritti giovanili di Galileo." An analysis of Galileo's juvenile studies on the centers of gravity of solids can be found in Riccardo Bellé and Beatrice Sisana, "Galileo Galilei and the Centers of Gravity of Solids: A Reconstruction Based on a Newly Discovered Version of the Conical Frustum Contained in Manuscript UCLA 170/624," *Archive for History of Exact Sciences* 76, no. 6 (2022): 551–588, <https://doi.org/10.1007/s00407-022-00289-4>.

¹² On this process, see Albrecht Heeffer, "From the Second Unknown to the Symbolic Equation," in *Philosophical Aspects of Symbolic Reasoning in Early Modern Mathematics*, ed. by Albrecht Heeffer and Maarten Van Dyck, 57–101, *Studies in Logic* 26 (London: College Publications, 2010).

symbolism greatly expands the expressive power of reasoning, but it does not, by itself, supply a conceptual idiom capable of replacing classical architecture.

The diversity and the lack of convergence of the examples just mentioned suggest that the difficulty is structural: not merely a matter of extending procedures, but of identifying a common ground on which new objects, new relations, and new standards of legitimacy could be coordinated. In particular, the classical requirement of homogeneous magnitudes and proportional comparability remains a powerful – and selective – constraint, while increasingly general inferential templates tend to outpace the repertoire of admissible objects.

Against this European backdrop – where internal intensification and external displacement coexist without resolution – Maurolico's position becomes especially revealing: it is a deliberately internal response that seeks to make the presuppositions of the classical architecture explicit.

3. *Maurolico and the Classical Tradition*

Problems from within Archimedean geometry

Maurolico works with full awareness of the language and methods of Greek geometry as his main tools of mathematical rigor and proof. His engagement with Archimedean mathematics spans his entire career and serves as a constant reference point for his mathematical thought.

This strict adherence to ancient models leads him to address foundational problems, which emerge most clearly in his extensive work of commenting on, rewriting, and systematically reorganizing Archimedean texts. Maurolico's work thus offers a privileged viewpoint on how sixteenth-century mathematics, while formally faithful to the classical tradition, increasingly questioned the conditions under which Archimedean-style arguments could be applied, just as it was beginning – more or less consciously – to move beyond them.

Before and after 1544: reconstructing Archimedes and confronting his foundations

A first phase of Maurolico's engagement with Archimedean geometry develops under conditions that are, from a textual point of view, severely constrained. A substantial part of his early Archimedean production – including the *De sphaera et cylindro*, the *De dimensione circuli*, the *Libellus de momentis aequalibus*, and the *Quadratura parabolae* – was composed before he had access to the authentic works of Archimedes.¹³

¹³ For Maurolico's engagement with Archimedean texts, the chronology of his reconstructions, and the impact of the Basel *editio princeps* of 1544, see specifically the "General Introduction"

The sources available to him prior to the middle of the century offered only a fragmentary and indirect image of Archimedean geometry. They consisted primarily of excerpts transmitted through humanist compilations, medieval reworkings of Archimedean results, and derivative traditions in which demonstrative rigor and foundational explicitness were unevenly preserved.¹⁴ Within this heterogeneous textual horizon, Maurolico's early reconstructions already display a remarkable technical competence, but they also rely on assumptions and argumentative shortcuts inherited from non-Archimedean contexts.

The situation changes decisively with Maurolico's encounter with the Basel *editio princeps* of Archimedes' works in 1544. For the first time, he gains access not only to treatises previously unknown to him – such as the *Spirals* and the *Conoids and Spheroids* – but also to the authentic structure of Archimedean demonstration, including its explicit use of definitions, postulates, and preliminary lemmas. This encounter forces a systematic comparison between Maurolico's own reconstructions and the demonstrative economy of Archimedes himself.

The outcome of this comparison is not uniform. In the case of *Quadratura parabolae* and *De dimensione circuli*, Maurolico's earlier reconstructions turn out to be remarkably accurate, requiring only limited adjustments. By contrast, his *De sphaera et cylindro* and *Libellus de momentis aequalibus* are judged – by Maurolico himself – to surpass the medieval tradition yet still to demand a thorough revision in the light of the authentic text. The *Spirals* are read with close fidelity, though not without significant interventions, whereas in the *Conoids and Spheroids* Maurolico not only redistributes the material into two books and makes explicit several conic-section arguments left implicit by Archimedes, but also develops an original treatment of the relation between sphere and spheroid, based on a systematic comparison of sections.¹⁵ As he would later emphasize in his letter to Juan de Vega (1554), Archimedes now appeared to him as the author of a single, coherently struc-

and the "Introduction to Tomus A" in Francesco Maurolico, *Archimedeae. Tomus A: Opera Archimedis Ex Traditione Maurolyci*, ed. by Riccardo Bellé, Pier Daniele Napolitani, and Beatrice Sisana, vol. 7, *Edizione Nazionale dell'opera matematica di Francesco Maurolico* (Pisa: Fabrizio Serra, 2022), XVII–LXVII and 3–34, <https://maurolico.it/Maurolico/sezione.html?path=7>.

¹⁴ On the printed sources and medieval traditions available to him before 1544, see in particular Giorgio Valla, *De Expetendis Et Fugiendis Rebus* (Venezia: Aldus Manutius, 1501), which transmits excerpts from Archimedes and Eutocius; Luca Gaurico, *Tetragonismus Idest Circuli Quadratura* (Venezia: Giovanni Battista Sessa, 1504), based on a corrupted version of William of Moerbeke's translation of *De dimensione circuli* and *Quadratura parabolae*; and the *Liber de curvis superficiebus* attributed to Iohannes de Tinemue (critical edition in Marshall Clagett, *Archimedes in the Middle Ages. Vol. I: The Arabo-Latin Tradition* (Madison: University of Wisconsin Press, 1964), 439–557).

¹⁵ On these different outcomes and on the constructive aspects of Maurolico's reworking of the Archimedean corpus see the relevant introductions and notes in Maurolico, *Archimedeae. Tomus A*.

tured corpus, while the *Libellus de momentis aequalibus* assumed the status of an original contribution rather than a mere reworking.¹⁶

From this point onward, Maurolico's engagement with Archimedean geometry is no longer limited to reconstruction or improvement of individual treatises. It acquires a distinctly foundational dimension. The task he increasingly sets himself is not merely to reproduce Archimedean results, but to clarify, systematize, and, where necessary, justify the principles that make those results demonstratively possible. This shift provides the immediate background for his later attempt to articulate an explicit introduction to Archimedean geometry, the *Praeparatio ad Archimedis opera*.

Implicit assumptions and demonstrative legitimacy: the case of *De sphaera et cylindro*

The foundational difficulties that Maurolico gradually comes to recognize are particularly clear in his treatment of *De sphaera et cylindro*. In its original Archimedean form, the treatise opens with a carefully articulated set of definitions, postulates, and preliminary lemmas, whose function is not merely formal, but constitutive of the validity of the demonstrations that follow. By contrast, the medieval tradition through which Maurolico first accessed this material transmits many of the results while leaving their foundational underpinnings largely implicit.

This difference becomes evident when Maurolico's early version of *De sphaera et cylindro*, completed in 1534, is compared with the authentic Archimedean text. Maurolico's reconstruction relies in several places on demonstrative patterns inherited from the medieval *Liber de curvis superficiebus*,¹⁷ which makes systematic use of assumptions concerning the existence of surfaces equal to a given surface,¹⁸ as well as principles of inclusion governing the comparison of curved surfaces.¹⁹

¹⁶ On the letter to Juan de Vega and its role in Maurolico's redefinition of his Archimedean project, see the "General Introduction," § 3.3.1, in Maurolico, *Archimedeae. Tomus A*. For the text: *Edizione Nazionale dell'opera matematica di Francesco Maurolico*, available at <https://maurolico.it/Maurolico/sezione.html?path=2.A>.

¹⁷ Marshall Clagett was the first to demonstrate Maurolico's dependence on the medieval text attributed to a certain Iohannes de Tinemue; see *Archimedes in the Middle Ages. Vol. III: The Fate of the Medieval Archimedes in the Sixteenth Century* (Philadelphia: American Philosophical Society, 1978), 798–812. On the unidentified author to whom this work is ascribed in some manuscripts, see Wilbur R. Knorr, "John of Tynemouth Alias John of London: Emerging Portrait of a Singular Medieval Mathematician," *The British Journal for the History of Science* 23, no. 3 (1990): 293–330.

¹⁸ "Rationis causa. Presentis demonstrationis hypothesis, et tota sequentium theorematum series, lineam rectam curva et superficiem rectam curva esse aequalem sibi postulat admitti" (Clagett, *Archimedes in the Middle Ages. Vol. I*, 452, 15–17).

¹⁹ "Superficies inclusa maior sit superficie includente, quod est impossibile" (Clagett, *Archimedes in the Middle Ages. Vol. I*, 458, 96–97). It should be noted that the medieval formulation is less

A revealing example is provided by Maurolico's proof of theorem X, asserting that the surface of the sphere is equal to the rectangle constructed on the diameter of the sphere and the circumference of its greatest circle. The argument proceeds by a double *reductio ad absurdum*, based on the inscription and circumscription of solids of revolution obtained by rotating polygons, and on the comparison of concentric spheres. For the proof to be legitimate, however, two assumptions are required, and Maurolico uses both of them as if they were immediately acceptable, inserting them into the flow of the proof as intuitive claims. First, he asserts that, if the surface generated from a given line is not equal to the surface of a given sphere, it will be equal to the surface of some greater or smaller sphere – that is, he presupposes the existence of a spherical surface equal to any given surface. Second, he assumes as impossible that an included surface should be greater than the surface that includes it, thereby relying on a principle of inclusion to guarantee that the surface of an inscribed solid cannot exceed that of the sphere containing it. In Maurolico's 1534 text, these assumptions are simply taken over as local working hypotheses, with no attempt to formulate them as general principles grounded in a preliminary system of definitions and postulates.

When confronted with the Archimedean text after 1544, however, Maurolico read *De sphaera et cylindro* as a highly structured model of demonstrative organization. In Archimedes' own formulation, the comparison of curved surfaces does not rest on ad hoc premises introduced only within the course of the proof, but on principles explicitly stated at the outset, together with precise conditions – such as concavity in the same direction – under which comparisons are admissible. Read in this way, the Archimedean text changed the status of the assumptions that had functioned as local working hypotheses in Maurolico's 1534 reconstruction: they could no longer remain merely implicit without raising questions of demonstrative legitimacy.

The contrast between these two approaches brings into focus a problem that is not merely textual, but structural. Demonstrative procedures that function within a given argumentative context cannot be freely generalized unless the principles on which they depend are explicitly stated and justified.

In the extant text of Maurolico's *De sphaera et cylindro* one can in fact discern traces of a first attempt to accommodate these requirements within the original framework of the treatise. Additional lemmas and local adjustments point to a partial reworking carried out in the light of the Basel edition, but the result is a composite text in which older demonstrative habits and new foundational demands still coexist in a somewhat unstable way.²⁰

precise than the Archimedean one, in which the condition that the two lines or surfaces be concave in the same direction is explicitly required.

²⁰ On the composite character of the surviving version of *De sphaera et cylindro* and on the traces of this reworking see Riccardo Bellé, "Il processo compositivo del *De sphaera et cylindro* ex traditione Maurolyci," *Bollettino di Storia delle Scienze Matematiche* 42, no. 2 (2022): 311–337.

The examination of the other Archimedean treatises transmitted by the Basel edition – in particular the *Spirals* and the *Conoids and Spheroids* – confirms that the difficulties brought to light by *De sphaera et cylindro* are not confined to a single work, but concern more generally the foundational presuppositions of Archimedean geometry.

The genesis of the *Praeparatio ad Archimedis opera*: foundational needs of a ‘general mathematics’

The shift described above from a local legitimacy of procedures to explicit foundations is described by Maurolico himself, reflecting on the work he undertook after gaining access to the complete Archimedean corpus:

Having examined all these works, I endeavoured to make their content easier to understand by adding many lemmas, by proving many results that Archimedes had left implicit, and, moreover, by treating the centres of solids in his investigation of equal moments, a topic that he had passed over.²¹

This passage makes clear that Maurolico conceives his task not simply as one of transmission or clarification, but as an intervention aimed at rendering explicit what had previously remained tacit within Archimedean demonstrations.

The need for such an intervention becomes particularly evident when Maurolico turns back to his own earlier work and explicitly acknowledges that, in his *De sphaera et cylindro*, he had adopted what he now calls a *facilior via*, relying on assumptions that might appear *inconcessibilia* if left without justification:

In my treatise *De sphaera et cylindro* I followed an easier path. In order that no one may think that I assumed inadmissible principles when I supposed that, for any given surface, there exists a spherical, conical, or cylindrical surface, under a given height, or the surface of a spherical segment, equal to it, we shall here demonstrate those very principles. Likewise, given two surfaces, we shall show that there exists a surface similar to one of the given and equal to the other; and, given two solids, that there exists a solid similar to one of the given and equal to the other. Since this requires the determination of two mean proportionals, we shall treat this problem following the tradition of the ancient philosophers, as Eutocius has recorded in his Commentaries.²²

²¹ *Praeparatio*, Proemium, § 18, in Maurolico, *Archimedeae. Tomus A*, 86: “Haec ego omnia cum vidissem, conatus sum ad faciliorem intellectum multa lemmata adiicere, multa ab Archimede omissa demonstrare, tum in aequalium momentorum negotio centra solidorum tractare, rem ab illo praetermissam.”

²² *Praeparatio*, Proemium, § 19, in Maurolico, *Archimedeae. Tomus A*: “In Libello De sphaera et cylindro usus sum faciliori via. In quo ne quis arbitretur me inconcessibilia principia postulasse

It is not the rejection of the classical framework, but the recognition that demonstrative validity cannot rest on inherited schemas alone. Procedures that were previously secured by local argumentative practice must now be grounded in explicit principles that specify the admissible operations and the conditions under which they apply. Composed in 1550, at a moment when Maurolico also presents Archimedes as the author of a unified corpus, the *Praeparatio* is thus conceived as a preliminary work in which the foundations of Archimedean geometry can be stated and examined in a systematic way.

Postulates and the regulation of comparison

A first and decisive step in Maurolico's attempt to make his vision of the geometry of measure fully coherent consists in the explicit formulation of a small but systematic set of postulates, designed to support both the *via facilior* adopted in his *De sphaera et cylindro* and, more generally, his reorganization of Archimedean geometry. The *Praeparatio* is not, in this respect, a mere preface, but a genuine attempt to lay down the principles on which his demonstrative procedures are to rest. Among these postulates, a central place is occupied, on the one hand, by those regulating admissible comparisons between magnitudes and, on the other, by the principle that reduces any ratio between magnitudes of the same kind to a ratio between lines – a principle that underlies the existence theorems discussed in the next subsection.

Within this framework, the postulates of inclusion play a clarifying rather than innovating role. In his *De sphaera et cylindro*, as well as in its medieval antecedent, principles of inclusion were already at work, but only in the form of local assumptions attached to specific arguments. In the *Praeparatio*, Maurolico rewrites them as general postulates governing the comparison of surfaces and solids. They state, in general terms, that a surface or solid which circumscribes or includes another is greater than the surface or solid that is inscribed or included, provided that the figures involved are concave in the same direction. What is crucial here is not the novelty of the claim itself – which closely follows Archimedean usage – but its elevation to the status of an explicit principle.

By making such conditions explicit, Maurolico addresses a structural difficulty already encountered in his earlier work. Demonstrations based on inscription and circumscription presuppose principles of inclusion in order to rule out certain inequalities as impossible. The *Praeparatio* turns this inherited practice into a general rule: comparisons between curved surfaces and solids are declared legitimate only when the relevant

si cuilibet superficiei aliquam sphaericam aut conicam sive cylindricam sub data celsitudine aut sphaericae portionis superficiem aequalem esse supponam, demonstrabimus et hic ipsa principia; item datis duabus superficibus superficiem esse uni datarum similem et alteri aequalem datisque duobus solidis aliquod solidum esse uni datorum simile et alteri aequale. Ad quod cum necessaria sit duarum mediarum proportionalium inventio, id ipsum problema ex veterum philosophorum traditione tractabimus, ut Eutocius memoratus in Commentariis scripsit.”

inclusion conditions are satisfied.

This move has an important methodological consequence. It does not extend the domain of admissible objects, nor does it introduce new forms of reasoning. Rather, it regulates the use of existing procedures by fixing in advance the conditions under which they may be applied. The postulates of inclusion thus function as boundary conditions for Archimedean-style arguments, delimiting the domain within which operations of comparison retain demonstrative force.

Existence theorems and the reduction of ratios to lines

Alongside the regulation of admissible comparisons, the *Praeparatio* introduces a second foundational move, which proves even more far-reaching: the systematic use of existence theorems grounded on the reduction of relations between magnitudes to relations between straight lines. This principle, stated explicitly among the postulates at the outset of the work, establishes that for any two magnitudes of the same kind there exist two lines standing in the same ratio:

Quibusbilibet duabus eiusdem generis magnitudinibus esse duas lineas proportionales
(*Praep.* Postulatum 1, § 21).

On this basis, relations between surfaces, solids, and other magnitudes can be treated indirectly through relations between segments. Subsequent propositions then assert the existence of lines, squares, circumferences, and spherical surfaces satisfying prescribed proportional relations.²³

A particularly instructive example is provided by the proof of Proposition XV, which asserts the existence of a circumference equal to any given straight line. Given a line A , Maurolico constructs a circle BED on an arbitrary diameter BD . If the circumference BED is not equal to the given line A , he invokes the existence of a fourth proportional CF , such that $BED:A=BD:CF$. The line CF is then taken as the diameter of the circle whose circumference is equal to the given line A .²⁴

²³ See *Praeparatio*, in Maurolico, *Archimedeae. Tomus A*. For instance, Prop. V: “Esse aliquam lineam ad quam data linea datam habet rationem”; Prop. VII: “Esse aliquod quadratum ad quod datum quadratum datam habeat rationem”; Prop. XV: “Cuivis datae lineae alicuius circuli peripheriam esse aequalem”; Prop. XX: “Cuilibet datae superficiei alicuius sphaerae superficiem esse aequalem.”

²⁴ The existence of this fourth proportional is guaranteed by *Praeparatio*, Prop. V, which in turn rests on Postulate 1.

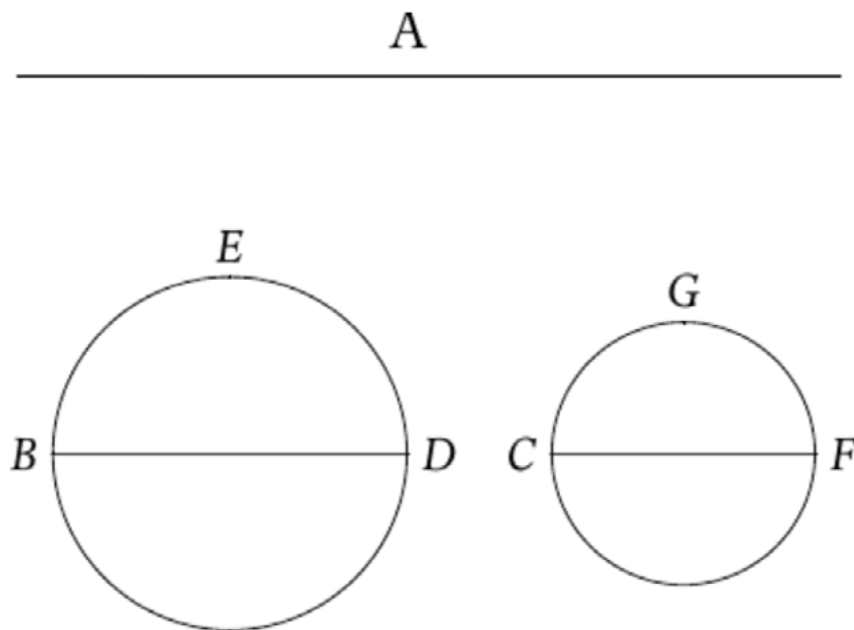


Figure 1 - *Praeparatio*, Proposition XV.

What matters here is not the introduction of new constructions, but the change in their status. In Archimedean practice, analogous results typically function as local tools, introduced to secure inequalities or approximations required for a specific proof – as in *De sphaera et cylindro* I.2 or in *Spirals* 4. As Reviel Netz has emphasized, such propositions do not aim to establish the ‘existence’ of objects in any philosophical sense, but to furnish instruments for obtaining inequalities between magnitudes and ratios, which can then be used to prove geometrical equalities.²⁵

In the *Praeparatio*, by contrast, the corresponding results are elevated to the status of general existence theorems. They guarantee in advance the availability of objects satisfying prescribed proportional conditions, thereby allowing demonstrations to proceed without repeated appeal to ad hoc constructions.

This shift has a clear methodological effect. By reducing questions about the relations between heterogeneous magnitudes to questions about ratios between lines, Maurolico creates a uniform medium in which existence and comparison can be handled systematically. At the same time, this strategy does not dissolve the classical constraints under

²⁵ Reviel Netz, *The Works of Archimedes. Vol. I: The Two Books on the Sphere and the Cylinder* (Cambridge: Cambridge University Press, 2004).

which those relations operate. The requirement of homogeneity is preserved, and proportional comparison remains the fundamental mode of reasoning. What changes is the level at which these constraints are enforced: no longer locally within individual proofs, but globally, through explicitly stated principles.

The result is a far-reaching reorganization of the classical Archimedean framework. The reduction of ratios to ratios between lines does not introduce a genuinely abstract notion of quantity independent of geometrical determination. Instead, it provides a general mechanism for translating relations between diverse magnitudes into a form compatible with classical proportional theory. In this sense, the existence theorems of the *Praeparatio* both extend the reach of Archimedean methods and reveal their internal limits: procedures acquire a new degree of generality, yet the ontology of admissible mathematical objects remains essentially unchanged.

At the same time, the very attempt to treat arbitrary magnitudes via proportional relations between lines brings into focus a new tension around the notion of quantity – a tension that Maurolico would try to address in various ways in the years following the composition of the *Praeparatio*, as the next section will show.

4. Quantity, Number, and Ratio: an Unfinished Synthesis

The Archimedean Dossier: a project that existed – and why it did not close

From the *Praeparatio ad Archimedis opera* (1550) onward, Archimedes remains a stable point of reference in Maurolico's mathematical horizon. This does *not* mean that his post-1550 writings are all devoted to Archimedes; rather, it means that Archimedean geometry continues to set a standard of demonstrative legitimacy and to provide a privileged testing ground for what counts as an admissible operation on magnitudes.

The documents that allow us to reconstruct Maurolico's attempt at a *comprehensive* and *definitive* arrangement of his Archimedean work, however, are of a specific kind and fall within a limited cluster: programmatic statements (above all the letter to Juan de Vega, drafted in 1554 with later additions dated 1556), short foundational essays written in June 1554 (the *Sermones*), the construction of a general operational framework for calculation in Book II of the *Arithmeticonum libri duo* (completed in 1557, printed in 1575), and the late compendium of Euclid's Book V (*Compendium Quinti Elementorum*, 1567). Taken together, these texts leave little doubt that a coherent project existed: Maurolico did not merely annotate Archimedes; he sought to reorganize the Archimedean corpus within an architectonic view of mathematics and of its demonstrative legitimacy.²⁶

²⁶ For the set of documents relevant to the evolution of Maurolico's Archimedean work, see the Introduction in Maurolico, *Archimedeae. Tomus A*, LIII–LV.

The dossier ‘fails to close’ not because Maurolico loses interest in Archimedes or lacks technical control. The dossier fails to close because, in the very act of trying to stabilize an Archimedean synthesis (and to keep it coherent with the foundational stance already taken in the *Praeparatio*), Maurolico is driven to a prior task: to make explicit a general framework for quantity, ratio, and proportion – a framework he does not find ready-made in Greek mathematics, neither in Euclid nor in Archimedes. It is this foundational detour, generated from within the Archimedean agenda itself, that repeatedly deflects the revision and prevents it from settling into a final synthesis.

That Maurolico himself perceived the connection is already clear in the *Sermo de divisione artium* (June 1554), where he explicitly adds to Euclid’s postulates and common notions precisely the principles he had placed as postulates in the *Praeparatio* – principles governing inclusion, proportionality, and the existence of equal, greater, or smaller magnitudes. He notes that they are required “not only in elementary demonstrations, but also in Archimedean and other demonstrations,” and that “upon these, as upon foundations, all propositions are constructed in successive demonstration.”²⁷

In search of a framework: the de Vega letter and the *Sermones*

The documents produced between 1554 and 1556 – above all the letter to de Vega and the *Sermones* of June 1554 – are best read not as isolated occasional pieces, but as Maurolico’s attempt to build a theoretical framework capable of hosting a heterogeneous body of work. By the mid-1550s, his scholarly output extends well beyond Archimedes: it includes Apollonius, spherical geometry, and a highly ambitious reorganization of arithmetic. What is missing is a context in which these domains can be integrated while preserving demonstrative legitimacy. The de Vega letter sketches this context at a programmatic level; the *Sermones* deepen it by identifying the notions that must carry the load – above all quantity, ratio, and proportion.

Maurolico uses the letter to frame his engagement with classical mathematics in terms of foundations and demonstrative legitimacy: before offering corrections or easier ways of proving, he insists on the need to “lay down the necessary foundations.”

In the *De quantitate sermo noster*, Maurolico proposes to consider quantity *simpliciter*, prior to its division into the specific species treated by arithmetic or geometry. Since quantity as such cannot be divided without being associated with qualities that differentiate its species, Maurolico introduces the idea of an undivided subject for an undivided mathematics – a *mathematica prima* concerned with operations and relations applicable

²⁷ *Sermo de divisione artium*, §§ 61–62: “His et illud addendum existimo ... Quae tria non solum elementariis, sed etiam Archimedaeis et aliorum demonstrationibus saepe usu veniunt. Super his enim tamquam fundamentis propositiones omnes successiva demonstratione construuntur.” (*Edizione Nazionale dell’opera matematica di Francesco Maurolico*, available at <https://maurolico.it/Maurolico/opera.html?path=2.C.1.1>).

to quantity in general: multiplication and division, ratio and proportion, commensurability and incommensurability. The point is not metaphysical. It is methodological: without such a level of generality, the kind of operational mathematics Maurolico is developing in the 1550s – above all in the second book of the *Arithmetica* – cannot claim the universality it demands.

An idea already hinted at in the de Vega letter is taken up again in the *De quantitate* and explicitly grounded: time, weight, mechanical moment, and *vis* are discussed as magnitudes that can be treated as continuous and compared *proportionaliter*. Time is framed as a continuous flux tied to an indivisible instant; weight and lightness, though ‘relative qualities’, are reduced to quantitative comparison by proportion; *momentum* is introduced as a distinct *potentia* exemplified by the balance and anchored in his doctrine of equal moments; and *vis* is treated as a further *potentia* whose instances can likewise be compared proportionally.²⁸

At this point, ratio becomes the bottleneck. In the *Sermo de proportione*, Maurolico introduces two distinct axes of classification: *rational/irrational* (relative to a *quantitas rationalis posita*) and *known/unknown*. Here *irrational* does not mean *unknown*, since ‘knowledge’ is a matter of *denominability* (by commensurability or by a law derived from it), not of commensurability as such.

On the first axis, Maurolico extends to *every* quantity a Euclidean-style classification into *rational* and *irrational* species. A quantity counts as rational if it is commensurable with the posited rational quantity either *in magnitude* or at least *in power*; it counts as irrational when it is commensurable in neither of these ways. Here “in power” is not a shorthand for the Euclidean quadratic case: Maurolico explicitly allows commensurability “in the first, second, or whatever power” and remarks that Euclid says nothing about quantities that are rational only in the cube, from which further families of irrational magnitudes and ratios could in principle be generated.²⁹

On the second axis, every ratio is, in the first place, either *known* or *unknown*. A ratio is *known* when it can be brought ‘into some knowledge’ by being bounded or stabilized *in numerical terms* – either directly by commensurability, or by some rule of relation that ultimately derives from commensurability. Incommensurability does not by itself block knowledge. An irrational ratio may still be known as long as the magnitudes can acquire

²⁸ *De quantitate sermo noster*, §§ 78–88 (Edizione Nazionale dell’opera matematica di Francesco Maurolico, available at <https://maurolico.it/Maurolico/opera.html?path=2.D.1.1>).

²⁹ On the open-ended scope of *in power*, see *Sermo de proportione*, § 126 (“*per potentiam primam, secundam, vel quotamcumque*”). On the extension beyond Euclid’s Book X (including ‘cube-rational’ quantities) and the potential proliferation of further species, see §§ 133–134; the appended table after § 137 (*Rationales, In potentia, Cubice, Mediales*) makes the intended grading explicit. (Edizione Nazionale dell’opera matematica di Francesco Maurolico, available at <https://maurolico.it/Maurolico/opera.html?path=2.D.1.2>).

‘names’ and be numerically signified through such a rule. Maurolico’s own example is telling: the ratio between the side of a pentagon and the diameter of the circle circumscribed about it is irrational, yet it is nonetheless a *known* ratio, precisely because both terms can be signified in numerical language (through their denomination and the laws that connect them).

By contrast, an *unknown* ratio is defined only privatively: it falls outside the domain of all denominable relations, admits no numerical naming, and is therefore *indemonstrable* in the required sense. The ratio of circumference to diameter is presented as the paradigmatic case – already hinted at in the de Vega letter. ‘Unknown’, in this sense, is not simply ‘incommensurable’: it is *anonymous*, *nullo pacto nominabilis*.

The upshot is sharper than a mere commensurable/incommensurable divide: the projected universality of the framework turns on a boundary between ratios that can be numerically denominated – even when irrational – and ratios that resist denomination altogether. At that point, universality becomes conditional rather than absolute. This tension will structure Maurolico’s later attempt to make *quantitas generalis* operative and, eventually, his return to Book V.

From concept to practice: *quantitas posita* and the arithmetization of magnitudes

The *Sermones* articulate an ambition, but they do not yet deliver a usable technique. To claim that mathematics may treat *quantitas simpliciter* is one thing; to show how such a notion can be made operative in concrete demonstrations is another. In Maurolico’s case, the bridge between the two is built around a single, deliberately instrumental device: the *posita* magnitude (or quantity), chosen *ad libitum* as a term of reference – and, in the explicit language of Book II of the *Arithmetica*, named *unitas* – used to *signify* magnitudes by numerical terms and thus make calculation on magnitudes demonstrably practicable.

A first, still local, manifestation of this device appears in the *Appendix* to the *Data* (7 April 1554), that is, only a few weeks before the *Sermones*.³⁰ There Maurolico makes explicit a principle that is easy to miss if one reads the *Sermones* in isolation: “no magnitude is given except with respect to a posited magnitude” (*nulla magnitudo datur, nisi respectu positae magnitudinis*). If one wants to perform constructions that amount to extracting roots or establishing mean proportionals, one must first fix a reference magnitude. In the geometrical setting of the *Data*, this reference is still typically a straight line, because it must be inserted into a construction. The point, however, is already unit-like: the posited magnitude functions as a general pole of comparison, in a way that is formally analogous to the unit in arithmetic.

³⁰ Veronica Gavagna and Rosario Moscheo, “I *Theonis Datorum libelli duo* di Francesco Maurolico,” *Bollettino di Storia delle Scienze Matematiche* 22, no. 2 (2002): 267–348. In Edizione Nazionale: <https://maurolico.it/Maurolico/sezione.html?path=4.3>.

By fixing a *magnitudo posita* as a term of reference, he shows how to construct the square root – and, mutatis mutandis, the cube root – of an arbitrary given magnitude. The procedure itself remains classical: for the square root he relies on the standard Euclidean construction via a mean proportional (*Elements*, VI.13 / II.14), while the cube root is obtained by inserting two mean proportionals. But the same move is then generalized and made systematic in Book II of the *Arithmeticon libri duo*.³¹

Here Maurolico states, in explicitly programmatic terms, that arithmetic is the instrument of all calculation and that every magnitude can be ‘signified’ through numbers; geometry, he adds, has a twofold practice – one by drawing, one by calculation – and the latter is derived from the former ‘as from a source.’ This distinction is introduced to account for procedures in which one searches, *by numbers*, ever closer to an irrational or even unknown magnitude. On this basis he announces a *novum demonstrandi genus*, tied not to the line as a privileged species, but to *general quantity* as the proper subject of a *mathematica prima*.

The operational core of this reorientation is the definition of *quantitas posita*. Maurolico characterizes it as the quantity freely chosen as the common measure for quantities of the same kind, and, as he puts it, “denominated *ab unitate*” – that is, it is *called unitas*, just as the unit is the common dimension of numbers. Everything that follows depends on this: at the definitional level, a magnitude is said to be *signified* numerically insofar as its relation to the posited quantity can be expressed through numbers in canonical forms. If the magnitude is a multiple of the posited one, it is signified by a single number; if it contains one or more parts of it, it is signified by a pair of numbers, numerator and denominator. In this sense, a fraction is not merely a numerical object; it is the canonical form under which a magnitude becomes arithmetically tractable.

Book II, however, makes clear that ‘numerical terms’ are not exhausted by integers and fractions. The practice Maurolico sets out to *demonstrate* explicitly includes operations on irrational quantities: extractions of square and cube roots, and, more generally, the handling of irrationals in calculation. The foundational definitions begin with the simplest signifying forms, but the operational dossier treats such irrational expressions as legitimate numerical terms, and develops rules for their use in computation.

Once magnitudes are ‘signified’ in this way, operations can be extended from numbers to magnitudes. This is most transparent in the definition of multiplication between quantities, which Maurolico defines through a proportional condition: a quantity *c* is the

³¹ On the *Arithmeticon libri duo* (Venezia, 1575), see Jean-Pierre Sutto, “Francesco Maurolico, mathématicien italien de la Renaissance (1494–1575)” (Thèse de doctorat, Université Paris VII–Denis Diderot, 1998), 317–384 (in particular pp. 350–370 regarding the arithmetic of *general quantity*). For the text of Book II: Edizione Nazionale, available at <https://maurolico.it/Maurolico/sezione.html?path=6.A.1>.

product of a and b , if c stands to b as a stands to the *posita* P .³² The definition shows how ratios are used to *control* the extension of numerical operations to magnitudes: multiplication becomes an operation on magnitudes only insofar as a proportional relation to P can be established and handled.

At this point the tension that will dominate the rest of the dossier becomes unavoidable. The entire apparatus presupposes that the ratio between a magnitude and the posited magnitude is itself a ratio that can be ‘named’ and treated as *cognita*. Maurolico can allow a very wide family of such cases – including incommensurable ratios, provided they are denominable *per magnitudinem* or *per potentiam quotamcumque*. But the procedure breaks down as soon as the relevant ratio is *incognita* in the stronger sense: *anonymous, nullo pacto nominabilis*. The *quantitas generalis* of the *Arithmetica* therefore comes with a built-in restriction: it is general with respect to *species* of magnitude (time, weight, line, surface, and so on), but not with respect to the *ratios* that arise within those species.

This is exactly the point at which the project begins to look like a structural detour rather than a straight road. The Archimedean corpus that Maurolico was trying to reorganize is saturated with comparisons that are controlled by proportional reasoning; and the promise of a *mathematica prima* is to turn that reasoning into a transferable calculus of quantity. Yet the moment the calculus is grounded on *posited quantity* and on numerical signification, it inherits the boundary drawn in the *Sermo de proportione*: not every ratio is a legitimate object of calculation. The more Maurolico pushes toward a general operational language, the more sharply the distinction between denominable ratios and anonymous ones becomes structurally decisive. It is against this background – and not merely as a late, isolated exercise – that his return to Book V must be read.

Recasting Book V: the 1567 compendium and its unresolved leap

The late *Compendium* of Book V is Maurolico’s most explicit attempt to neutralize the friction that had become unavoidable in the *Sermones* and in his arithmetization of magnitudes: how to reason *on ratios* without remaining trapped in the full Euclidean apparatus of equimultiples – and, at the same time, how to treat ratios as objects capable of carrying foundational work in a projected *mathematica prima*.³³

³² In a schematic modern shorthand: $a \times b = c \Leftrightarrow a : P = c : b$.

³³ Maurolico’s *Compendium of Book V* belongs to the broader set of *Elementorum compendia*, in which he produced a synopsis of the first ten books of Euclid’s *Elements*. This enterprise should be situated within Maurolico’s collaboration, in the last years of his life, with the Jesuits of Messina, a collaboration that aimed at producing a complete mathematical *cursus* for teaching purposes. On this context, see Rosario Moscheo, *I gesuiti e le matematiche nel secolo XVI. Maurolico, Clavio e l’esperienza siciliana* (Messina: Società Messinese di Storia Patria, 1998). On the *Compendium of Book V* in particular, see Jean-Pierre Sutto, “Le Compendium du 5^e livre des Éléments d’Euclide de Francesco Maurolico,” *Revue d’histoire des Mathématiques* 6 (2000): 59–94.

The core move is definitional, and it is best read as a deliberate step toward the autonomy of ratio. Proportion is no longer grounded in the classical comparison of equimultiples of the terms, but is recast as an *equality of ratios*.³⁴ In Maurolico's compendium, ratios are first sorted by 'name': two ratios may be said to be equal if they share the same denomination. When they do not, equality is decided by a systematic comparison with *named* ratios: $r_1 = r_2$ if, for every named ratio r , r_1 and r_2 occupy the same position with respect to r (i.e. whenever $r_1 \leq r$, then also $r_2 \leq r$). In this way, the criterion of equality no longer passes through the four terms that instantiate the ratios. The ratio itself is treated as the primary object of comparison, endowed with an order structure relative to a privileged class of ratios.

The crucial claim is Proposition 3: "every ratio of magnitudes is either named, or it falls between named ratios." Read on its face, this looks like the decisive closure Maurolico needs. It suggests that non-named ratios can always be bracketed by named ones and therefore controlled by comparison. In a framework built on denominability, this is the natural substitute for the excluded equimultiple machinery: one secures the governance of arbitrary ratios by anchoring them between ratios that can be named.

But this is precisely where the *Compendium* shows its limits. As Sutto notes,³⁵ the proof is exceptionally short and relies on the unargued existence of the magnitudes required to produce the bracketing ratios. Establishing that existence would demand substantial work on multiples (or an Archimedean-type lemma), or else a return to Euclid's missing material on proportion; yet the *Compendium* has eliminated the relevant equimultiple apparatus and does not provide substitute lemmas of comparable strength. Proposition 3 asserts the very control over arbitrary ratios that the project needs, but it does so by presupposing, rather than reconstructing, the labour that makes such control legitimate.

Even if we set aside this logical vulnerability, a more fundamental barrier still persists. The *Compendium*'s attempt to autonomize ratios *could* have led toward a recasting of number as the general surrogate for ratio – but Maurolico cannot take that step. 'Number' remains, for him, ultimately anchored in the classical idea of a plurality of units, even when

³⁴ The idea of defining proportion in terms of an equality of ratios goes back to Campanus's edition of the *Elements* (Book V, Def. 4), and Maurolico had already adopted it in the *Sermo de proportionibus*, § 94: "Videtur igitur ratio esse quodammodo relatio. Et tamen conferuntur rationes quo ad aequalitatem vel inaequalitatem, unde proportio est rationum aequalitas sive identitas." Already in 1554, therefore, Maurolico was thinking in terms of a definition of proportion that would "autonomize" ratios; not by chance, immediately after the passage quoted he proposes to use the Euclidean algorithm: "Posset et proportionalia aliter diffiniri sic: tunc enim bini hic termini binis illic positos proportionales esse dicuntur, cum quoties hic toties illic minor de maiori auferri potest, et quoties hic toties illic residuum de minori; itemque et quoties hic residuum de ablato, toties illic residuum de ablato, itaque in infinitum" (§§ 117–118).

³⁵ Sutto, "Le Compendium du 5^e livre des Éléments d'Euclide de Francesco Maurolico," 73–74.

numerical practice is widened to include fractions and radical expressions. This is stated with particular clarity in the *Arithmetica*, where he insists on the need to distinguish the two branches of geometry precisely because, “by means of numbers, we track ever closer the term or limit of an irrational or unknown magnitude” (*per numeros, irrationalis aut ignotae magnitudinis terminum seu limitem magis ac magis propinquantes vestigamus*) – for instance, in approximating the side of the equilateral triangle or square inscribed in a circle from a given diameter. The procedure licenses an asymptotic numerical pursuit; it does not produce a number that can signify the ratio that remains *incognita* in the stronger sense: *nullo pacto nominabilis*.

The result is a structural loop. Book II of the *Arithmetica* needs ratios to define operations on magnitudes via the *posita/unitas*; but a fully general arithmetic of *quantitas generalis* would require a theory of ratio that is autonomous enough to handle, as legitimate objects of calculation, precisely those relations that escape numerical denomination. The *Compendium* moves in that direction by treating ratios as objects with comparison and composition; yet, without re-importing the justificatory work it set aside, and without introducing a numerical surrogate capable of signifying all ratios, it cannot complete the leap.

In this sense, the *Compendium* does not close the Book V problem; it relocates it. And the relocation is fully consistent with the broader picture suggested by the *Sermones* and by the arithmetization of magnitudes: the more Maurolico leans on ratios to secure a calculus of *quantitas generalis*, the more sharply the internal boundary between denominable ratios and *incognitae* becomes structurally decisive.

5. Labyrinths

Maurolico: *quantitas generalis* and a structural loop

On the basis of the *Sermones* and the *Compendium*, and taking into account the previously discussed arithmetization of quantities, a single project becomes apparent, in which ratio is asked to mediate between two demands that are not easily reconciled. On the one hand, ratios are pushed toward a measure of autonomy – they must be comparable, orderable, and composable in ways that make them usable across domains. On the other hand, they must remain classically grounded as relations among magnitudes, governed by the constraints that guarantee demonstrative legitimacy. But a *quantitas generalis* needs ratios as the operative pivot that links the *posita/unitas* to multiplication, division, and the transport of procedures.

This is what we will call a *labyrinth*: a chain of internally justified dependencies in which each step is rational and locally productive, yet the sequence as a whole forces the argument back into the very structure it was trying to reorganize. A labyrinth, in this sense,

is a productive impasse: it yields clarifications and workable local devices, but it does so by sharpening the boundary conditions of the classical grammar rather than by dissolving them.

In Maurolico's case, the loop tightens around a single requirement: numerical denomination. The operative programme can advance as long as ratios can be brought under a form of naming that makes them usable in calculation; but the same programme also brings into focus relations that resist such naming altogether. The mismatch is generated from within the attempt to turn proportional comparison into a transferable calculus of general quantity, while staying inside the conditions that make ratio a legitimate object of demonstration within the Euclidean–Archimedean structure.

Galileo as a test case: physical qualities and proportion

What becomes of the Book V constraints when proportional reasoning is used to render physical 'qualities' quantitatively tractable? Galileo's work provides an ideal testing ground, and we appeal to it here only in this limited sense.

Galileo's entry into 'new magnitudes' is the outcome of a late-Renaissance formation shaped by Archimedean texts, Euclidean proportion theory, and the problem-oriented culture of mixed mathematics. In the writings of the Pisan period – the *De motu antiquiora* and the early work surrounding the *Bilancetta* – and still in *Le meccaniche* (early 1590s), Galileo is more interested in identifying the conditions under which a quality can be handled as a magnitude within proportion theory than in defining new physical quantities. The result is a *syncretic Archimedeanism*: a willingness to mobilize Archimedean-style devices (balances, moments, hydrostatic situations) together with Euclidean proportion schemata.

This juvenile laboratory also clarifies why the classical grammar can look, at once, powerful and fragile. It is powerful because it offers a stable and reliable way of operating with a target concept through comparisons among homogeneous magnitudes – spaces with spaces, times with times, weights with weights. But it is also fragile because this stabilization relies on a chain of mediations that may break down either when the phenomenon calls for a derived or compound quantity (whose magnitude-status and admissible operations are not secured within the classical ontology – for instance, specific weight), or when it calls for a practicable handle on variation in time, where no finite-interval comparison can directly stabilize the relevant magnitude. In this sense, Galileo's later successes and impasses are different paths inside the same Italian garden, where the hedges are set by Book V.

When this requirement – that ratios bind homogeneous magnitudes – is carried into natural philosophy, it shapes the approach to physical phenomena. Galileo enters this terrain with a clear working rule, already prepared by his early meditations: a physical quality becomes mathematically tractable only insofar as it can be stabilized as a magnitude

governed by proportion. In several domains the rule is productive. Uniform motion is the cleanest case: in *De motu aequabili* (conceived around 1609; published in 1638) ‘speed’ is not introduced as a numerical quotient, but as a magnitude that can be compared under explicit constraints, by transporting into kinematics a classical proportional scheme.³⁶ Once the direct and inverse proportionalities are fixed (spaces with spaces for equal times; times with times for equal spaces; etc.), Galileo can move beyond the narrowly Archimedean template that guided his earliest investigations and state operative rules in the idiom of Book V – compounded ratios and inversions do the technical work, while each step remains anchored in comparisons among homogeneous magnitudes.³⁷

A comparable reconstruction is available in hydrostatic contexts, where weights, volumes, and specific weights (*gravità in specie*) become manageable only through controlled comparisons (weights of equal volumes; volumes corresponding to equal losses of weight in immersion), as in the *Discorso intorno alle cose che stanno in sull’acqua* (1612).³⁸ In these cases the Euclidean-Archimedean pattern is stressed, but it can work, because homogeneity is preserved at every step.

The point of maximum stress is accelerated motion, because the problem now demands an operational grasp of variation. Here the ‘new magnitude’ is the instantaneous speed – a quantity that does not persist for any finite time and therefore resists direct stabilization. Galileo’s solution is a characteristic mediation: he anchors the instant to a measurable effect, interpreting instantaneous speed as a ‘moment’ or efficacy at impact (the *percossa* on a yielding medium). The strategy gives him a handle, but at a price. In the 1604 letter to Sarpi the search for a *principio totalmente indubitabile* leads to a false linear relationship between the ‘grades of speed’ (v) and distance (d) from the starting point: $v \propto s$. This move illustrates a deeper mechanism of the proportional framework: monotone dependence is naturally pressed into (linear) proportionality. When the correct dependence is later identified ($v \propto t$), the structural difficulty is displaced rather than removed: one still needs a surrogate that ties the evanescent instant to admissible magnitudes. The ensuing ‘integration’ of motion is therefore not a natural calculus but a tortuous chain of transports. Since the theory of proportions does not allow one to multiply heterogeneous

³⁶ Winifred L. Wisan, “The New Science of Motion: A Study of Galileo’s *De Motu Locali*,” *Archive for History of Exact Sciences* 13 (1974): 102–306, especially 281–286. More generally, for pre-classical conceptions of speed and the difficulties surrounding its definition and measurability, see the studies collected in Pierre Souffrin, *Écrits d’histoire des sciences. L’Âne d’or* (Paris: Les Belles Lettres, 2012).

³⁷ Giusti, *Euclides Reformatus*, ch. 2; and Enrico Giusti, “Ricerche galileiane: Il trattato *de Motu aequabili* come modello della teoria delle proporzioni,” *Bollettino di Storia delle Scienze Matematiche* 6, no. 2 (1986): 89–108.

³⁸ Napolitani, “La geometrizzazione della realtà fisica: il peso specifico in Ghetaldi e in Galileo,” esp. 190–232.

magnitudes, Galileo does not sum infinitesimal spaces of the form $v \cdot dt$; he sums ‘grades of speed’ (figures, triangles of velocities), obtaining a macroscopic velocity that must then be re-related to space through further proportional steps. The procedure yields results, but it does not dissolve the conceptual knot – and the final demonstrations inevitably rely on bridges that are as much rhetorical as they are mathematical.³⁹

A shared labyrinth – and a cut through it

Late-Renaissance mathematics can be pictured as a garden whose most carefully tended feature is, paradoxically, a labyrinth. The paths are themselves orderly, even elegant: Euclid’s Book V fixes what it means to compare magnitudes; Archimedean geometry shows how far one can go by inscription, circumscription, and controlled approximation; composed ratios and proportional schemata make procedures portable across an expanding repertoire of problems. Yet the very discipline of this layout ensures that certain turns have no exit. The garden is not ‘Italian’ in the sense of a free play of perspectives; it is ‘Greek’, in the sense that its rules of admissibility are built into the terrain.⁴⁰

Maurolico enters this space from a position that is, in a strict sense, more primitive. At the beginning of the sixteenth century the Archimedean corpus is not yet a stable object; it must be recovered, reconstructed, and organized. Maurolico does precisely this – and in doing so he tries to give Archimedean geometry of measure a more transparent foundation. His postulates of inclusion, his systematic reduction of ratios to ratios between lines, and, later, the device of a *quantitas posita* are all rational, locally productive moves: they aim to secure demonstrative legitimacy while making procedures transferable and operational. But the price of this internal strengthening is that ratio becomes the bottleneck. The more Maurolico leans on an operative notion of *quantitas generalis*, the more decisive becomes the boundary between ratios that can be denominated (and hence inserted into calculation) and ratios that remain *incognitae* in the stronger sense. The late return to Book V attempts to neutralize the friction by ‘autonomizing’ ratios, finding a way to control arbitrary relations. Maurolico, in short, helps build the labyrinth and simultaneously discovers why it closes.

³⁹ On Galileo and the laws of motion, see the introduction to Galileo Galilei, *Discorsi e dimostrazioni matematiche intorno a due nuove scienze attinenti la meccanica e i movimenti locali*, ed. by Enrico Giusti (Torino: Einaudi, 1990).

⁴⁰ The metaphor should not be taken to imply that this was the only cultivated ground of sixteenth-century mathematics. Around the Euclidean-Archimedean garden there developed algebraic and abacus-based traditions, ultimately indebted to Arabic mathematics. The point here is more limited: the garden considered in this article is the one whose rules of admissibility continued to govern the treatment of magnitudes, even when other mathematical plants were already growing along the walls of the garden and beginning to test the strength of those boundaries.

The same constraint will reappear, in a different key, once the Archimedean paradigm is carried into natural philosophy.

Galileo, unlike Maurolico, does not need to construct Archimedes. He inherits a late-Renaissance Archimedean landscape already populated by texts, commentaries, and problem-oriented practices, and he coordinates them into a distinctive synthesis. In several domains, this syncretic Archimedeanism proves extraordinarily effective: where the relevant qualities can be stabilized through controlled comparisons among homogeneous magnitudes, the Book V grammar can be made to carry real operative weight. But the hard case is accelerated motion. Here the demand is not merely to compare magnitudes but to grasp variation operationally, and the Euclidean restriction becomes structural: the very quantities one would like to treat as autonomous objects (instant, instantaneous speed, 'efficacy') must be reconstructed indirectly. The results are genuine, but the route remains tortuous; and precisely at the point where the new physics seems to require a general calculus, the classical proportional framework forces Galileo back into the same kind of detour that Maurolico had met from within geometry.

That both men, moving from radically different formations and aiming at radically different ends, are driven toward reforms of the theory of proportions is therefore not an accident but a symptom.

In this sense, Maurolico and Galileo occupy two distant entrances to a shared labyrinth. The first tries to secure the classical architecture from within, making explicit what Archimedean practice had left tacit and pushing toward a general operational treatment of quantity; the second tries to extend the same architecture outward, using it to render nature mathematically tractable. Their impasse is not merely personal, nor is it reducible to a lack of technical ingenuity. It belongs to a broader late-Renaissance configuration: Benedetti's reforms of proportion, Guidobaldo's mechanics, Valerio's generalization of Archimedean procedures, Cavalieri's instrumental dissolution of form – all are, in different ways, attempts to stretch the same grammar beyond the point where it can remain both rigorous and fully operative.

Viète marks a clear break: his symbolism makes algebra portable into geometry; but even Viète remains bound by the *lex homogenorum* and by proportional schemata that continue to regulate admissible comparisons when algebra is made to speak about magnitudes.

If these developments can be read, retrospectively, as anticipations, this is because they mark the outer limits of classical grammar, not because they disclose a hidden path out of it.

In the seventeenth century, mathematicians would find a way out of this labyrinth, not by following its paths more cleverly – this labyrinth, in fact, had no way out – but by applying new tools that allowed them to 'cut through the hedges' instead of walking along them. It is from this cutting-through that the journey of modern mathematics begins.

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Clavius' edition of Euclid's *Elements*: A computational approach

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Abstract

Christoph Clavius' Latin translation of Euclid's *Elements* is studied by applying standard and less standard tools of computational linguistics. Clavius' lexical choices are compared with the relevant Greek and Latin editions and translations. We shall thereby assess Clavius' treatment of Latin sources, as well as his specific, scholarly aims and his general strategy.

Keywords

Euclid's *Elements*, Christoph Clavius, Latin translations, computational linguistics

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Basics on Christoph Clavius and his edition of the Elements

Christoph Clavius' (1538–1612)¹ edition of Euclid's *Elements* stands out as one of the achievements of the “Italian Renaissance of mathematics.”² After the first edition of 1574,³ this monumental work was repeatedly reprinted during Clavius' lifetime, sometimes substantially enhanced by the author, and was finally set to open his *Opera Mathematica* of 1612.⁴ Clavius' edition comprises a Latin version of the Greek text and an overwhelming scholarly apparatus: additions to the original text in the form of definitions, lemmas, corollaries, and propositions, and again commentary, *praxeis*, and digressions on subjects that were not included in the original.

Clavius had a strategic aim in producing this kind of edition, and in fact in designing most of his works:⁵ establishing mathematical sciences as a scholarly discipline, suited to being included in the curriculum of the Jesuits' academies and, more generally, consolidating the Society of Jesus as a leading academic authority within the early modern Republic of Letters.⁶ Consequently, Clavius' aims in his Euclid edition were completeness and clar-

¹ A detailed biographical account of Clavius can be read in Clavius, Christoph. *Corrispondenza*, ed. by Ugo Baldini and Pier Daniele Napolitani (Pisa: Università di Pisa, Dipartimento di Matematica, 1992), vol. I, parte 1, 33–58. For the scientific context in Rome, with special emphasis on Clavius' contributions to the Gregorian reform of the calendar, see Ugo Baldini, “Christoph Clavius and the Scientific Scene in Rome,” in *Gregorian Reform of the Calendar. Proceedings of the Vatican Conference to Commemorate Its 400th Anniversary. 1582-1982*, ed. by George V. Coyne, Michael A. Hoskin, and Olaf Pedersen (Città del Vaticano: Pontificia Academia Scientiarum, 1983), 137–169. For Clavius' role in the last days of Ptolemaic astronomy see James M. Lattis, *Between Copernicus and Galileo. Christoph Clavius and the Collapse of Ptolemaic Cosmology* (Chicago and London: The University of Chicago Press, 1994). For overviews of Jesuit Science after Clavius see Mordechai Feingold, ed., *The New Science and Jesuit Science: Seventeenth Century Perspectives* (Amsterdam: Kluwer, 2003), and Michael John Gorman, *The Scientific Counter-Revolution. The Jesuits and the Invention of Modern Science* (London; etc.: Bloomsbury Academic, 2020).

² This is the title of an excellent study published half a century ago: Paul L. Rose, *The Italian Renaissance of Mathematics* (Genève: Librairie Droz, 1975).

³ *Euclidis Elementorum libri XV. Accessit XVI de Solidorum Regularium comparatione. Omnes perspicuis demonstrationibus, accuratisque scholijs illustrati. Auctore Christophoro Clavio Bambergensi Societatis Iesu. Romae, Apud Vincentium Accoltum. 1574.* On Clavius' edition see Sabine Rommevaux, *Clavius : une clé pour Euclide au XVI^e siècle* (Paris: Librairie Philosophique J. Vrin, 2005), with focus on Book V.

⁴ *Christophori Clavii Bambergensis e Societate Iesu Opera Mathematica V. Tomis distributa [...]. Moguntiae Sumptibus Antonij Hierat excudebat Reinhardus Eltz [...]. Anno M. DCXII*, 13–638.

⁵ A complete list of first editions is in Clavius, *Corrispondenza*, vol. I, parte 3, 5–11.

⁶ See the studies collected in Ugo Baldini, *Legem Impone Subactis. Studi su filosofia e scienza dei Gesuiti in Italia. 1540-1632* (Roma: Bulzoni, 1992); Romano Gatto, *Tra scienza e immaginazione. Le matematiche presso il collegio gesuitico napoletano (1552-1670 ca.)* (Firenze: Olschki, 1994); Rosario Moscheo, *I gesuiti e le matematiche nel secolo XVI. Maurolico, Clavio e l'esperienza siciliana*

ity of the proofs, exhaustiveness as to the proven results,⁷ completeness of the scholarly apparatus,⁸ emphasis on constructions (the *praxeis*) and on practical applications.⁹

It thus comes as no surprise that Clavius' Latin text of the *Elements* proper is not a translation from the Greek, but a complex mixture of (unavowed) borrowings and systematic rewriting.¹⁰ It is not even clear whether Clavius was able to read Greek, or, if indeed he could read it,¹¹ whether he ever bothered to check Grynaeus' *princeps* of 1533, let alone any Greek manuscript.¹² This approach was not new, to some extent: in fourteenth- and fifteenth-century Byzantium, every copyist-scholar shaped his own edition of the Greek text of the *Elements* by conflating material drawn from several models and by more or less heavily modifying the text.¹³ Grynaeus' edition – and in fact any edition of the Greek *Ele-*

(Messina: Società Messinese di Storia Patria, 1998); Antonella Romano, *La Contre-réforme mathématique. Constitution et diffusion d'une culture mathématique jésuite à la Renaissance* (Roma: École française de Rome, 1999), 35–178. Clavius' syllabus is edited and discussed in Romano Gatto, "Christoph Clavius' 'Ordo Servandus in Addiscendis Disciplinis Mathematicis' and the Teaching of Mathematics in Jesuit Colleges at the Beginning of the Modern Era," *Science & Education* 15 (2006): 235–258.

⁷ Read, at the beginning of the 1574 edition, the 48-page *Index problematum, ac theorematum, quae praeterea, quae continentur in Euclidis propositionibus, in his elementorum libris demonstrantur*. Following a mediaeval tradition, Clavius also proved all Euclidean "common notions," which included two postulates (see note 14 below): Vincenzo De Risi, "The Derivability Theory of Axioms. Logic and Mistranslation in the Middle Ages and the Renaissance," in *Pre-Modern Mathematical Thought. The Latin Discussion (13th–16th Century)*, ed. by Clelia V. Crialesi (Leiden: Brill, 2025), 237–288.

⁸ In this case, Clavius appears regularly to mention his sources, if only to contradict them. The issue is not irrelevant, for recent scholarship has sometimes regarded refuting the opposite claim as necessary: Eberhard Knobloch, "Sur la vie et l'œuvre de Christophore Clavius (1538–1612)," *Revue d'histoire des sciences* 41 (1988): 331–356, at 334–335, and Eberhard Knobloch, "L'œuvre de Clavius et ses sources scientifiques," in *Les jésuites à la Renaissance. Système éducatif et production du savoir*, ed. by Luce Giard (Paris: Presses Universitaires de France, 1995), 263–283, at 267–270. A blatant case of unavowed borrowing escaped Knobloch: see below.

⁹ On this feature see in particular Audrey Price, "Pure and Applied: Christopher Clavius's Unifying Approach to Jesuit Mathematics Pedagogy" (PhD diss., University of California, San Diego, 2017).

¹⁰ Yet, Clavius also calls it *interpretatio* (*Ordo*) and *traditio* (*Ordo brevior*): Gatto, "Christoph Clavius," 249, 252–253. More on this crucial point in our Conclusion(s).

¹¹ In his edition of the *Elements*, Books I–VI, Clavius mentions 3 Greek words: βεβηκέναι, ἀπολαμβάνω, ἀναλογία. The contexts of the mentions might suggest that Clavius' command of Greek was at best suboptimal. See also note 80 below.

¹² ΕΥΚΛΕΙΔΟΥ ΣΤΟΙΧΕΙΩΝ ΒΙΒΛ. ΙΕ'. ΕΚ ΤΩΝ ΘΕΩΝΟΣ ΣΥΝΟΥΣΙΩΝ. Εἰς τοῦ αὐτοῦ τὸ πρῶτον, ἐξηγημάτων Πρόκλου βιβλ. δ'. [...]. Basileae Apud Ioan. Hervagium Anno M. D. XXXIII. Mense Septembri.

¹³ On this widespread Byzantine practice of appropriation by rewriting see Fabio Acerbi, "Rewriting Mathematical and Astronomical Treatises," in *The Routledge Handbook for Rewriting in Byzantium*, ed. by Juan Signes Codoñer, Martin Hinterberger, and Inmaculada Pérez Martín (London: Routledge, 2025), 407–419.

ments until Peyrard's in 1814 – inherited a striking example of such a tampering with the text: the last two postulates were printed among the “common notions.”¹⁴

Clavius could check plenty of Latin translations: the most successful Arabo-Latin translation, and the first that was printed, namely, Campanus';¹⁵ the first Renaissance translation from the Greek, authored by Bartolomeo Zamberto,¹⁶ first published in 1505 along with the translation of Euclid's minor works and subsequently, and aptly, reprinted propositionwise, intermingled with Campanus', by Lefèvre d'Étaples in 1516;¹⁷ Federico Commandino's translation, printed two years before Clavius' edition.¹⁸ Other translations, sometimes associated with the Greek text,¹⁹ were available, and even abridged editions comprising only the principles and the enunciations of the propositions. Of the latter kind was Stephanus Gracilis' edition, printed in 1557.²⁰ The latter is exactly the only obvious source of Clavius' Latin text of the *Elements*: for reasons that escape us and without recognising his debt, he lifted everything he could lift from Gracilis' booklet, making small, cosmetic changes, if any, and modifying the wording in a few cases only.²¹ As for the

¹⁴ See Fabio Acerbi, Stefano Martinelli Tempesta, and Bernard Vitrac, “Gli interventi autografi di Giorgio Gemisto Pletone nel codice matematico Marc. gr. Z. 301,” *Segno e Testo* 14 (2016): 411–456. This is the reason why, in Bolyai's groundbreaking work on the parallel postulate, this is referred to as *axioma XI euclidei*, and not as *postulatum V euclidei*.

¹⁵ The colophon is *Opus elementorum euclidis megarensis in geometriam artem In id quoque Campani perspicacissimi Commentationes finiunt. Erhardus ratdolt Augustensis impressor solertissimus. venetiis impressit. anno salutis M.cccc.lxxxij. Octavis. Calendas. Junii.*

¹⁶ *Euclidis megarensis philosophi platonici Mathematicarum disciplinarum Janitoris [...]* Bar. Zamberto. Vene. Interprete. [...] *Impressum Venetiis [...]* in aedibus Ioannis Tacuini librarii accuratissima diligentia recognitum. Anno reconciliatae divinitatis. M.D.VIII. Kalendas novembris. For Zamberto's manuscript sources, see Bernard Vitrac, “A propos de l'histoire du texte des *Éléments* d'Euclide : Préalables à une nouvelle édition critique” (hal-03328161, 2022).

¹⁷ *Euclidis Megarensis Geometricorum elementorum libri XV, Campani Galli transalpini in eosdem commentariorum libri XV, Theonis Alexandrini Bartholamaeo Zamberto veneto interprete, in tredecim priores, commentariorum libri XIII, [...]* Parisiis in officina Henrici Stephani e regione scholae Decretorum (1516).

¹⁸ *Euclidis Elementorum libri XV. Unà cum scholijs antiquis, À Federico Commandino urbinatè nuper in latinum conversi, commentarijsque quibusdam illustrati, Pisauri, M D LXXII.* For Commandinus' manuscript sources see Bernard Vitrac, “La traduction latine des *Éléments* d'Euclide par Federico Commandino : sources, motivations” (hal-03328386, 2021).

¹⁹ See the website Médecée at <https://thamous.univ-rennes1.fr/docs/Medee/gb/index.html#> (Bernard Vitrac and Alain Herreman).

²⁰ *Euclidis Elementorum libri XV. Graecè & Latinè, Quibus, cum ad omnem Mathematicae scientiae partem, tum ad quamlibet Geometriae tractationem, facilis comparatur aditus [...]* Lutetiae, Apud Gulielmum Cavellat [...] 1557.

²¹ See Francesca Salvi, “Le fonti degli *Elementi* di Euclide di Cristoforo Clavio” (Tesi di Laurea, Università degli Studi di Firenze, 2005), part 2, and, for Book V, Enrico Giusti, “La théorie des proportions au XVI^e siècle : entre philologie et mathématiques,” in *Liber amicorum Jean Dhom-*

constructions and the proofs, Clavius made eclectic choices:²² in most cases he loosely followed the deductive progression of the Greek text while adding cases; sometimes, he reorganised the proof; in a few cases, he radically changed it.²³ In every instance, Clavius did not translate the text (read his explicit statement in our Conclusion(s)), but provided a new formulation of almost all sentences. He likewise felt free to redraw almost all diagrams; his propositions may also adopt a different lettering (first example in Prop. I.3).

The aims of the present contribution

In our perspective,²⁴ Clavius' choice of lifting the principles and the enunciations from Gracilis' edition while translating the rest *suo Marte* has a capital consequence: the lexicon of his edition is redundant. There are two striking examples. One comes from Prop. I.11, where the enunciation has Gracilis' *excitare* for ἀγαγεῖν ("to raise"), whereas 2 lines later, the setting-out, which is intended to replicate the lexical content of the enunciation and is absent in Gracilis' edition,²⁵ has *erigere*. The other is from Book V, where λόγος is Gracilis' *ratio* in the enunciations but (often, not always!) Clavius' own preferred translation *proportio* in the proofs.

Well, what is our perspective, and what are our aims? We shall automatically process the lexical content of Euclid's text in Clavius' edition and compare it with the lexical content of some reference texts by using standard and less standard stylometric tools. We

bres, ed. by Patricia Radelet-de Grave (Turnhout: Brepols, 2008), 173–193, at 185 and 193.

²² See, by clicking on "Clavius 1574," the structural correlations listed in the Médée website mentioned in note 19 above.

²³ The partitions into these three categories of the proofs of the propositions in Books I and XI are (28,19,1) and (27,11,2), respectively: Salvi, "Le fonti," 93 and 129, and the detailed comparisons at 93–128 and 129–151, respectively.

²⁴ The stress on "our perspective" also means that the lexical redundancy of Clavius' Latin text of the *Elements* does not affect the mathematical soundness and readability of the latter, nor does it undermine Clavius' work as a mathematician. We shall return on this point in the Conclusion(s). An extreme example of lexical redundancy of a (mediaeval) Latin translation from Greek is given by Henry Aristippus' version of Plato's *Phaedo*: Lorenzo Minio-Paluello, "Henri Aristippe, Guillaume de Moerbeke at les traductions latines médiévales des Météorologiques et du De Generatione et Corruptione d'Aristote," *Revue Philosophique de Louvain* 45 (1947): 206–235, at 213–214 = Lorenzo Minio-Paluello, *Opuscula. The Latin Aristotle* (Amsterdam: Adolf M. Hakkert, 1972), 64–65.

²⁵ For the proof-theoretical terminology used in this paper, and in general for all features of the demonstrative stylistic code of Greek mathematics, see Fabio Acerbi, *The Logical Syntax of Greek Mathematics* (New York; Heidelberg: Springer, 2021). See our Conclusion(s) on Clavius programmatically disregarding all features – even such obvious features as the exact and systemic correspondence between enunciation and setting-out – of the Euclidean demonstrative stylistic code.

conceived and realised both the database (xml coding, tagging, and lemmatisation of all works mentioned below – in fact, of the entire Greek and Byzantine mathematical and astronomical corpus – supplemented by a three-tier morphosyntactic categorisation of the lemmas)²⁶ and, unless otherwise stated, the Python scripts that allow creating and automatically processing it. We also conceived the several notions introduced and the scripts that implement them. Neither our database nor our scripts are in the public domain.

The reference texts we shall use are the Greek text of the *Elements* in Heiberg's standard edition, the 12th-century Graeco-Latin translation in Busard's edition, and Commandinus' translation.²⁷ We did not include any Arabo-Latin translation because the lexical record is blurred by the passage through Arabic. We did not include any other Renaissance translation of the Greek *Elements* because the task of producing and checking an OCR of any such text is time-consuming, life is short, and skilled contractors apparently hide behind Pluto's orbit.²⁸ In particular, both editions of Zamberto's translation are an OCR nightmare, with the additional hassle that one must disentangle the thousands of references to previous results, by book and proposition number, that Zamberto introduced in the text. For the same reasons we limited our investigation to Books I–VI of the *Elements*.

Our primary aim is to provide a quantitative, multi-focus assessment of the aforementioned lexical redundancy of Clavius's text. Our strategic aim is to establish a protocol for automatically processing corpora made of highly regularised scientific texts written in natural language, possibly involving translations. One must stress that stylometry as applied to this kind of texts cannot provide anything that is not obvious to any well-informed and perceptive reader. It does, however, provide a quantitative approach – and hence numer-

²⁶ According to our forms-to-lemma database, which amounts to a perfect parser by its very conception, the standard Greek parsers applied to technical texts have a failure rate of about 30%. The performance rate for technical Latin is higher but still far from optimal.

²⁷ The Graeco-Latin translation had almost no circulation: two extant witnesses only are known, one of them incomplete – but recall that the quotes from the *Elements* in Fibonacci's *Liber abaci* (1228) are taken from this translation: Menso Folkerts, "Leonardo Fibonacci's Knowledge of Euclid's *Elements* and of Other Mathematical Texts," *Bollettino di Storia delle Scienze Matematiche* 24 (2004): 93–113. The Graeco-Latin translation is edited in Hubert L. L. Busard, *The Mediaeval Latin Translation of Euclid's Elements made directly from the Greek* (Stuttgart: Franz Steiner Verlag, 1987); it is an output of the so-called "Sicilian school" of translation (a bit more on this in note 76 below). The e-text of this translation has been downloaded from the website <https://www2.hf.uio.no/polyglotta/index.php>, checked (just a few mistakes have been spotted), and converted to an e-text suitable for our purposes. Commandino's translation and Clavius' edition have been OCR'd, checked, and converted.

²⁸ The production of the regularised e-texts of Commandinus' translation and Clavius' edition was partly funded by the project Sin-aps (Alexander von Humboldt-Professorship, FAU Erlangen-Nürnberg). Getting regularised e-texts of all Latin translations of the *Elements* is one of the goals of the ANR-DFG project EUCLIDES (PI Vincenzo de Risi), see <https://sphere.cnrs.fr/en/projets-en-cours/euclides-anr-project-2024-2027/>.

ical data and powerful visualisation tools – to such obvious yet vaguely apprehended features: in our case, that Clavius’ text of the *Elements* is not a translation. In a sense, then, the notions, the tables, the graphs, the diagrams, the bulk and intensive indicators presented in what follows are not simply a tool, but the goal, and for this reason they are a good many and varied: they *are* a significant quantitative description of the involved texts according to a carefully devised protocol, and hence they speak for themselves and are, at least in our present perspective, more important than any moral we may derive from them – we shall derive one in the final conclusion, don’t worry.

As said above, the entire lexical record of the above-listed four works has been recorded and tagged via lemmatisation. As the frequency distributions of word tokens and types in a text are statistical ensembles, we shall frequently use notions borrowed from statistical thermodynamics and define a number of statistical features and indicators accordingly.²⁹ In particular, we shall systematically use suitably defined “entropy” and “temperature” parameters as indicators of lexical disorder,³⁰ and a set of matrices and associated numerical indicators that display and measure, respectively, the lexical reorganisation that occurs in translation. The discussion focuses in succession on defining, computing, presenting, and comparing the relevant statistical features and indicators of the lexicon of the selected works. These features and indicators fall into five broad categories:

- *General data related to lexical content.* We discuss the progressive creation of the dictionary of a given work; the extent to which a power law is a good fit to the frequency distribution of the lemmas; the distribution of *hapax*, one-form lemmas, and most frequent lemmas; the form spread within one and the same lemma; the distribution of lemmas, forms and occurrences among the parts of speech (POS).
- *Location and density of the information content within a proposition and a Book, by morphosyntactic types.* We introduce the intensive indicators X-entropy and X-cloud for categorial types X that generalise the standard POS; some representative computations of these indicators are carried out.
- *Lexical reorganisation consequent on translation.* We discuss in detail the lexical spread consequent on translation by introducing the correlation tables and the correlation and dispersion matrices of a translation, along with the notion of “metalemma.” On these grounds, we measure the lemmatic reorganisation that has occurred in a given translation by introducing several indicators that process the data set out in the correlation tables and in the dispersion matrix.

²⁹ In the same perspective, see the breakthrough study Pierluigi Contucci, Claudio Giberti, Godwin Osabutey, and Cecilia Vernia. “Statistical properties of the rooted-tree encoding of $\mathbb{N}$,” *Physica A* 686 (2026), art. 131361, <https://doi.org/10.1016/j.physa.2026.131361>.

³⁰ Entropy was famously introduced in information theory, and thereby and later in computational linguistics, in Claude E. Shannon, “A Mathematical Theory of Communication,” *The Bell System Technical Journal* 27 (1948): 623–656.

- *Textual proximity.* After this, standard stylometric tools are used to determine the relations of proximity between the selected works: cluster analysis, consensus trees, network analysis, and principal component analysis (PCA).³¹
- *Syntactic and deductive depth.* A dedicated Section goes deeper into syntax by displaying and discussing the “diagrams of subordination” of Book I and by comparing a few bulk and intensive indicators of deductive depth.

A warning is in order before beginning with the lexical analysis. Even strictly analogous texts in Greek and Latin – even more so if the Latin texts are translations of a Greek text – need not display similar lexical data. The main reason, and a crucial point in the process of naturalisation of a rigid stylistic code as the one employed in Greek demonstrative texts,³² is the absence of the definite article in Latin. This may be partly compensated by an increase of the (pro)nominal content of the Latin translations,³³ for the nominalising article in Greek technical texts also allows omitting plenty of nouns. As we shall see in the next Section, the relative frequency of the article in the Greek *Elements* is very high. However, denotative letters are an essential feature of the demonstrative code and at least one article must figure in each designation. Consequently, most of the articles can be safely omitted in translation,³⁴ even if they may be replaced by nouns and pronouns.³⁵ Add that the verb morphology is richer in Greek than in Latin, and that ambiguous word forms are less frequent in Greek than in Latin: think of the ambiguity of such widespread forms as *quam* and *quod*, a real nightmare for someone engaged in lemmatising a Latin text.

³¹ We used the `stylo` package for dendrograms, consensus trees and PCA plots. See the descriptions of the package in Maciej Eder, Jan Rybicki, and Mike Kestemont, ‘*Stylo*: a package for stylometric analyses’ (2015); Maciej Eder, Jan Rybicki, and Mike Kestemont, ‘Stylometry with R: A Package for Computational Text Analysis,’ *R Journal* 8 (2016): 107–121. The package can be downloaded at <https://github.com/computationalstylistics/stylo>.

³² On the meaning of “naturalisation” in this context see Abdelhamid I. Sabra, “The Appropriation and Subsequent Naturalization of Greek Science in Medieval Islam: A Preliminary Statement,” *History of Science* 25 (1987): 223–243.

³³ In the Graeco-Latin translation, the article is frequently replaced by conjoined forms of *is* and *qui*; Commandinus often employs forms of *ipse* to the same end. As a matter of fact, these pronouns “translate” the case of the denotative letter, as the nominalising article in Greek has first and foremost this function. For this reason, we did not regard Commandinus’ *ipse* as a translation of the Greek article. On William of Moerbeke’s “Latin” translation of the Greek nominalising article with the French article *le* – one of the highest points of his linguistic genius – see Lorenzo Minio-Paluello, “Guglielmo di Moerbeke traduttore della *Poetica* di Aristotele (1278),” *Rivista di Filosofia Neo-Scolastica* 39 (1947): 1–17, at 5–7 = Id., *Opuscula*, 44–46.

³⁴ In the *Elements*, only 23% of the articles are not attached to denotative letters.

³⁵ Just check the frequency of forms of *recta* in the first propositions of Book I in the Graeco-Latin translation.

Lexical analysis

By applying the tools of computational linguistics described in the previous Section, we now compare the lexical content of Books I–VI of the Greek *Elements* (*Elem.* henceforth), of the Graeco-Latin translation (*Gr.-Lat.*),³⁶ of Commandinus’ translation (*Comm.*), and of Euclid’s text in Clavius’ 1574 edition (*Clavius*).³⁷ These texts will be collectively referred to as “the four works;” the three Latin texts as “the Latin translations” – even if we have seen that *Clavius* is not a real translation.

Some definitions are in order. Every continuous sequence of signs in a text is an “occurrence;” identical occurrences are a “form;” the set of forms related to one and the same term is a “lemma;” the set of lemmas is the “dictionary” of the text. A lemma is designated by writing one of its forms without diacritics, if any is required in the intended language: ἀπὸ, ἀπό, ἀπ, and ἀφ’ are forms of the Greek lemma *απο*; *quotcunque* and *quotcumque* are forms of the Latin lemma *quotcumque*. The headwords of any lexicon follow the same principle, while keeping diacritics. Words resulting from a crasis such as καὶ ἐκεῖνον are split into καὶ + ἐκεῖνον, which are recorded separately as forms of the lemmas *καί* and *εκείνος*; a Latin word to which enclitic *que* is attached is split as *andque* + *word*, which are assigned to different lemmas. In order to eliminate lexical noise, the denotative letters, the numbers, and the transliterated Greek words in Latin texts count as one single lemma each.

Raw data, creating the dictionary

The following table compares the raw lexical data of the four works (a heading like, e.g., “% l/o” means the ratio lemmas/occurrences in percentage).

	lemmas	forms	occurrences	% l/o	% f/o	% l/f
<i>Elem.</i>	348	1773	52894	0.66	3.35	19.63
<i>Gr.-Lat.</i>	415	1743	46067	0.90	3.78	23.81
<i>Comm.</i>	380	1665	45176	0.84	3.69	22.82
<i>Clavius</i>	619	2400	47371	1.30	5.07	25.79

³⁶ This translation merely transliterates 29 forms and wavers between transliterating and translating 28 forms; see Busard, *The Mediaeval*, 12–15. Busard does not mention *esto*, which in no instance is the Latin future imperative of *sum*, but a transliteration of the Greek present imperative ἔστω; we shall treat this as a Latin form, but normally we count the transliterated forms that figure in Books I–VI as one single lemma, see just below.

³⁷ Disentangling Euclid’s text from the commentary in *Comm.* and *Clavius* is easy, since the commentary is expressly marked as such. Happily, both editions also put the book-and-proposition references to previous results in the margins. We did not include these references in the lemmatised text.

The lower number of occurrences in the Latin translations is entirely due to the absence of the article. *Elem.* is famously the lexically driest piece of literature ever written; *Comm.* is more restrained than *Gr.-Lat.*; the lexicon explodes with *Clavius*, which also engages its lemmas in a wider range of syntactic configurations (check the ratio forms/occurrences). The best linear regression of the Latin data with respect to size (occ.) is given by the ratio lemmas/forms ($\times 10^4$), with slope -0.1365 and a coefficient of determination $R^2 = 0.9933$.

We may trace the progressive creation of the dictionary of an assigned work by computing the relative (that is, normalised to the size of the dictionary) number of lemmas engaged up to any given textual unit. In our case, “textual unit” means either a principle (definition, postulate, or common notion, the latter two in Book I only) or a proposition (regular proposition, lemma, or corollary).

The patterns of the progressive creation of the dictionary in the four works are displayed as Figures 1a–d of the Appendix; the textual units are counted sequentially starting from def. 1 of Book I; the red vertical bars mark the beginning of a Book. The steep increase in *Clavius*, textual unit 61, coincides with the beginning of the propositions of Book I. Otherwise, the curve in *Clavius* displays, more than the three other works, the sublinear behaviour typical of texts written in natural language, as codified in Heaps' law.³⁸ Conversely, the increase that coincides with the beginning of Book V is less marked in *Clavius* than in the three other works. As the words that denote a mathematical object – whose first occurrence is normally found in a definition – are a small fraction of the entire dictionary, there is no sharp step at the beginning of each Book.

Zipf's law

The most important statistical item associated with the lexicon of a text is the distribution of the number of occurrences referred to each lemma (this is the “frequency”). We can study the rank distribution of the lemmas as a function of their frequency,³⁹ in order to check to what extent they follow Zipf-Mandelbrot's law.⁴⁰ This law states that, for any

³⁸ Harold S. Heaps, *Information Retrieval. Computational and Theoretical Aspects* (New York; San Francisco; London: Academic Press, 1978), sect. 7.5.

³⁹ The least frequent lemma has rank 1, the second most frequent one has rank 2, etc.; consecutive ranks are associated with lemmas that have the same number of occurrences. Note that the literature is divided between using either the frequency or the rank as the independent variable. We adopt the same convention as in Newman's papers cited in the next note; this convention is to be preferred on theoretical grounds. Zipf and Mandelbrot put the rank as the independent variable.

⁴⁰ See Benoît Mandelbrot, “Information Theory and Psycholinguistic,” in *Scientific Psychology*, ed. by Benjamin B. Wolman (New York: Basic Books, 1965), 550–562; Mark E. J. Newman, “Power Laws, Pareto Distributions and Zipf's Law,” *Contemporary Physics* 46 (2005): 323–335; Aaron Clauset, Cosma R. Shalizi, and Mark E. J. Newman, “Power-law distributions in empirical data,”

sufficiently large linguistic *corpus*, the rank r of each lemma as a function of its frequency f (very nearly) follows an inverse power law like $r = Af^{-\alpha}$, where A and α are positive real numbers and $\alpha = 1$ in this case. If we take the logarithm of the two variables, the distribution of the data in a log-log graph is (very nearly) a straight line. Modifications made by Benoît Mandelbrot to Zipf's law include the introduction of a rank shift that amounts to setting a lower bound $f_{\min}$ to the scaling range and the generalisation to scaling exponents α other than 1.

The rank distributions of the lemmas as a function of the frequency in the four works are plotted in the log-log graphs displayed as Figures 2a–e of the Appendix (the scale is the same); the elongated “dots” in the graphs represent lemmas that have the same frequency. We also included the data of the complete text (that is, including all sorts of commentary) of Clavius' 1574 edition, Books I–VI; this text is noted *Clavius**. Two distributions appear to merge in each graph: an almost-linear yet sparsely populated high-frequency regime (normally dominated by function words, see below) merges with a less neatly delineated middle- and low-frequency regime. This means that middle-frequency lemmas are over-represented in the geometric sectorial lexicon, a fact that is also confirmed by the sparseness of the highest-frequency lemmas. As a mathematical treatise has two naturally embedded scales (the proposition and the Book), the superposition of at least two regimes is the behaviour that is to be expected.⁴¹ The distribution of *Clavius** appears to be less hunched than the others; this will be confirmed by the numerical data introduced and set out in the next paragraph.

The table below sets out the following statistical parameters, in the hypothesis that a power law does fit the actual frequency data: the frequency $f_{\min}$ that is the lower bound of the scaling range; the maximum likelihood estimate of the scaling exponent α ; the expected error σ on it; the p value of the goodness-of-fit KS test taken as a confirmatory parameter, which means that, in the indicated range, the power law is a good fit if $p < S$ (and the lower is p , the better is the fit), where the significance level S is usually taken to be 0.1 or 0.05. The hypothesis that the data follows a power law is thus in general weakly supported. A comparison with other candidate distributions by means of a suitably

SIAM Review 51 (2009): 661–703. In the computations described below, we used Jeff Alstott's package `powerlaw` (https://nbviewer.org/github/jeffalstott/powerlaw/blob/master/Manuscript_Code.ipynb), presented in Jeff Alstott, Ed Bullmore, and Dietmar Plenz, “powerlaw: a Python package for analysis of heavy-tailed distributions” (arXiv:1305.0215v3, 2014); see the resources in <https://aaronclauset.github.io/powerlaws/>.

⁴¹ For a similar phenomenon occurring in large corpora (> 2000 lemmas) see Ramon Ferrer i Cancho and Ricard V. Solé, “Two Regimes in the Frequency of Words and the Origins of Complex Lexicons: Zipf's Law Revisited,” *Journal of Quantitative Linguistics* 8 (2001): 165–173; Marcelo A. Montemurro, “Beyond the Zipf–Mandelbrot law in quantitative linguistics,” *Physica A* 300 (2001): 567–578, which appears largely to draw on the previous.

defined p value shows that a power distribution is a better candidate (“Y” in the table) than an exponential distribution in all cases, whereas it is a worse candidate (“N” in the table) than a lognormal distribution in all cases bar *Clavius**. The high value of $f_{\min}$ for any text except *Clavius** confirms the fact that, in point of style, *Clavius** is more akin to a literary text than to mathematical text.

	$f_{\min}$	α	σ	p	power vs.	
					exp	lognorm
<i>Elem.</i>	147	2.108	0.146	0.063	Y	N
<i>Gr.-Lat.</i>	182	2.049	0.158	0.070	Y	N
<i>Comm.</i>	187	2.257	0.176	0.049	Y	N
<i>Clavius</i>	139	2.217	0.149	0.054	Y	N
<i>Clavius</i> *	1	1.398	0.010	0.044	Y	Y

Uniqueness of use: 1-occurrence and 1-form lemmas

Let us now look closely at the head of the frequency distribution of lemmas and forms. The data related to the lemmas that have 1 occurrence only (“hapax” without qualification, henceforth) or 1 form only (indeclinable words are included, the hapax are not) is as follows:

	lemmas	hapax	%	1 form	%	tot.	%
<i>Elem.</i>	348	51	14.66	88	25.29	139	39.94
<i>Gr.-Lat.</i>	415	78	18.80	126	30.36	204	49.16
<i>Comm.</i>	380	56	14.74	136	35.79	192	50.53
<i>Clavius</i>	619	131	21.16	199	32.15	330	53.31

This data is not easy to model. The relative frequency (lemm.) of hapax is fairly stable, as its spread, which is 44.61%, is lower than the size (lemm.) spread, which amounts to 77.59%. *Comm.* has fewer hapax than the other Latin texts but more 1-form lemmas.

The absolute and relative frequencies (lemm.) of the hapax by POS are as in the following table; the columns shaded grey contain the relative frequency of the overall lemmas referred to each POS (see the sub-Section *Distribution of lemmas, forms, and occurrences by POS* for the complete data). *Clavius*' metadiscursive drive is made obvious by the high relative frequencies of verbs and adverbs.

	<i>Elem.</i>			<i>Gr.-Lat.</i>			<i>Comm.</i>			<i>Clavius</i>		
	#	%		#	%		#	%		#	%	
adjective	15	29.41	25.86	19	24.36	26.02	15	26.79	25.00	30	22.90	20.84
adverb	7	13.73	10.92	14	17.95	12.29	14	25.00	15.00	33	25.19	18.42
noun	11	21.57	19.54	16	20.51	18.80	8	14.29	12.31	21	16.03	16.48
particle	4	7.84	8.62	6	7.69	8.43	2	3.57	2.46	5	3.82	8.89
preposition	1	1.96	4.60	4	5.13	5.30	1	1.79	1.26	1	0.76	4.20
pronoun	1	1.96	4.60	0	0.00	3.37	1	1.79	4.38	2	1.53	2.58
verb	12	23.53	25.29	19	24.36	25.06	15	26.79	25.29	39	29.77	28.27

We also give the distribution of the absolute and relative frequencies of the 1-form lemmas bar *hapax* by POS. The partly filled column gives the number of Greek indeclinable words in *Elem.* that exhibit at least two forms because of accentual phenomena (bar-ytonesis, enclisis accent) or elision.

<i>cat.</i>	<i>Elem.</i>				<i>Gr.-Lat.</i>				<i>Comm.</i>			<i>Clavius</i>				
	#		%		#		%		#		%		#		%	
adjective	15		17.05	25.86	18	14.29	26.02	17	12.50	25.00	16	8.04	20.84			
adverb	25	6	28.41	10.92	37	29.37	12.29	43	31.62	15.00	80	40.20	18.42			
noun	12		13.64	19.54	11	8.73	18.80	9	6.62	12.31	10	5.03	16.48			
particle	13	13	14.77	8.62	29	23.02	8.43	37	27.21	2.46	46	23.31	8.89			
preposition	7	8	7.95	4.60	16	12.70	5.30	16	11.76	1.26	25	12.56	4.20			
pronoun	5		5.68	4.60	4	3.17	3.37	3	2.21	4.38	4	2.01	2.58			
verb	11		12.50	25.29	11	8.73	25.06	11	8.09	25.29	18	9.04	28.27			

High-frequency lemmas

If the *hapax* are the head of the rank/frequency distribution of lemmas, the most frequent lemmas are its tail. The 40 most frequent lemmas of the four works are listed in the tables below. The lemma “o” is the Greek article, “A” the lemma “denotative letter;” the colour shadings identify the POS.⁴² We set out the data of *Elem.* first:

⁴² The lemmas “article” and “denotative letter” are excluded from shading because they always occupy the first places; the adverbs are also unshaded. For the correspondence colour-POS, see the first table after this sequence of four.

lemma	occ.	%	lemma	occ.	%	lemma	occ.	%	lemma	occ.	%
ο	11446	21.64	ευθεια	623	1.18	μεν	350	0.66	οτι	248	0.47
A	9584	18.12	απο	611	1.16	ορθος	338	0.64	δεω	247	0.47
ειμι	2622	4.96	αυτος	583	1.10	γαρ	320	0.60	οσπερ	236	0.45
και	2190	4.14	τριγωνος	570	1.08	ουτως	313	0.59	λοιπος	228	0.43
ισος	1899	3.59	κυκλος	483	0.91	πλευρα	285	0.54	εαν	228	0.43
υπο	1350	2.55	δυο	416	0.79	σημειον	269	0.51	εκατερος	227	0.43
προς	1174	2.22	επι	389	0.74	δη	264	0.50	διδωμι	223	0.42
αρα	1154	2.18	ως	377	0.71	δεικνυμι	263	0.50	βασις	217	0.41
δε	858	1.62	επει	359	0.68	εχω	260	0.49	κεντρον	214	0.40
γωνια	781	1.48	μειζων	352	0.67	τετραγωνος	250	0.47	ελασσων	208	0.39
tot.	33058	62.50	tot.	37821	71.50	tot.	40733	77.01	tot.	43009	81.31

Then the data of *Gr.-Lat.*:⁴³

lemma	occ.	% lemma	occ.	% lemma	occ.	% lemma	occ.	%			
A	9620	20.88	is	896	1.95	punctum	336	0.73	oporteo	232	0.50
sum	3041	6.60	quoniam	618	1.34	quidem	330	0.72	habeo	228	0.50
rectus	2783	6.04	a	579	1.26	autem	324	0.70	do	227	0.49
et	2368	5.14	in	558	1.21	enim	314	0.68	tetragonum	222	0.48
qui	2243	4.87	trigonus	548	1.19	si	312	0.68	ipse	221	0.48
aequalis	1795	3.90	vero	485	1.05	ita	307	0.67	multiplex	210	0.46
ergo	1465	3.18	quantitas	480	1.04	idem	290	0.63	reliquus	209	0.45
ad	1136	2.47	circulus	471	1.02	ut	290	0.63	uterque	209	0.45
angulus	933	2.03	duo	464	1.01	latus	268	0.58	basis	206	0.45
sub	908	1.97	maior	373	0.81	quam	254	0.55	parallelo- grammum	202	0.44
								dico	202		
tot.	26292	57.07	tot.	31764	68.95	tot.	34789	75.52	tot.	36955	80.22

⁴³ An interesting quirk introduces a bias in the Latin data: Latin *rectus* (adj.) corresponds to both *ευθεια* (noun) and *ορθος* (adj.). See note 49 below.

Then the data of *Comm.*:

lemma	occ.	% lemma	occ.	% lemma	occ.	% lemma	occ.	%			
A	9841	21.78	triangulum	622	1.38	habeo	366	0.81	minor	265	0.59
sum	3182	7.04	igitur	611	1.35	quoniam	356	0.79	a	258	0.57
et	2000	4.43	linea	611	1.35	latus	354	0.78	is	256	0.57
aequalis	1912	4.23	circulus	484	1.07	si	348	0.77	reliquus	251	0.56
ad	1507	3.34	ex	442	0.98	idem	347	0.77	proportio	245	0.54
angulus	1347	2.98	maior	432	0.96	ita	342	0.76	oporteo	241	0.53
ipse	1118	2.47	quadratus	419	0.93	ergo	310	0.69	parallelogram- mum	240	0.53
qui	1097	2.43	duo	406	0.90	quam	302	0.67	do	230	0.51
rectus	941	2.08	ut	395	0.87	enim	287	0.64	vero	224	0.50
in	658	1.46	autem	387	0.86	punctum	280	0.62	basis	222	0.49
tot.	23603	52.24	tot.	28412	62.89	tot.	31704	70.18	tot.	34136	75.56

And finally the data of *Clavius*:

lemma	occ.	% lemma	occ.	% lemma	occ.	% lemma	occ.	%			
A	10132	21.39	ut	554	1.17	circulus	379	0.80	centrum	249	0.53
sum	3516	7.42	duo	517	1.09	linea	376	0.79	totus	247	0.52
aequalis	1651	3.49	maior	504	1.06	cum	329	0.69	minor	241	0.51
rectus	1513	3.19	latus	486	1.03	ita	316	0.67	hic	231	0.49
ad	1454	3.07	idem	478	1.01	-que	315	0.66	duco	231	0.49
et	1419	3.00	ipse	457	0.96	ex	296	0.62	autem	231	0.49
angulus	1308	2.76	igitur	454	0.96	dico	293	0.62	inter	230	0.49
qui	961	2.03	si	413	0.87	multiplex	264	0.56	parallelogram- mum	229	0.48
in	723	1.53	quadratus	407	0.86	enim	262	0.55	do	219	0.46
triangulum	609	1.29	quam	382	0.81	se	261	0.55	proportio	218	0.46
tot.	23286	49.16	tot.	27938	58.98	tot.	31029	65.50	tot.	33355	70.41

Clavius' lexical drift towards low frequency lemmas highlights again his metadiscursive drive. The distribution by POS of the 40 most frequent lemmas bar article and denotative letter shows the increase of the pronominal content in the Latin translations, consequent on the absence of the article (the colour shadings of the POS are the same as in the previous tables):

	<i>Elem.</i>			<i>Gr.-Lat.</i>			<i>Comm.</i>			<i>Clavius</i>		
	#	%		#	%		#	%		#	%	
adjective	7	18.42	25.86	7	17.50	26.02	6	15.38	25.00	7	17.95	20.84
adverb	1	2.63	10.92	1	2.50	12.29	1	2.56	15.00	1	2.56	18.42
noun	9	23.68	19.54	9	22.50	18.80	10	25.64	12.31	9	23.08	16.48
particle	10	26.32	8.62	10	25.00	8.43	10	25.64	2.46	9	23.08	8.89
preposition	4	10.53	4.60	4	10.00	5.30	4	10.26	1.26	4	10.26	4.20
pronoun	2	5.26	4.60	4	10.00	3.37	4	10.26	4.38	5	12.82	2.58
verb	5	13.16	25.29	5	12.50	25.06	4	10.26	25.29	4	10.26	28.27

We may measure the distance between the frequency distributions (lemm.) of hapax, 1-form lemmas, and most frequent lemmas, as given in the previous table and in the two similar tables above, and the rescaled relative frequency of the overall occurrences by using standard distances such as Eder’s Simple and Canberra.⁴⁴ The results are as in the table below; the *hapax* are a sample more representative of the overall distribution than the other two:

	<i>Elem.</i>	<i>Gr.-Lat.</i>	<i>Comm.</i>	<i>Clavius</i>	
<i>hapax</i>	2.766	3.163	3.013	3.717	<i>Eder</i>
	1.116	1.340	1.165	1.590	<i>Canb</i>
1-form lemmas	7.023	10.186	12.896	11.435	<i>Eder</i>
	1.801	2.456	3.476	2.933	<i>Canb</i>
40 m. f. lemmas	7.767	9.003	13.418	10.829	<i>Eder</i>
	2.157	2.580	3.729	2.993	<i>Canb</i>

The following table compares the 10 most frequent lemmas of the four works (the colour shadings identify identical lemmas in different works, and also see the Sankey diagram displayed as Figure 3 of the Appendix):

⁴⁴ See note 65 below for these distances.

<i>Elem.</i>			<i>Gr.-Lat.</i>			<i>Comm.</i>			<i>Clavius</i>		
lemma	occ.	%	lemma	occ.	%	lemma	occ.	%	lemma	occ.	%
ο	11446	21.64	A	9620	20.88	A	9841	21.78	A	10132	21.39
A	9584	18.12	sum	3041	6.60	sum	3182	7.04	sum	3516	7.42
ειμι	2622	4.96	rectus	2783	6.04	et	2000	4.43	aequalis	1651	3.49
και	2190	4.14	et	2368	5.14	aequalis	1912	4.23	rectus	1513	3.19
ισος	1899	3.59	qui	2243	4.87	ad	1507	3.34	ad	1454	3.07
υπο	1350	2.55	aequalis	1795	3.90	angulus	1347	2.98	et	1419	3.00
προς	1174	2.22	ergo	1465	3.18	ipse	1118	2.47	angulus	1308	2.76
αρα	1154	2.18	ad	1136	2.47	qui	1097	2.43	qui	961	2.03
δε	858	1.62	angulus	933	2.03	rectus	941	2.08	in	723	1.53
γωνια	781	1.48	sub	908	1.97	in	658	1.46	triangulum	609	1.29
tot.	33058	62.50	tot.	26292	57.07	tot.	23603	52.24	tot.	23286	49.16

This is a fairly good index of lexical dispersion. Three general facts must be taken into account when reading this data. First, and obviously, the absence of the article in Latin accounts for at least 8000 missing occurrences, if we want to be generous with allowing the article to be replaced by pronouns in some instances (lemmas in red; *Clavius* did not try to translate the Greek article). This explains the flattening of the distribution of the occurrences in all Latin translations:⁴⁵ 39.76% of the lexical content of *Elem.* is just made of articles and denotative letters. Second, the constant presence of the article and of particles and prepositions is a structural trait of the Greek language, which is partly lost in Latin: one counts (6,4,3,3) such items among the 10 most frequent lemmas of the four works. The relative frequency flow between the 20 most frequent lemmas is represented by the Sankey diagram displayed as Figure 3 of the Appendix, where the height of the coloured rectangles is proportional to a suitable rounding of the relative frequency.⁴⁶ The loops are attached to lemmas whose position in the ranking of the other works is lower than the 20th place. Likewise, contributions to non-looping lemmas in the diagram that come from lemmas lower than the 20th place are neglected; this problem arises conspicuously for the Greek particles δε and αρα, and to a lesser extent for και and προς. The fictitious boxes “new occurrences” and “lost occurrences” take also account of the fact that the relative frequencies are not normalised to a common scale.

⁴⁵ For comparison's sake, the 10 most frequent lemmas of Archimedes' *De sphaera et cylindro* and of Pappus' *Collectio* represent 57.65% and 60.21% of their lexical content, respectively.

⁴⁶ The temporal ordering of the Latin works adopted in the table and in the chart is lexically irrelevant, for they are all independent editions or translations of *Elem.* So, the Sankey does not depict a real frequency flow between the Latin works, but the rearrangement of the ranking.

Form spread

As Greek and Latin are inflectional languages, it is interesting to study the form spread within one and the same lemma. The 26 lemmas of *Elem.* that have at least 10 forms are as follows (see below for the colour shadings):

lemma	# forms	lemma	# forms	lemma	# forms
A	511	επιzeugνυμι	16	συνιστημι	13
ειμι	33	εφαπτομαι	16	λαμβάνω	11
τεμνω	23	λοιπος	15	μειζων	11
αυτος	21	ος	15	ορθος	11
ο	19	διδωμι	14	πλευρα	11
ελασσων	18	ισος	14	αφαιρεω	10
εχω	18	αγω	13	εγγραφω	10
περιεχω	18	ποιεω	13	ευθεια	10
αναγραφω	16	προσπιπτω	13		

The 17 lemmas of *Gr.-Lat.* and of *Comm.* that have at least 10 forms are as follows:

Gr.-Lat.

lemma	# forms	lemma	# forms
A	526	is	12
seco	16	rectus	12
habeo	14	facio	11
contineo	13	ipse	11
sum	13	qui	11
contingo	12	coapto	10
describo	12	constituo	10
divido	12	sumo	10
idem	12		

Comm.

lemma	# forms	lemma	# forms
A	527	sum	13
seco	21	constituo	12
describo	17	contingo	12
contineo	15	habeo	12
qui	15	rectus	12
is	14	duco	11
sumo	14	reliquus	11
idem	13	uterque	11
ipse	13		

The 41 lemmas of *Clavius* that have at least 10 forms are as follows:⁴⁷

lemma	# forms	lemma	# forms	lemma	# forms	lemma	# forms
A	514	hic (pron.)	16	qui	14	efficio	11
divido	26	idem	16	alius	13	exsisto	11
seco	23	sumo	16	comprehendo	13	quicumque	11
counter	22	describo	15	compono	12	reliquus	11
sum	20	habeo	15	ille	12	applico	10
constituo	19	is	15	ipse	12	cado	10
aufero	17	ostendo	15	oppono	12	consequor	10
tango	17	subtendo	15	pono	12	do	10
contineo	16	dico	14	rectus	12	produco	10
duco	16	facio	14	demonstro	11	totus	10
						uterque	10

One must not forget that accents and elisions tend to increase the number of forms: 5 lemmas of *Elem.* enter the ranking thanks to this feature of the Greek language: αφαιρειω, ευθεια, ορθος, μειζων, πλευρα.

The colour shadings identify lemmas that are identical or such that the Latin lemma is a translation of the Greek lemma; the colours in the column of the number of forms mark two lemmas that translate a single lemma: αντος > *idem*, *ipse*; εφαπτομαι > *contingo*, *tango*; περιεχω > *comprehendo*, *contineo*; τεμνω > *divido*, *seco*; ευθεια, ορθος < *rectus*. Both uniquely related pairs of Greek and Latin lemmas and each of these complexes will be called “meta-lemmas;” more on them below. With this qualification, there are 12 core metalemmas common to the four texts, (0,1,2,1) metalemmas common to three texts, (0,0,2,0,0,1) common to two texts only.⁴⁸ *Elem.* has 8 unique lemmas with at least 10 forms (30.77%), *Gr.-Lat.*, 1 (5.88%), *Comm.*, 0 (0.00%), *Clavius*, 19 (46.34%). *Clavius*’ rhetorical education is apparent from this data: lexical redundancy, form spread.

It comes as no surprise that the 12 core metalemmas comprise the denotative letters, the noun and the adjective for “straight,” “right” (ευθεια, ορθος / *rectus*),⁴⁹ the most com-

⁴⁷ The lemma “counter” collects the ordinals that feature in the headings of principles, propositions, and Books, which are part of the text in *Clavius*.

⁴⁸ The entries in each parenthesis give the numbers that correspond, in a suitable order, to the 4 combinations of 3 objects out of 4 and to the 6 combinations of 2 objects out of 4, respectively.

⁴⁹ In principle, the forms of the lemma ευθεια (noun) should be assigned to the adjective ευθς, which, however, is scarcely used. In Latin, *recta* etc. are indeed forms of *rectus*, which is frequently used in the masculine for the “right” angle. So, we decided to separate ευθεια from ευθς but not *recta* from *rectus*. Otherwise, we would have lost the noun of the most important math-

mon pronouns (αὐτός / *idem, ipse*; οὗς / *qui*), 5 verbs with an obvious geometric connotation (ἀναγράφω / *describo*; ἐφαπτομαι / *contingo, tango*; λαμβάνω / *sumo*; περιέχω / *comprehendo, contineo*; συνιστήμι / *constituo*; τέμνω / *divido, seco*), and the all-purpose verbs “to be” and “to have.”

We represent the form spread of the core metalemmas bar denotative and numeral letters in the charts displayed as Figures 4a–b of the Appendix, where the core metalemmas are located in the tick marks of the abscissae and the numbers of forms have been normalised to the total number of forms in each text;⁵⁰ we took the arithmetic mean of the form numbers of the two lemmas that translate a single lemma, if both of the former are present in the list. In chart 4a, the order of the metalemmas in *Elem.* is also adopted for the other works; this highlights an initial peak of the Latin translations (form spread of *sum* much lower than the form spread of εἶμι), and the oscillations of *Comm.* and *Clavius*. In chart 4b, the order is by decreasing number of forms in each text separately; the form spread of *Elem.* and the striking similarity of the patterns of *Comm.* and *Clavius* are apparent.

Let us also compare the above overall data:

	≥ 10 forms	% lemmas	occ.	tot. occ.
<i>Elem.</i>	26	7.47	30046	52894
<i>Gr.-Lat.</i>	17	4.10	20086	46067
<i>Comm.</i>	17	4.47	18640	45176
<i>Clavius</i>	41	6.62	21109	47371

These parameters are an index of lexical concentration: the same lemma is used in several syntactic situations. The occurrences engaged by the lemmas with the highest number of forms are linearly correlated with the size (occ.) of the text. The correlation is very good: the best linear regression of the data, with slope 0.6703, has a coefficient of determination $R^2 = 0.9936$. Of course, there is a natural upper bound to the number of forms a lemma can display in a sectorial lexicon: mainly pronouns or adjectives and, above all, verbs naturally admit a high number of forms – yet the limited number of semantic fields accessible to a sectorial lexicon introduces a bias in the number of available POS.

Distribution of lemmas, forms, and occurrences by POS

The following table sets out the distribution of the occurrences, in our reference texts, among categories that by and large coincide with the standard POS. As said, specific items

ematical object of Greek mathematics, or categorised (Latin) two crucial mathematical objects in an inconsistent way.

⁵⁰ That is, for εἶμι / *sum*, (33, 13, 13, 20) > (33/1774, 13/1743, 13/1665, 20/2400) = (0.0186, 0.0075, 0.0078, 0.0083).

as numerals, denotative letters, and Greek words are each assigned a separate category. The column shaded yellow gives the frequencies of *Elem.* rescaled as if the text did not contain any article.

	<i>Elem.</i>			<i>Gr.-Lat.</i>		<i>Comm.</i>		<i>Clavius</i>	
	occ.	%		occ.	%	occ.	%	occ.	%
adjective	6308	11.93	15.23	8631	18.74	7548	16.71	8683	18.33
adverb	1313	2.48	3.15	1179	2.56	1255	2.78	1911	4.03
article	11446	21.64	0.00	0	0.00	0	0.00	0	0.00
denot. lett.	9584	18.12	23.12	9620	20.88	9841	21.78	10132	21.39
Greek	0	0.00	0.00	8	0.02	0	0.00	0	0.00
noun	5089	9.62	12.28	4957	10.76	6188	13.70	6382	13.47
number	0	0.00	0.00	10	0.02	0	0.00	84	0.18
particle	7385	13.96	17.82	7515	16.31	6640	14.70	6110	12.90
preposition	4416	8.35	10.66	3903	8.47	3399	7.52	3774	7.97
pronoun	1248	2.36	3.01	3875	8.41	3143	6.96	2713	5.73
verb	6105	11.54	14.73	6369	13.83	7162	15.85	7582	16.01

Even compared with the rescaled relative frequencies of *Elem.*, it is apparent that the pronominal content of the Latin texts increases as a partial counterbalance to the absence of the definite article. The lower relative frequency (occ.) of particles in Latin texts points to a loosening of the deductive structure more than to structural facts underlying the two languages, for the relative frequencies (lemm.) of this POS are nearly identical in the four works, see the tables just below. The relative frequency data of the above table is displayed in the chart displayed as Figure 5 of the Appendix, where the lemmas “Greek” and “number” have been eliminated.

Let us finally set out a complete table of lemmas, forms, and occurrences distributed among POS. The reader is urged to spend some time comparing the numbers in the tables. The first table sets out the data of *Elem.*:

	lemmas	%	forms	%	occ.	%	% l/o	% f/o	% l/f
adjective	90	25.86	336	18.95	6308	11.93	1.43	5.33	26.79
adverb	38	10.92	44	2.48	1313	2.48	2.89	3.35	86.36
article	1	0.29	19	1.07	11446	21.64	0.01	0.17	5.26
denot. lett.	1	0.29	511	28.82	9584	18.12	0.01	5.33	0.20
noun	68	19.54	235	13.25	5089	9.62	1.34	4.62	28.94

particle	30	8.62	48	2.71	7385	13.96	0.41	0.65	62.50
preposition	16	4.60	32	1.80	4416	8.35	0.36	0.72	50.00
pronoun	16	4.60	68	3.84	1248	2.36	1.28	5.45	23.53
verb	88	25.29	480	27.07	6105	11.54	1.44	7.86	18.33
total	348	100.00	1773	100.00	52894	100.00	0.66	3.35	19.63

The same table for *Gr.-Lat.*:

	lemmas	%	forms	%	occ.	%	% l/o	% f/o	% l/f
adjective	108	26.02	337	19.33	8631	18.74	1.25	3.90	32.05
adverb	51	12.29	52	2.98	1179	2.56	4.33	4.41	98.08
denot. lett.	1	0.24	526	30.18	9620	20.88	0.01	5.47	0.19
Greek	1	0.24	7	0.40	8	0.02	12.50	87.50	14.29
noun	78	18.80	237	13.60	4957	10.76	1.57	4.78	32.91
number	1	0.24	7	0.40	10	0.02	10.00	70.00	14.29
particle	35	8.43	35	2.01	7515	16.31	0.47	0.47	100.00
preposition	22	5.30	24	1.38	3903	8.47	0.56	0.61	91.67
pronoun	14	3.37	66	3.79	3875	8.41	0.36	1.70	21.21
verb	104	25.06	452	25.93	6369	13.83	1.63	7.10	23.01
total	415	100.00	1743	100.00	46067	100.00	0.90	3.78	23.81

The same table for *Comm.*:

	lemmas	%	forms	%	occ.	%	% l/o	% f/o	% l/f
adjective	95	25.00	320	19.22	7548	16.71	1.26	4.24	29.69
adverb	57	15.00	57	3.42	1255	2.78	4.54	4.54	100.00
denot. lett.	1	0.26	527	31.65	9841	21.78	0.01	5.36	0.19
noun	66	17.37	205	12.31	6188	13.70	1.07	3.31	32.20
particle	40	10.53	41	2.46	6640	14.70	0.60	0.62	97.56
preposition	19	5.00	21	1.26	3399	7.52	0.56	0.62	90.48
pronoun	12	3.16	73	4.38	3143	6.96	0.38	2.32	16.44
verb	90	23.68	421	25.29	7162	15.85	1.26	5.88	21.38
total	380	100.00	1665	100.00	45176	100.00	0.84	3.69	22.82

The same table for *Clavius*:

	lemmas	%	forms	%	occ.	%	% l/o	% f/o	% l/f
adjective	129	20.84	442	18.42	8683	18.33	1.49	5.09	29.19
adverb	114	18.42	115	4.79	1911	4.03	5.97	6.02	99.13
denot. lett.	1	0.16	514	21.42	10132	21.39	0.01	5.07	0.19
noun	102	16.48	313	13.04	6382	13.47	1.60	4.90	32.59
number	1	0.16	22	0.92	84	0.18	1.19	26.19	4.55
particle	55	8.89	57	2.38	6110	12.90	0.90	0.93	96.49
preposition	26	4.20	28	1.17	3774	7.97	0.69	0.74	92.86
pronoun	16	2.58	107	4.46	2713	5.73	0.59	3.94	14.95
verb	175	28.27	802	33.42	7582	16.01	2.31	10.58	21.82
total	619	100.00	2400	100.00	47371	100.00	1.31	5.07	25.79

Information content by categorial type: X-entropy

We have also introduced an intensive indicator, which measures how much lexical information of a specific kind is provided in a given text or in any of its relevant subdivisions. In order to do this, we have categorised each lemma by means of a three-tier typology that is a refinement of the standard partition into POS. The broadest categories are the POS as given above, the finer ones are partly semantically oriented.⁵¹ From this categorisation we have extracted a cross-section of categorial types,⁵² which are as follows (capital letters refer to the sigla in the charts displayed in the Appendix): INFERential and coordinant (among particles), METadiscursive (among any POS), OPERational (among nouns and verbs), RELational (among nouns and verbs), NOMinal (all nouns + pronouns), DENotative letters, QUALifiers (among adjectives), GENerality determiners (among adjectives). To exemplify the notions we are going to introduce, the computations in this and the following subsection mainly refer to the textual unit made by a Book.

In each proposition of each Book of the four works, we counted all the occurrences related to the lemmas that belong to any given categorial type among those in the list above, and we normalised to the total number of words in the assigned proposition. For

⁵¹ An almost complete list of these sub-categories is given in Fabio Acerbi and Ramon Masìà, *George of Trebizond, Introduction to the Almagest. Greek and Latin Text* (Leiden: Brill, 2026) Introduction, sect. 4.

⁵² The selected cross-section does not realise a partition of the lemmas: the categorial types given below may not be exclusive, and do not cover all lemmas (for instance, the prepositions are not included).

any given categorial type X and any given proposition i , we further normalised the output of the previous step to the sum of the outputs associated with all the propositions of the given Book. For each Book j , the resulting numbers p_{ij}^X are a probability distribution, for their sum is 1 by definition. Consequently, we define the intensive indicator X -entropy of the categorial type X in the assigned Book as $H_j^X \equiv -\sum p_{ij}^X \log p_{ij}^X$.⁵³ The results are as in the X -entropy vs. Book charts displayed as Figures 6a–h of the Appendix.

For all categorial types and almost all Books, the X -entropy confirms the wider lexical spread of *Clavius* (an exception is GEN in Book II, for *Clavius* omits all the generalising determiners that qualify the way a given straight line is cut into segments) while highlighting the compartmentation of the lexicon by Book and the monothematic character of Books II and IV. The data of *Elem.* and those of *Gr.-Lat.* are normally hard to distinguish; note, however, the discrepancy as for NOM: *Gr.-Lat.* may have a lower entropy because it tends to “saturate” the nominal lexicon by frequently supplying nouns like *recta*, *angulus*, *ratio*.

Location and density of categorial types: X -clouds

Within the word space of a mathematical proposition, word-tokens belonging to specific POS or to categorial types as those defined in the previous sub-Section may tend to cluster or, conversely, to display a wide spread. We measured the location and density of some categorial types by introducing a four-leg indicator, which we call “ X -cloud.” This indicator is constructed as follows.

Compute the position, in number of word-tokens (the first word has position 1, the second has position 2, etc.), of each of the occurrences of lemmas belonging to an assigned categorial type X . Divide each of these positions by the number of words contained in the assigned proposition: these are the “normalised positions” (hence, the last word has normalised position 1). Take the arithmetic mean of these normalised positions: this is the “ X -centroid” of the assigned proposition. Take the minimum of the normalised positions: this is the “ X -beginning” of the assigned proposition. Take the maximum of the normalised positions, and subtract from it the X -beginning as computed in the previous step: this is the “ X -range” of the assigned proposition.⁵⁴ Count the number of occurrences of lemmas belonging to the categorial type X in the assigned proposition. Divide this number by the overall number of words counted from the first to the last occurrence (inclusive of the extremes) of lemmas belonging to the categorial type X in the assigned proposition: this is the “ X -density” within the X -range of the assigned proposition.

⁵³ In our entropy computations here and below, we use the base 10 logarithm.

⁵⁴ Thus, the X -range of a unique occurrence belonging to categorial type X is 0. On the one hand, this makes the definition of “ X -density” cumbersome, and yields X -range $\neq 1$ for a proposition that begins and ends with a word-token belonging to categorial type X . On the other hand, including the X -beginning in the range would have made the X -range of an X -singleton depend on the size of the proposition, which we deemed a drawback more annoying than the one just pointed out.

All these proposition indicators can be lifted to a Book or to a set of Books by taking suitable means. The bookwise behaviour of the four indicators for the categorial types X = DEN, INF, NOM is plotted in the charts displayed as Figures 7a–l of the Appendix.

We also selected the categorial types X = INF, NOM and plotted three of the above indicators propositionwise (the propositions are numbered sequentially from prop. 1 of Book I) in the charts displayed as Figures 8a–f of the Appendix; the gaps in the interpolating lines correspond to autonomous deductive units – mainly corollaries and additional propositions – that do not figure in the associated text. The mean values for the entire segment of Books I–VI are as follows:

	INF-cen	INF-beg	INF-ran	NOM-cen	NOM-beg	NOM-ran
<i>Elem.</i>	0.5211	0.1159	0.7526	0.5087	0.0557	0.9280
<i>Gr.-Lat.</i>	0.5490	0.1246	0.7900	0.5030	0.0478	0.9371
<i>Comm.</i>	0.5244	0.1527	0.6977	0.4985	0.0524	0.9054
<i>Clavius</i>	0.5830	0.2574	0.6325	0.4995	0.0622	0.9110

The first three charts show that *Clavius* starts deducing later and goes on doing this less long. The second three charts highlight the considerable extension of the NOM-range; the systematic exception is Book V, where adjectives normally replace nouns in the enunciations and in the setting-outs.

Correlation table and correlation and dispersion matrix of a translation

The difficulty of comparing the lexicon of a Greek and a Latin text that are one the translation of the other lies in the fact that there is no natural one-to-one correlation between Greek and Latin lexemes, and that many possible correlations are context-dependent. Of course, there are obvious instances of bijective correlation (think of the adjective “two”), and their number increases in technical texts, which tend to make the lexicon univocal.⁵⁵ However, a number of facts blurs the correlation:

- A one-to-one correlation is lost between many categories of lexemes just because the Latin translators did not replicate the rigid conventions of the Greek stylistic code. Take coordinant and inferential particles: ἀρα and δη, whose use is strictly differentiated in Greek, more or less randomly correspond to *ergo*, *idcirco*, *ideo*, *quare*, *quocirca* (just check the proof of Prop. I.1 in *Clavius*); δε can be *autem*, *vero*, *verum*; γαρ can be *enim*, *nam*, *namque*, *quia*; και and τε can be *et*, *ac*, *atque*, enclitic *que*, adverbial *και* can be *etiam* and *quoque*.

⁵⁵ Technical texts written in natural language normally fall short of this requirement. The *Elements* are no exception: see the discussion in Acerbi, *The Logical Syntax*, 35–36.

- Verbs with two preverbal prepositions are common in Greek and exceedingly rare in Latin: the only specimens in *Gr.-Lat.* are *adimpleo* and *adinvenio*, the latter is the only specimen in *Clavius*; no specimen in *Comm.* On the other hand, if *Gr.-Lat.* and *Clavius* have both *invenio* and *adinvenio*, this occurs because *Elem.* has both *εὑρίσκω* and *προσεὑρίσκω*. This example shows that the dilution of the technical lexicon already took place in the Greek language.
- A lemma in Greek may correspond to a form in Latin; a case in point are the verbal adjectives (Greek), which correspond to a verb form (Latin). Likewise, some correlated lemmas may belong to different grammatical categories, for instance, adjective vs. noun.
- Most of *Clavius*’s Euclid is not a translation, so strictly speaking we cannot compare its lexicon with the lexicon of *Elem.*: we might more properly state that we draw a comparison between Gracilis’ partial translation and *Elem.* It remains that the setting-outs in *Clavius* reproduce most of Gracilis’ lexicon – and we are interested in the evolution of the lexical record as a witness to the evolution of the way Greek mathematics was appropriated and naturalised.

Paying due attention to these features, we have separately established univocal and multivocal correlations between lemmas of *Elem.* and lemmas of *Gr.-Lat.*, *Comm.*, and *Clavius*. The lemmas and occurrences (overall numbers and percentage with respect to the entire lexical content) engaged for each of the three combinations Greek/Latin are given in the following tables.⁵⁶ *Clavius*’ huge lexical spread is obvious, for it employs a lesser amount of its own lexicon in the “translation.”

# lem.	<i>Gr.-Lat.</i>	<i>Comm.</i>	<i>Clavius</i>	# occ.	<i>Gr.-Lat.</i>	<i>Comm.</i>	<i>Clavius</i>
	380	342	328		45820	45034	45943
<i>Elem.</i>	334	323	294	<i>Elem.</i>	41254	41083	41078

% lem.	<i>Gr.-Lat.</i>	<i>Comm.</i>	<i>Clavius</i>	% occ.	<i>Gr.-Lat.</i>	<i>Comm.</i>	<i>Clavius</i>
	91.57	89.74	52.99		99.46	99.69	96.99
<i>Elem.</i>	95.98	92.82	84.48	<i>Elem.</i>	77.99	77.67	77.66

⁵⁶ This means, for instance, that *Gr.-Lat.* uses 380 lemmas to translate 334 lemmas of *Elem.*, and the occurrences these lemmas engage in the two works are 45820 and 41254, respectively. The second table gives the percentages. At least 96.87% of the percentage (occ.) loss of *Elem.* with respect to the Latin translations comes from the absence of the article.

As several Latin lemmas may translate one and the same Greek lemma,⁵⁷ we can compute how many Latin lemmas are ranked in a given position in the sequences that translate one and the same Greek lemma.⁵⁸ The distribution, which is of course independent of the actual ordering of any such sequence of Latin lemmas, exhibits a typical Pareto-law-like behaviour, see Figure 9 of the Appendix.

Complex networks of correlation normally come to be established in translation for some key categories of lemmas, often comprising function words. For instance, the “forward” Greek inferential particles give rise in *Clavius* to the following *correlation matrix*, which includes all (source) Greek lemmas that are translated by a given maximal set of (target) Latin lemmas.⁵⁹

δε	autem	vero			
απα	igitur	ergo	quocirca	quare	itaque
δη	igitur	autem			
ουν	itaque				
μην	vero				

A further example comes from *Gr. Lat.*: a few constellations of verbs that are naturally related – namely, the verbs meaning tangency, motion, generation and intersection of lines – are also linked to the indefinite adjective οποιουσιν “whichever” by the several translations of the determiners of arbitrariness involving forms of the verb τυγχάνω “to happen.”

⁵⁷ At most 6 Latin lemmas in the case of *Gr.-Lat.*, 9 for *Comm.*, 7 for *Clavius*. The percentages of lemmas in *Gr.-Lat.*, *Comm.*, and *Clavius* that correspond to one single lemma in *Elem.* are 58.55, 65.91, and 33.17, respectively. This confirms that *Clavius*’ lexical richness is mainly a consequence of its rewriting the proofs.

⁵⁸ In the first correlation table below, these numbers are 5, 3, 1, 1, 1.

⁵⁹ That is, if a Greek lemma in the correlation matrix is translated by a number of Latin lemmas, all these Latin lemmas and all Greek lemmas they translate are also included in the matrix. The correlation matrices are easily constructed by picking a Greek lemma and applying the definition recursively.

τυγχανω	<i>accido</i>	<i>fortuitus</i>	<i>quicumque</i>	<i>quilibet</i>	<i>contingo</i>	<i>quomodolibet</i>
προσπιπτω	<i>accido</i>	<i>venio</i>				
προσαναγραφω	<i>adscribo</i>					
παραβαλλω	<i>adscribo</i>	<i>constituo</i>	<i>describo</i>	<i>educo</i>		
βαινω	<i>consisto</i>	<i>constituo</i>	<i>insido</i>			
συνιστημι	<i>consisto</i>	<i>constituo</i>	<i>sisto</i>			
απτομαι	<i>contingo</i>					
εφαπτομαι	<i>contingo</i>					
αναγραφω	<i>describo</i>	<i>adscribo</i>				
αγω	<i>duco</i>	<i>protraho</i>				
διαγω	<i>educo</i>	<i>protraho</i>	<i>produco</i>			
εκβαλλω	<i>emitto</i>	<i>educo</i>	<i>ejicio</i>			
προσεκβαλλω	<i>produco</i>	<i>educo</i>				
ερχομαι	<i>transeo</i>	<i>venio</i>				
οποιοσουν	<i>unusquislibet</i>	<i>quicumque</i>				
ηκω	<i>venio</i>					

Setting all target lemmas as headers of a suitable number of columns and reordering the rows and the columns, a correlation matrix can be regularised to give a perspicuous representation of the correlation between the source and target lemmas, and thereby as it were “diagonalised,” where by “diagonal” we mean here the cell diagonal coming down from the upper left corner. Depending on the distribution of the target lemmas, there may not exist an ordering of the source and target lemmas that allows filling the entire cell diagonal. In the present instance, only 15 of the 16 possible cells can be filled. The reason is that the two verbs *απτομαι* and *εφαπτομαι* are translated by one and the same lemma, namely, *contingo*. A diagonalised form of the just-presented correlation matrix – which in general is not unique – is set out below.

[illegible]

The correlations between Greek and Latin lemmas can be classified as in the *correlation table* below, where numerical pairs such as n/m mean that n different lemmas of *Elem.* are correlated with m different lemmas of the intended Latin translation. The first row gives the total number of lemmas involved in a given kind of correlation. The second row gives the kind of the correlation; we shall give this correlation a name later. The third row gives the scaling factor between the pairs located in the first two rows, namely, the number of the correlations of a given kind. For instance, from the second column of the first table one gets that *Gr.-Lat.* contains 19 correlations (third row) in which 2 Greek lemmas are translated by 1 Latin lemma (second row), hence these 19 correlations involve 38 Greek lemmas and 19 Latin lemmas (first row). To any of the pairs in the second row there corresponds a correlation matrix of the kind described above. We set out the correlation table for *Gr.-Lat.* first:

161/161	38/19	6/2	4/1	32/64	3/9	1/4	1/5	30/30	12/18	4/8	6/4	3/3	3/4	3/6	5/4	6/10	16/21
1/1	2/1	3/1	4/1	1/2	1/3	1/4	1/5	2/2	2/3	2/4	3/2	3/3	3/4	3/6	5/4	6/10	16/21
161	19	2	1	32	3	1	1	15	6	2	2	1	1	1	1	1	1

The same table for *Comm.*:

154/154	34/17	15/5	17/34	5/15	18/18	2/3	4/8	3/3	3/4	3/5	4/6	5/2	4/10	5/7	7/16	8/7	10/14	22/22
1/1	2/1	3/1	1/2	1/3	2/2	2/3	2/4	3/3	3/4	3/5	4/6	5/2	4/10	5/7	7/16	8/7	10/14	22/22
154	17	5	17	5	9	1	2	1	1	1	1	1	1	1	1	1	1	1

The same table for *Clavius*:

151/151	26/13	6/2	15/30	3/9	1/4	1/5	1/6	12/12	4/6	2/4	3/2	3/3	3/4	3/5	4/2	4/8	5/2	5/4	5/7	7/9	7/10	9/10	14/20
1/1	2/1	3/1	1/2	1/3	1/4	1/5	1/6	2/2	2/3	2/4	3/2	3/3	3/4	3/5	4/2	4/8	5/2	5/4	5/7	7/9	7/10	9/10	14/20
151	13	2	15	3	1	1	1	6	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1

We may visualise the data set out in a correlation table by introducing an $r \times s$ -cell *dispersion matrix* D , where r and s are the numbers of lemmas of the texts R and S that are involved in the translation, respectively. In our case, $R = Elem.$ and $S = Gr.-Lat., Comm.,$ or *Clavius*. The lemmas index the rows and the columns of the dispersion matrix according to their order in the associated correlation table; the right lower submatrix is occupied by lemmas that do not have a correlate. A cell D_{ij} of the dispersion matrix is filled by lemma j if this is the translation of lemma i . As seen above, given a maximal set of Latin lemmas that translate a set of Greek lemmas, their ordering can be arranged in such a way as to maximise the number of cells filled along any block diagonal. The result is a block

matrix of sorts,⁶⁰ whose larger blocks coincide with the diagonalised correlation matrices introduced above. The results are displayed as Figure 10 of the Appendix; the black cells contain words, the cells in colour are empty and contribute to identifying different diagonalised correlation matrices. The order from top to bottom is *Gr.-Lat.*, *Comm.*, *Clavius*; zoom in to see details.

We measure the lemmatic reorganisation that has occurred in a given translation by introducing five indicators that process the data set out in the correlation table and in the dispersion matrix. The idea is to measure the extent to which the lexicon is reorganised by the interplay of lexical sinks (several source lemmas are translated by fewer target lemmas) and sources (the opposite occurs). The indicators are:

- The “temperature” T_t of the given translation, defined, as is customary in statistical thermodynamics, by means of the Equipartition Theorem, but read the other way around.⁶¹ We select the first row of a correlation table and compute $\delta_i = |n_i - m_i|$ for n_i and m_i in each pair n_i/m_i , where i is the position of the pair in the sequence of the table. We measure the “mean thermal energy” associated with the dynamics of lemmatic diversity occurring in translation by taking the expectation value of the square of δ_i w.r.t. a discrete Maxwell-Boltzmann probability distribution featuring the same square in the mass function:⁶² $T_t \equiv \sum_i \delta_i^2 \exp(-\delta_i^2)$. A bijective translation has $i = 1$ only, $\delta_i = 0$, and hence $T_t = 0$; the temperature of a silly translation in which all source lemmas are translated by one and the same target lemma rapidly converges to 0 when the number of source lemmas increases.⁶³ A translation that is “hotter” than another has carried out a more extensive lemmatic reorganisation.
- A “translation entropy,” $H_t \equiv -\sum_i \partial_i \log \partial_i$, where ∂_i is δ_i normalised to $\sum_i \delta_i$. Here again, a bijective translation has $H_t = 0$, and a translation having a greater entropy than another also carried out a more extensive lemmatic reorganisation. Because of the small sample size, most of the contribution to the first two indicators comes from small δ_i .
- The “mixing index,” which is a bulk indicator of lemmatic reorganisation. We select

⁶⁰ It is easy to see that the result cannot be a real block matrix.

⁶¹ Check for instance Ravinder R. Puri, *Modern Thermodynamics and Statistical Mechanics. A Comprehensive Foundation* (Cham: Springer, 2024), sect. 3.2. The approach in Sasuke Miyazima and Keizo Yamamoto, “Measuring the Temperature of Texts,” *Fractals* 16 (2008): 25–32, takes the underlying physics wrong; it just amounts to computing the slope of a trivial reshaping of the log-log graph of Zipf’s law.

⁶² There is no need to introduce constants of normalisation because we set a definition and the temperature scale is arbitrary.

⁶³ A totally fuzzy translation, in which each source lemma is translated by all target lemmas, can be impossible to achieve, depending on the size of the text and on the distribution of the occurrences.

the second row of a correlation table and compute $\Delta_i = |n_i - m_i|$ for n_i and m_i in each pair n_i/m_i , where i is the position of the pair in the sequence of the table. We thus define the mixing index $M_t = (\sum_i \Delta_i) / (\text{average of the numbers of source and target lemmas})$. Clearly, $0 \leq M_t \leq 1$, the upper bound being approximated by the silly translation and attained by an empty translation. Here again, a bijective translation has $M_t = 0$.

- The normalised value S_t of the lexical span of the given translation as a subset of the dispersion matrix D . The idea is to compute the normalised cell size of the minimal rectangle spanned by an n/m correlation between lemmas. Taking for example *Gr.-Lat.*, this means that the lexical span of a 1/1 correlation is 1 (one single cell is involved) times 161 (the number of such cells), the lexical span of a complex correlation, for instance 16/21, is 16×21 (the rectangle spanned by the lemmas involved in the correlation, whatever their distribution within the rectangle, see the next indicator) times 1 (just one correlation is involved). We normalise to the cell size of the entire dispersion matrix. With reference to the three correlation tables above, set u_i/v_i for a pair in the second row and w_i for the corresponding number in the third row, where i is the position of the pair in the sequence. The normalised lexical span is defined as $S_t \equiv \sum_i u_i v_i w_i / rs$. One has $1/\min\{r,s\} \leq S_t \leq 1$.
- As the rectangle spanned by the lemmas involved in a given correlation may be partly empty, we compute the degree of lexical reorganisation F_t of the given translation as the ratio between the sum of the filled cells within all rectangles spanned by all n/m correlations such that both m and $n \neq 1$ and the sum of the cell sizes of the rectangles spanned by the same correlations.⁶⁴ Clearly, $0 \leq F_t \leq 1$, where we set $F_t = 1$ whenever there are no correlations such that both m and $n \neq 1$.

The values of these indicators are set out in the following table. *Clavius* is hotter and more mixing than the other translations; within the rectangles spanned by the correlation matrices, its lexicon is nearly as tightly reorganised as in *Comm.*

	T_t	H_t	M_t	S_t	F_t
<i>Gr.-Lat.</i>	0.812	0.958	0.0962	0.006983	0.584
<i>Comm.</i>	1.325	1.015	0.1159	0.011031	0.504
<i>Clavius</i>	1.914	1.181	0.1576	0.009654	0.511

Measuring the textual distances among the four texts: first- and second-order meta-lemmas

As the (ordered) set of the relative frequencies of the lemmas of any text is a vector, we may measure the distance between two texts by defining a suitable distance between the

⁶⁴ The excluded correlations cannot give rise to empty cells.

relative frequency vectors associated with them.⁶⁵ In order to compute the distance between *Elem.* and any of its Latin translations, we must preliminarily reduce the multivocal correlations between lemmas described in the preceding sub-Section to one-to-one numerical correlations between occurrences of single lemmas. This necessarily entails creating fictitious lemmas, any of which we call (first-order) “metalemma;” the number of metalemmas for any given correlation in the second row of each of the three correlation tables above are those set out in the third row of the same tables. The reduction procedure that gives the occurrences of a metalemma is as follows:

- if the correlation is 1/1, the metalemma coincides with the lemma on either side;
- if the correlation is n/m , with $n > 1$ and $m > 1$, we create the metalemma by adding the occurrences on the Greek side and on the Latin side, respectively.

In this way, a set of Greek-Latin metalemmas is associated with each of the three Latin translations. These three sets of metalemmas need not coincide; this raises a problem, because measuring textual distances requires operating on a stable lexicon. To overcome this problem, we introduced second-order metalemmas, defined as the set-theoretical union, over all translations, of all Greek lemmas that are translated by a given maximal set of Latin lemmas. Clearly, the number of second-order metalemmas is smaller than the number of metalemmas associated with each translation; conversely, there exist huge second-order metalemmas.⁶⁶ Note that a given second-order metalemma need not be shared by all translations, but there is – in fact, there must be – a core of shared second-order metalemmas. The outcome of the reduction procedure just outlined are 188 second-order metalemmas, 167 of which are shared by the four works.

Cluster analysis, consensus trees, networks

We first perform a cluster analysis applied to second-order metalemmas using a number of standard distances, even if stylometric tools grounded on textual distances are not tailored to process a corpus comprising four texts only. For some of the distances available in the package *stylo*, the results are charted in dendrograms as the ones displayed as Figures

⁶⁵ If the relative frequency vectors of the two texts do not refer to the same set of lemmas, just upgrade them to the set-theoretical sum of the sets of lemmas and put 0 for the frequency of a lemma that is absent in either set. Recall that a distance between two vectors is a non-negative symmetric function of them which is zero iff the two vectors coincide and such that the triangle inequality holds. A summary of the currently used textual distances can be found in Eder, Rybicki, and Kestemont, ‘*Stylo*’, 15–17. An assessment of some stylometric techniques is in Maciej Eder, “Visualization in stylometry: Cluster analysis using networks,” *Digital Scholarship in the Humanities* 32 (2017): 50–64; see also Stefan Evert et al. “Understanding and explaining Delta measures for authorship attribution,” *Digital Scholarship in the Humanities* 32 (2017): ii4–ii16.

⁶⁶ The richer second-order metalemma, which swallows most of the verb content both in Greek and in Latin, contains 54 Greek lemmas and 102 Latin lemmas.

11a–d of the Appendix. MFW are the Most Frequent Words (in fact, second-order meta-words) in each work used to compute the distance, and the “culling” is the percentage of texts that must contain a given lemma in order for the latter to be included in the computation; a 0% culling means that no culling is operative, so all lemmas are included; “Culled @ 0–100%” means that culling is weighted out by consensus. The dendrograms confirm that the organisation of the lexicon in *Clavius* is farther from the organisation in *Elem.* than *Gr.-Lat.* and *Comm.* are, and that the latter two translations compete for being the most faithful rendering of the Greek original.

As is easy to see, the outcome of the cluster analysis depends on the selected distance, on the number of MFW, and on culling, sometimes in ways that may be bewildering. To overcome this problem, which can be perceived as a drawback, a “consensus tree” providing the most likely outcome of the dendrograms that result from varying the number of MFW and the culling can be constructed for each distance. The caption “Consensus 0.5” in the graphs means that a node is validated in the consensus tree as soon as at least 50% of the dendrograms have it; the number of MFW ranges from 100 to 160, culling from 0% to 100%. The consensus trees for the 10 distances available in the package `styl0` highlight a partition: 7 distances make *Gr.-Lat.*, and 3 *Comm.*, as the translation nearest to *Elem.* Two examples are displayed as Figures 12a–b of the Appendix.

The data displayed in a dendrogram can be also visualised by means of a weighted network, which represents a kind of “lexical interaction” between texts in the form of a network whose edges have a weight that is proportional to lexical proximity; this is measured, in the networks displayed as Figures 13a–b of the Appendix and in the same order from left to right, by the same two distances and parameters as in the two dendrograms displayed as Figures 11a–b of the Appendix.

Principal component analysis

A different approach, the “principal component analysis” (PCA) as applied to frequency distributions of lemmas in a set of works,⁶⁷ looks at a representative low-dimensional cross-section of the multidimensional space distribution of the relative frequency vectors f_{ij} , $i = 1 \dots n$, $j = 1 \dots k$, of n lemmas associated with the k works. “Representative” means here that most of the variance of the distribution is captured by the cross-section. This amounts to diagonalising the covariance matrix or the correlation matrix of the relevant vectors and by selecting the span of the eigenvectors that have the highest eigenvalues; each of the latter is the variance of a “principal component.” For the purpose of representing the result in a graph, two such vectors are selected. This bidimensional cross-section projects the distribution cloud, which is located in the k -dimen-

⁶⁷ On PCA see Ian T. Jolliffe, *Principal Component Analysis* (New York; Berlin; Heidelberg: Springer, 2002).

sional space of the selected lemmas or in the n -dimensional space of the works,⁶⁸ onto the plane that contributes most to the lexical variation. The number of MFW is kept fixed, culling can be weighted out by consensus. The variations along the two principal components PC1 and PC2 are represented in a plane graph, which plots both the MFW and the titles of the works, each referred to axes lying on opposite sides of the plot square. A separate histogram in the plot gives the ratio of the variance “explained” – that is, projected onto the associated eigenvectors – by the several PCs. The works can also be set out in a separate graph.

The four plots displayed as Figures 14a–b and 15a–b of the Appendix select two MFW numbers (lower numbers give less crowded plots) and accordingly the two ways of presentation just mentioned, and operate independently on the covariance matrix and on the correlation matrix.⁶⁹ Metalemmas on the border of the cloud contribute to the variance of the distribution more than tokens near its center; the same applies to works. The plot for the covariance matrix of the metalemmas is extremely crowded towards the center, as is natural for a work and its translations – the plot for the correlation matrix better displays the fine structure of the central core. PCA as applied to the covariance and correlation matrices exhibits an extremely flattened distribution of lemmas and works along the two principal components: PC1 and PC2 “explain” a very high ratio of the variance.

Syntactic analysis

The deductive progression of the four works has been quantified by assigning a level of “logico-syntactic subordination,” or “syntactic depth,” to each autonomous clause of each proposition of a segment of text as representative as that of the entire Book I. The “autonomous clauses” are main and subordinate clauses, genitive and ablative absolutes, non-attributive participles; in terms of discourse logic, postposed explanations and identifications of objects introduced by “that is” are counted as subordinates.

The main clauses of each proposition of Book I are assigned level 0; each subordinate raises the level by one unit; nested subordinates allow reaching higher levels. The result is a numerical sequence that can be represented by a “subordination diagram,” where the dots are joined by line segments. In principle, a Greek mathematical proposition in pure demonstrative style should exhibit a subordination diagram flattened on level 0, with the following exceptions:⁷⁰

⁶⁸ To a lemma is attached the k -component vector of its relative frequencies in the k works, to a work is attached the n -component vector of the relative frequencies in it of the n lemmas.

⁶⁹ In the PCA plots, every second-order metalemma gets an abridged designation that is made of the first two lemmas it includes, taken in alphabetic order Greek-Latin and preceded by letter M.

⁷⁰ On all the parts of demonstrative discourse listed below see Acerbi, *The Logical Syntax*, sects. 1.1, 4.1, 4.4, 4.5.3, 5.2.1, 5.3.2.

- the protasis of the conditional clause that formulates the enunciation of most theorems;
- the substantive clause that, in diorisms, states in instantiated form what is to be demonstrated;
- the causal subordinate that opens the proof or a sub-proof;
- the protasis of the conditional clause that opens a *reductio ad absurdum*, and the relative clause – ὅπερ ἄτοπον “which is absurd” or the like – that closes it;
- the QED formulas ὅπερ ἔδει δεῖξαι and ὅπερ ἔδει ποιῆσαι at the end of the conclusion of a proposition; this is also an example of a nested subordination, which reaches to level 2 (this fact will be further assessed presently).

One must not forget that a canonical text like the *Elements* is already an exception to the just-described optimal behaviour: in the whole treatise as it is handed down by the manuscript tradition, there are more than 200 postposed explanations introduced by γάρ (main clause, which is assimilated to a subordinate in point of discourse logic), 40 by ἐπειδήπερ (causal subordinate), and 36 by διὰ τό + infinitive (causal subordinate). Each of these clauses raises the level by 1.

We decided to process two sets of data: (1) as described above; (2) the subset of (1) that results from eliminating all conclusions, inclusive of the QED formula. The reasons for this choice are that (a) the conclusions – which by default repeat the enunciation word-for-word or are just a truncation of it – and the QED formulas do not have any deductive import; (b) *Gr.-Lat.* almost randomly curtails both the conclusions and the QED formulas (writing, for instance, just *quod oportebat* and omitting *facere*, which amounts to eliminating a level-2 subordinate), whereas Heiberg's edition of the Greek text homogenises the manuscript evidence and prints all of the QED formulas in full; (c) perfectly well-formed Latin QED formulas like *quod demonstrandum erat* count as one single level-1 subordinate; they are used almost randomly in *Clavius*. All of this shows that the fully-fledged conclusions introduce a bias in the measurement of syntactic depth.

The subordination diagrams of the four works are displayed as Figures 16a–h of the Appendix; for each work, the graph for dataset (1) precedes that for dataset (2).

The raw data set out in the subordination diagram may also be processed to give a number of bulk or intensive indicators of syntactic depth and information availability; all these indicators bar the second and the next-to-last in the list below increase with syntactic depth.

- The overall number of autonomous clauses in Book I, which is an index of deductive dilution, for the same result is established in a greater number of steps.
- The overall and relative (that is, normalised to the overall number of autonomous clauses) numbers of level 0 clauses in Book I, which is an index of adherence to the standard stylistic code.

- The overall and relative (that is, divided by the overall number of level $\neq 0$ clauses) sum of the levels of level $\neq 0$ clauses in Book I,⁷¹ which are the area under the subordination diagram, if this were set out as a histogram, and its mean height, respectively, and measure the strength of the “explanatory brake.”⁷²
- The mean value for a proposition of the indicators defined in the previous item.
- The first intensive indicator is the “temperature” T_A of text A , defined by means of the Equipartition Theorem. We measure the “mean thermal energy” associated with the level fluctuations of the clause sample by computing the discrete first derivative of the level function and by taking the expectation value of the square of this derivative w.r.t. a discrete Maxwell-Boltzmann probability distribution featuring the same square in the mass function: if a_{ij} is the i -th autonomous clause in the j -th proposition of work A starting from a_{op} , for proposition j we compute $\delta_{ij} = |a_{ij} - a_{i-1,j}|$, where $i \geq 1$, and we set $T_A \equiv \sum_j \sum_i \delta_{ij}^2 \exp(-\delta_{ij}^2)$. As is to be expected, a stronger explanatory brake makes a text “hotter” than another. There is no need to introduce constants of normalisation because we set a definition and the temperature scale is arbitrary; see the next indicator. Any text flattened to one single level – which is necessarily level 0 – has $T_A = 0$.
- The “temperature” as defined in the previous item, but normalised to $T_A = 100$ for *Elem.*
- The second intensive indicator is a kind of entropy, which measures the degree to which information is “buried” in deep syntactic levels; if a_i is the relative (that is, divided by the overall sum of the levels of the clauses in Book I) sum of the levels of the clauses in proposition i of Book I of work A , we define its “proposition entropy” $H_p \equiv -\sum_i a_i \log a_i$. We set $H_p = 0$ for any text flattened to one single level.
- Likewise, if k_i is the relative (that is, divided by the number of clauses in Book I) frequency of level $i \geq 0$ clauses in Book I of work A , we define its “book entropy” as $H_b \equiv -\sum_i k_i \log k_i$. We set $H_b = 0$ for any text flattened to one single level.

The following tables set out the indicators of dataset (1) first, then those of dataset (2).

(1)	# cl.	# level 0			sum levels		mean prop.		T	T _n	H _p	H _b
<i>Elem.</i>	1332	796	59.760	628	1.172	13.083	2.441	253.243	100.000	1.6393	0.3741	
<i>Gr.-Lat.</i>	1266	773	61.058	562	1.140	11.708	2.375	228.523	90.239	1.6344	0.3606	
<i>Comm.</i>	1486	808	54.374	816	1.204	17	2.507	321.302	126.875	1.6269	0.4024	
<i>Clavius</i>	1518	670	44.137	1157	1.364	24.104	2.842	363.306	143.461	1.5845	0.4639	

⁷¹ Of course, this coincides with the sum of the levels of all clauses in Book I.

⁷² This metaphor means that syntactic depth, frequently in the form of postponed explanations, slows down the deductive progression.

(2)	# cl.	# level 0		sum levels		mean prop.		T	T _n	H _p	H _b
<i>Elem.</i>	1118	732	65.474	423	1.096	8.812	2.283	190.851	100.000	1.6173	0.3281
<i>Gr.-Lat.</i>	1129	732	64.836	440	1.108	9.167	2.309	195.194	102.275	1.6175	0.3354
<i>Comm.</i>	1217	738	60.641	555	1.159	11.562	2.414	238.898	125.175	1.5984	0.3673
<i>Clavius</i>	1396	632	45.272	1065	1.394	22.187	2.904	337.040	176.598	1.5687	0.4683

We may also compute the relative proposition entropy $H(A||B) \equiv \sum_i a_i \log(a_i/b_i)$ for both datasets, with a_i and b_i defined as for the proposition entropy. This yields the following values:

(1)	<i>Elem.</i>			(2)	<i>Elem.</i>		
<i>Gr.-Lat.</i>	0.0061	<i>Gr.-Lat.</i>		<i>Gr.-Lat.</i>	0.0043	<i>Gr.-Lat.</i>	
<i>Comm.</i>	0.0130	0.0140	<i>Comm.</i>	<i>Comm.</i>	0.0185	0.0208	<i>Comm.</i>
<i>Clavius</i>	0.0796	0.0676	0.7291	<i>Clavius</i>	0.0973	0.0936	0.9500

The first table portrays *Gr.-Lat.* as “cooler” and syntactically tighter than *Elem.* The second table shows that the above-mentioned abbreviations of QED formulas in *Gr.-Lat.* by and large explain away this bewildering state of affairs: the syntax of *Gr.-Lat.*, Book I dataset (2), is marginally “hotter” and less tight than the syntax of *Elem.* The subordination diagrams and all indicators make it obvious that *Clavius* slams on the explanatory brake and locates more information in deep syntactic levels. *Comm.* always occupies an intermediate position.

Conclusion(s)

After this full immersion in technicalities, we would like to make two final points, touching on translation theory and on a literary-theoretic assessment of *Clavius’* edition of the *Elements*.

First, we believe that the present paper might contribute to showing that a strictly technical, digital humanities-oriented approach to (early modern) scientific translation may lead to rigorous results that corroborate factually- and historically-rooted assessments. One must not forget that early translators very seldom indulge in metadiscourse about their own translation methods;⁷³ present-day historians must normally extract such methods –

⁷³ A celebrated exception is Burgundio’s praise of the *de verbo ad verbum* method in the preface of his own translation of John Chrysostomos’ homilies on John’s Gospel: Peter Classen, *Burgundio von Pisa. Richter – Gesandter – Übersetzer*. (Heidelberg: Carl Winter Universitätsverlag, 1974), 84–102, English translation of the relevant passages in Dimitri Gutas, Charles Burnett,

if any can be perceived – from selected features of a given translation, as for instance the way specific classes of function words are translated.⁷⁴ These focused investigations may also settle problems of attribution. In the case of the mediaeval translators of Aristotle, this approach was brought to perfection by Lorenzo Minio-Paluello and is currently pursued by the Leuven school that is in charge of the editions of the *Aristoteles Latinus*.⁷⁵ As we have seen on the example of *Gr.-Lat.*, such an approach was also used by the editors of the mediaeval Latin translations of Greek scientific works.⁷⁶ On the other hand, translation studies appear to have started systematically interacting with digital humanities only very recently; we hope that our study will contribute to widening this interaction.⁷⁷

and Uwe Vagelpohl, eds., *Why Translate Science? Documents from Antiquity to the 16th Century in the Historical West (Bactria to the Atlantic)* (Leiden; Boston: Brill, 2022), 493–497; this book conveniently collects, both in the original and in English translation, preliminary statements of translators that operated before the modern era (neither of the statements to be read below is reported). Burgundio lists Boethius' version of Nicomachus' *Introductio arithmetica* among the *de verbo ad verbum* translations, but Boethius states exactly the contrary: Classen, "Burgundio," 91.139–92.141 = Gutas, Burnett, and Vagelpohl, *Why Translate Science?*, 495 versus Boethius, *De institutione arithmetica*, 4.27–5.4. In his philosophical translations, Boethius indeed acted as a *fidus interpres* (as Horace, *Ars poetica* 133–134, calls the author of a word-for-word version): Gutas, Burnett, and Vagelpohl, *Why Translate Science?*, 32, 44. One might also wish to compare Burgundio's praise with Vladimir Nabokov's brilliant defence of his own translation of Pushkin's *Onegin*, see Vladimir Nabokov, "Problems of Translation: *Onegin* in English," *Partisan Review* 22 (1955): 496–512.

⁷⁴ The obvious fact that content words cannot be used to establish filiations between the Latin translations of the *Elements* (by the way, these filiations were already established, to the satisfaction of everyone, in the Médée website) has apparently escaped the author of Benjamin Wardhaugh, "Euclidean terms in European languages, 1482–1703," *Historia Mathematica* 68 (2024): 22–37.

⁷⁵ See Minio-Paluello, *Opuscula*, and the *prolegomena* to the several volumes of the *Aristoteles Latinus* series, listed at <https://hiw.kuleuven.be/dwmc/research/al>.

⁷⁶ For *Gr.-Lat.*, see already John E. Murdoch, "Euclides Graeco-Latinus: A Hitherto Unknown Medieval Latin Translation of the Elements Made Directly from the Greek," *Harvard Studies in Classical Philology* 71 (1967): 249–302. Lexical analyses of other products of the "Sicilian school" of translation (on which see at least Charles H. Haskins, *Studies in the History of Mediaeval Science* (Cambridge, MA: Harvard University Press, 1924), 155–193) can be read in Wilfred R. Theisen, "Liber De Visu: The Greco-Latin Tradition of Euclid's Optics," *Mediaeval Studies* 41 (1979): 44–105, at 51–54; Shuntarō Itō, *The Medieval Latin Translation of the Data of Euclid* (Tokyo; Boston; Basel; Stuttgart: University of Tokyo Press; Birkhäuser, 1980), 23–38.

⁷⁷ See Julie McDonough Dolmaya, *Digital Research Methods for Translation Studies* (London and New York: Routledge, 2024); Hilary Brown, Regina Toepfer, and Jörg Wesche, *Early Modern Translation and the Digital Humanities* (Berlin: J. B. Metzler, 2025); Raluca Tanasescu, "Translation Studies and Digital Humanities," in *The Routledge Handbook of Translation Technology and Society*, ed. by Stefan Baumgarten and Michael Tieber (London and New York: Routledge, 2025), 192–205.

Second, our analysis appears to support a historiographical conclusion. To get there, we first remark that the Latin mathematical lexicon did not get stabilised by a sustained tradition as the Greek lexicon did. By Clavius' times, the mediaeval translation practice *de verbo ad verbum* had fallen out of fashion; as for the geometric lexicon, it was the result of the conflation of the lexica stemming from the mediaeval Arabo-Latin tradition and from the much more recent Graeco-Latin tradition. Prospective scholars received a *trivium*-based education, with glimpses, if any, of quadrivial territory.⁷⁸ This entails, among other things, that a varied lexicon and a complex syntax were employed as a matter of course and even valued, thereby diluting the lexical and syntactic rigidity of technical Greek.

Clavius' well-thought, mathematically sound, far-reaching educational project aims at blending the theoretical mathematics handed down from ancient Greece (Tome I of his *Opera Mathematica*), the practical mathematics that characterised the Middle Ages and the early modern era (Tome II), and the mathematics traditionally applied to astronomy and chronology in spherical and plane trigonometry (Tomes I and III), *astrolabica* (Tome III), *gnomonica* (Tome IV), and *Easter Computi* (Tome V). The intended readership of this project mainly point to *trivium*-trained, young students. Let us first read how Conrad Dasypodius – the author with Christian Herlinus of a translation of *Elem.* I–VI featuring a “translation” of all proofs into Aristotelian syllogisms – put it a few years before Clavius published his own edition:

[an Euclidean proof] *ex multis constat syllogismorum argumentis seu mediis, ad tollendam itaque obscuritatem & difficultatem: syllogismorum propositiones seiunguntur: & quæ à Theone acutissima breuitate sunt connexa: hic fusius sunt dilatata, & apertiora reddita. [...]* Erunt fortassis qui dicent hæc nimis scholastica & puerilia esse: fateor & apertè atque candidè dico pueris me hæc scribere, & nostris discipulis qui ex classibus post cognitionem linguarum, adhæc Rhetoricæ & Dialecticæ deducuntur ad geometras, & descendunt in arenam geometrarum, ut ibi se exerceant: & iudicium quod tenerum adhuc est, confirment: animum quoque suum hisce contemplationibus iuuent: denique ipsam memoriam in qua multum est situm corroborent.⁷⁹

To summarise: this is trivial stuff devised for our students: we needed to give the Euclidean proofs a form (apparently, no Euclidean form was perceived): this form is the Aristotelian syllogism, by means of which we also make explicit all deductive steps that the original text keeps implicit.

⁷⁸ Clavius, a self-taught mathematician, certainly did not get any glimpse: he was famously flashed by geometry when attending a course on Aristotle's *Posterior Analytics* (Clavius, *Correspondenza*, vol. I, parte 1, 38).

⁷⁹ *Analyseis geometricæ sex librorum Euclidis. Primi et quinti factæ à Christiano Herlino : reliquæ unâ cum commentariis, & Scholiis perbreuibis in eosdem sex libros Geometricos: à Cunrado Dasypodio. [...]* Pro schola Argentinensi [...]. Excudebat Iosias Rihelius. M.D.LXVI, aiii.

Clavius probably did not have students he could call *pueri*, but he also needed to give his Euclid a form suited to such a readership and attendance: a form in which the tightly intertwined steps of the logical fabric of a proof *fusius sunt dilatata, & apertiora reddita*. Let us read the statement of Clavius' programme, which includes a statement of the method underlying his version of the *Elements*:⁸⁰

Demonstrationes aliorum, maxime Theonis, quas quidem ipsius esse Euclidis, non leuibus argumentis adducti quidam asseuerant, & Proclus etiam testatur, breuiiores, quantum per rei difficultatem licuit, uel certe planiores, quando illud non potuimus, delucidioresque reddere conati sumus. Non enim illas nude, ac totidem uerbis, quot erant scripte, proposuimus. Etenim ea est interdum illarum breuitas, ut illud accadat, quod ab elegantissimo poeta dictum est. Breuis esse laboro, obscurus fio.⁸¹ Interdum etiam, cum breuius, atque succinctius efferri possint, magna, ob longiorem, quam satis est, sermonem, affertur molestia legenti. Quare utrunque uitantes, eas, uelut παραφραστικῶς, atque ad eum fere modum tradidimus, quem, cum publice Euclidem interpretaremur, obseruauimus; hac etiam re auditorum desiderio, & uoluntati, quantum est in nobis, satisfacere cupientes. Ita enim, nostra sententia, Euclides facilius a studiosis, ijs præsertim, qui ceu tyrones, hæc Mathematica studia nunc primum auspicantur, ac maiore uoluptate, utilitateque cognoscetur.

Thus, Clavius' edition of the *Elements*, and in fact most of his works, are first and foremost scholarly products tuned to a readership of students.

In the perspective of the present study, the result of all of this is the dissolution of one of the stylistic codes that made Greek mathematics a distinctive literary product and the *Elements* one of its gems. Starting with Clavius, and despite his boasting about adopting an intermediate approach, this code was replaced by a mix of pedantry and rhetorical flourish, both witnessed, as highlighted in the previous Sections, by the conspicuous lexical spread and by the increasing syntactic complexity of Clavius' text – not a real translation,

⁸⁰ *Euclidis Elementorum libri XV, Ad Lectorem*. A similar statement can be read in the short text introducing the five riddles in form of epigrams contained in the Appendix of Clavius' *Algebra* (*Algebra Christophori Clauii Bambergensis e Societate Iesu. Romae, Apud Bartholomæum Zannetum. Anno M. DC. VIII.*, 378; the enthusiastic praise of algebra on pages 1–3 is worth a reading): Clavius states that they are followed by *eorum interpretatione, non quidem de verbo ad verbum, sed ut res clarior fieret, expresso illorum senso dumtaxat*. Note that this does not prove that Clavius himself translated these poems; the fact that he proposed three versions of the last epigram suggests the opposite. Clavius' preface to his own edition of the *Elements* was slightly modified in the subsequent reprints: most notably, Commandinus was downgraded from *geometra peritissimus* (1574) to *non vulgaris* (1589 and later).

⁸¹ Horace, *Ars poetica* 25–26. This quote is also an allusion to a broader debate, for the topos of the *fidus interpres*, amply attested in Latin sources as far as late antiquity, frequently alludes to Horace's programmatic poem (see note 73 above).

let us stress this point once more – of the *Elements*. The dissolution of the demonstrative stylistic code paved the way to establishing the modern historiographical assessment of Greek mathematical works through their “mathematical content” only, and to entrenching the belief that scientific texts are lowest-brow paraliterature amply deserving the contempt of purportedly high-brow classicists.

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Fig. 1 – The progressive creation of the dictionaries of the four works.

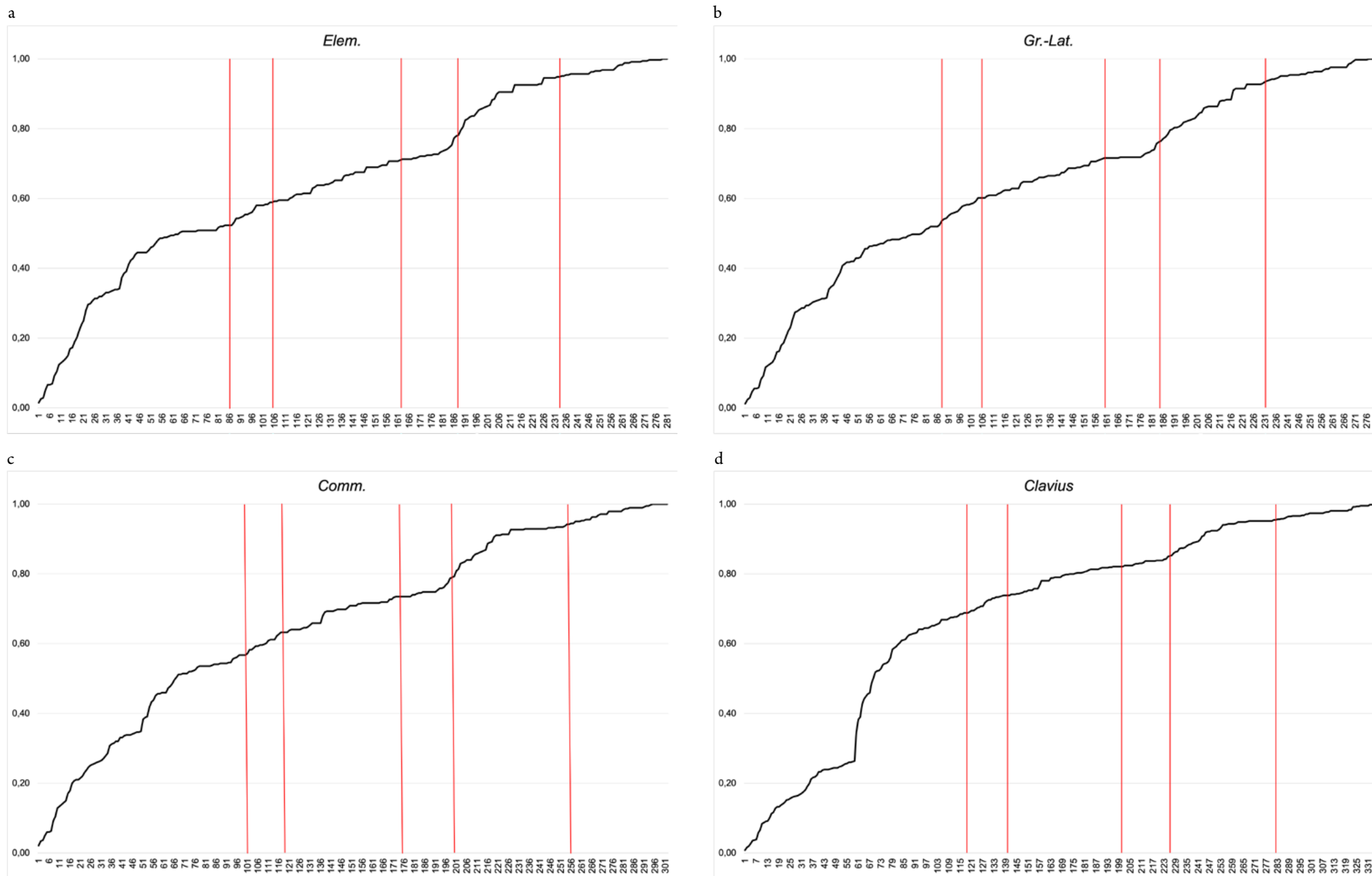


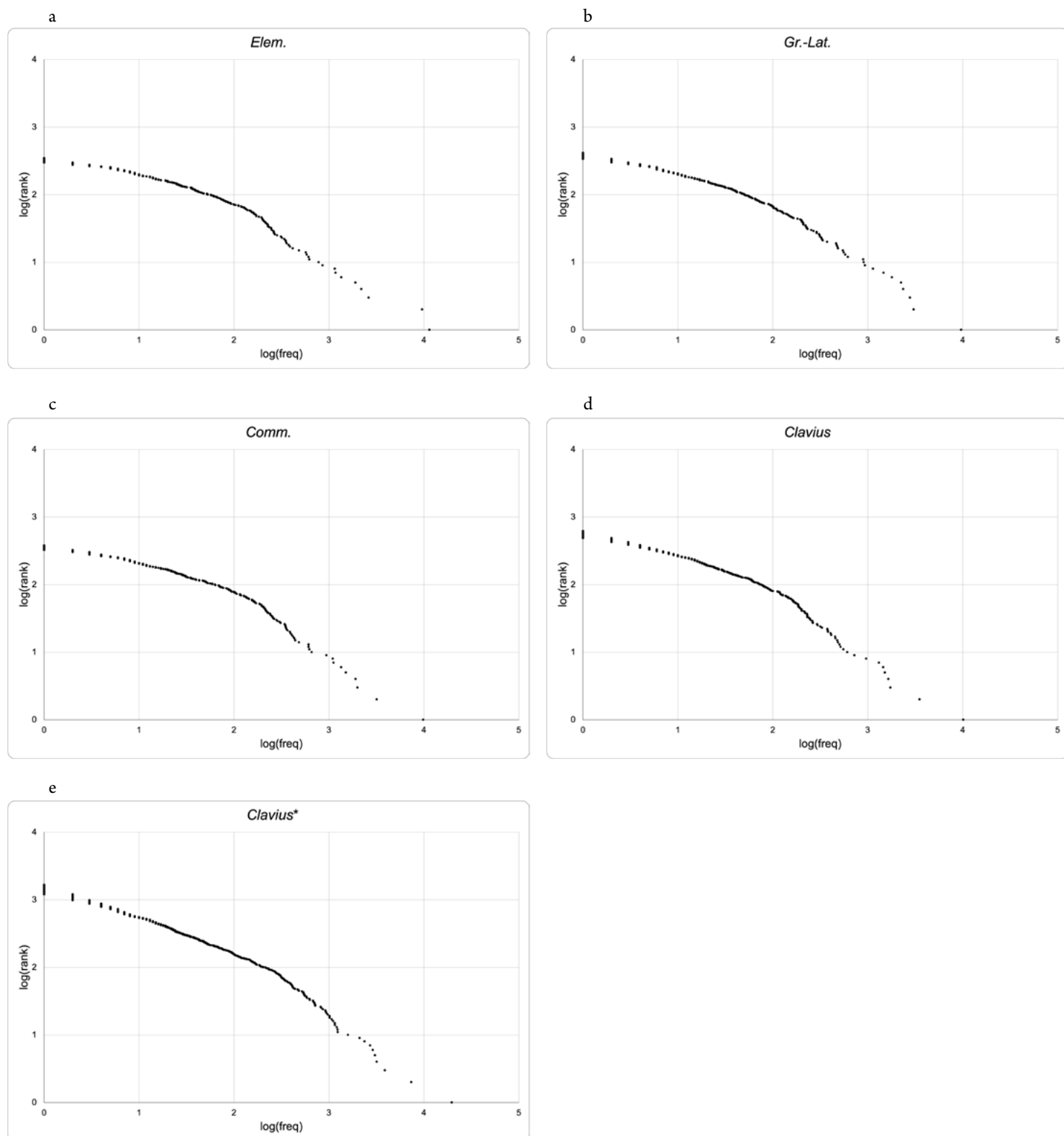
Fig. 2 – The rank distributions of the lemmas as a function of the frequency in the four works and in *Clavius** (Zipf's law).

Fig. 3 – The Sankey diagram that displays the relative frequency flow between the 20 most frequent lemmas of the four works.

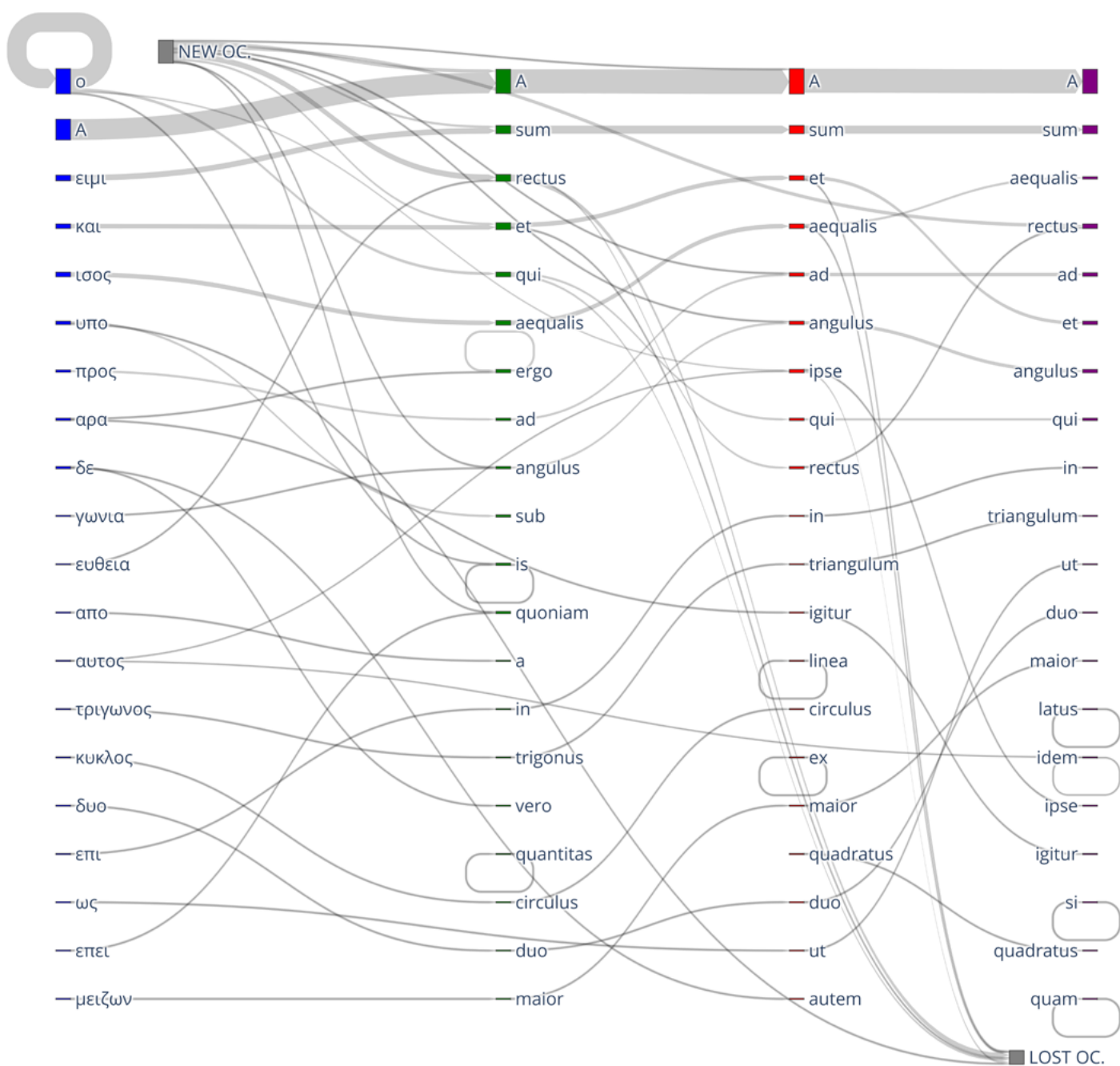


Fig. 4 – The form spread of the core metalemmas of the four works.

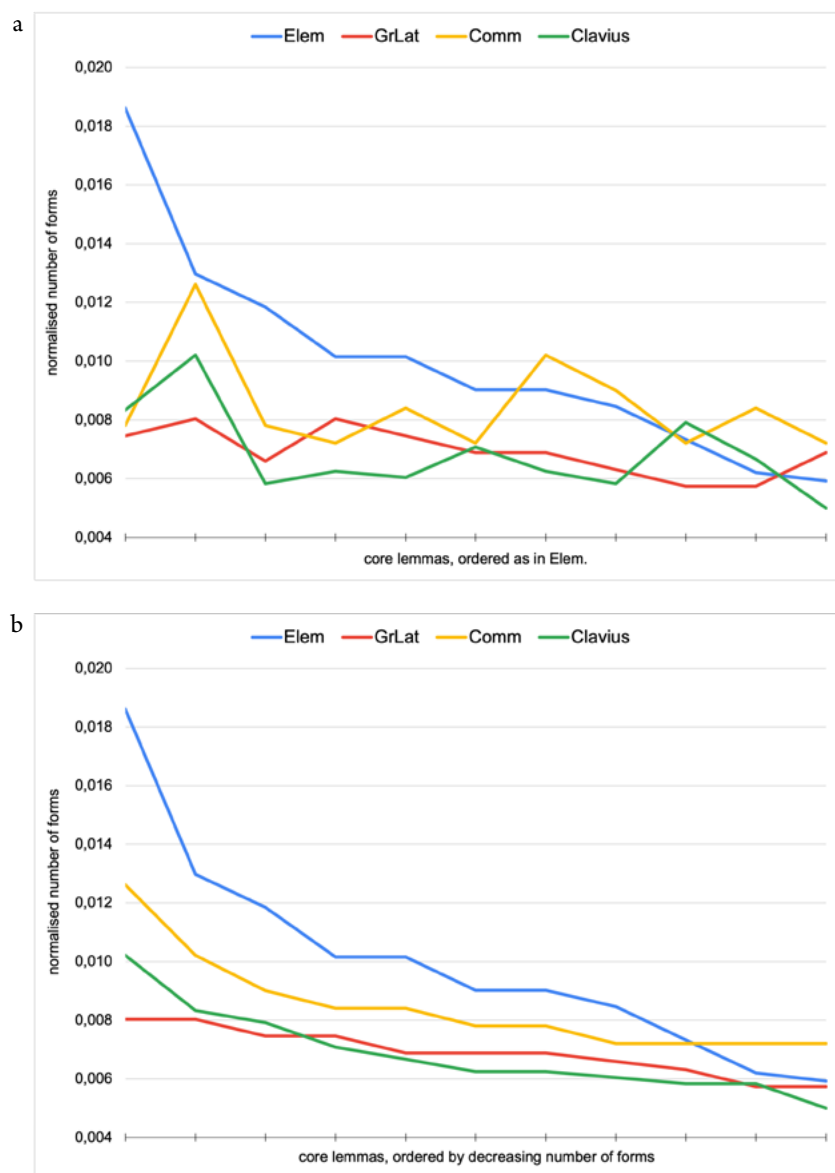


Fig. 5 – The distribution by POS of the relative frequency of the occurrences in the four works.

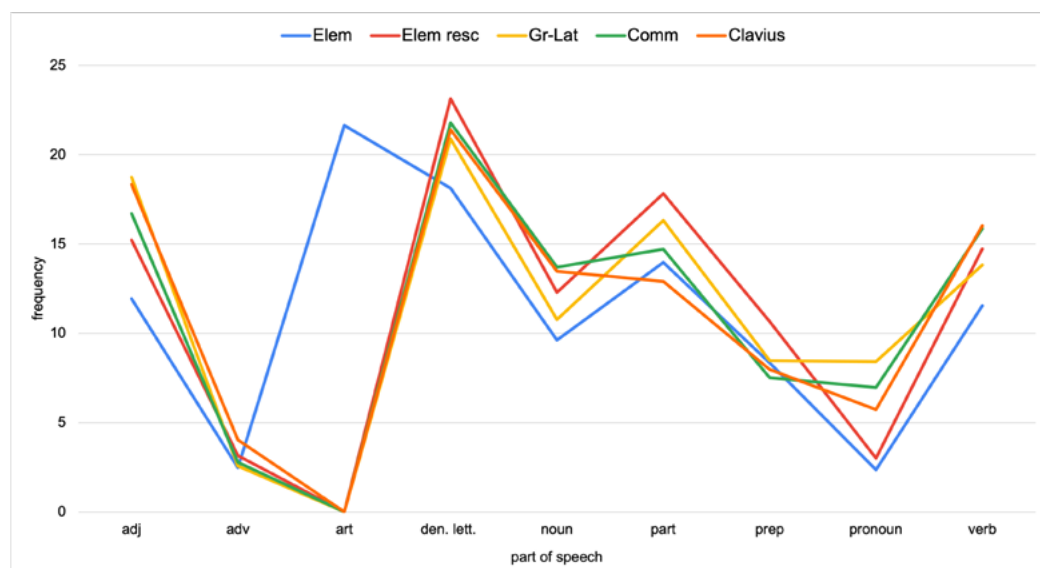


Fig. 6 – The behaviour of the X-entropy by Book in the four works.

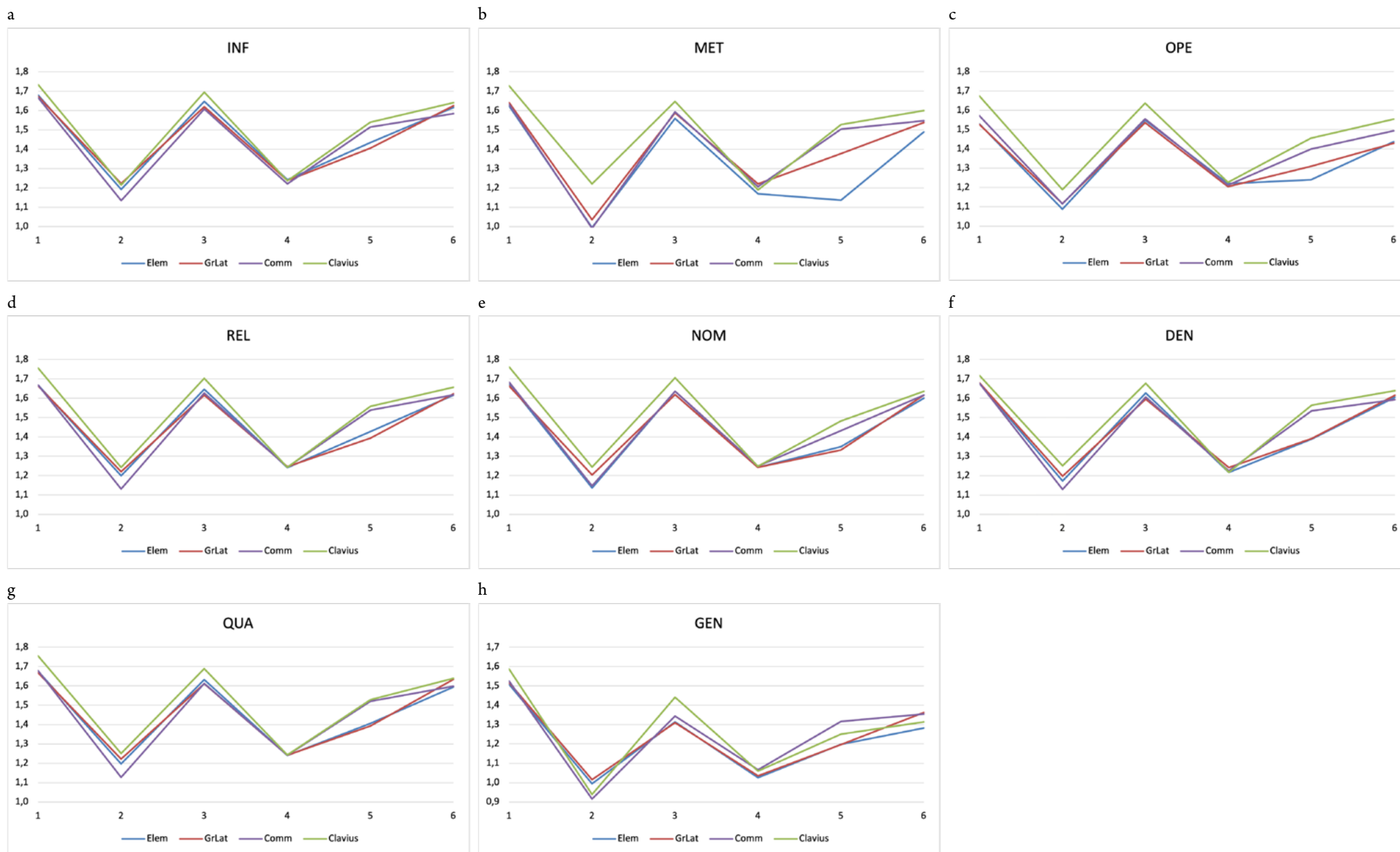


Fig. 7 (a-d) – The bookwise behaviour of the four X-cloud indicators in the four works for three selected categorial types.

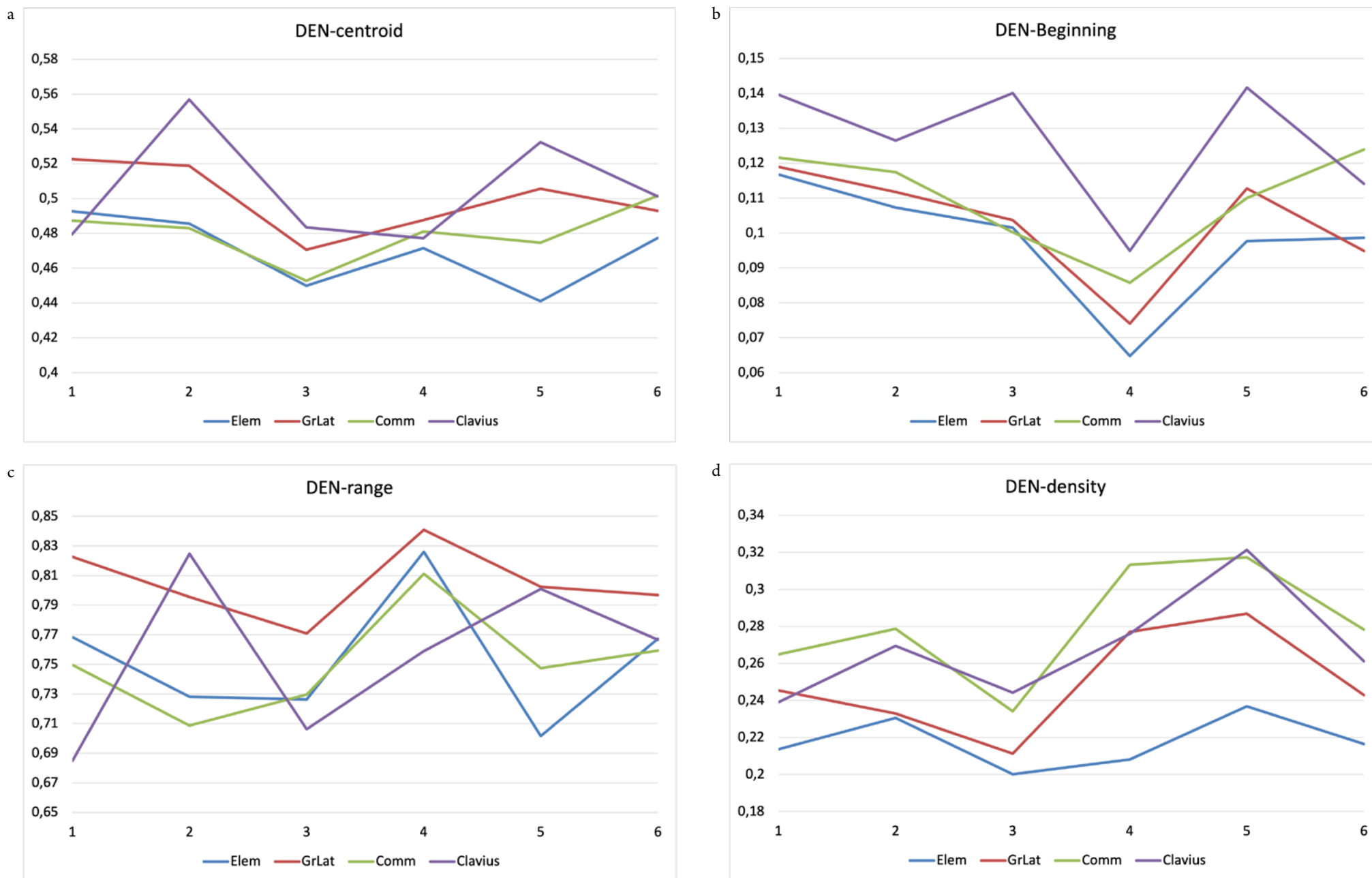


Fig. 7 (e-h) – The bookwise behaviour of the four X-cloud indicators in the four works for three selected categorial types.

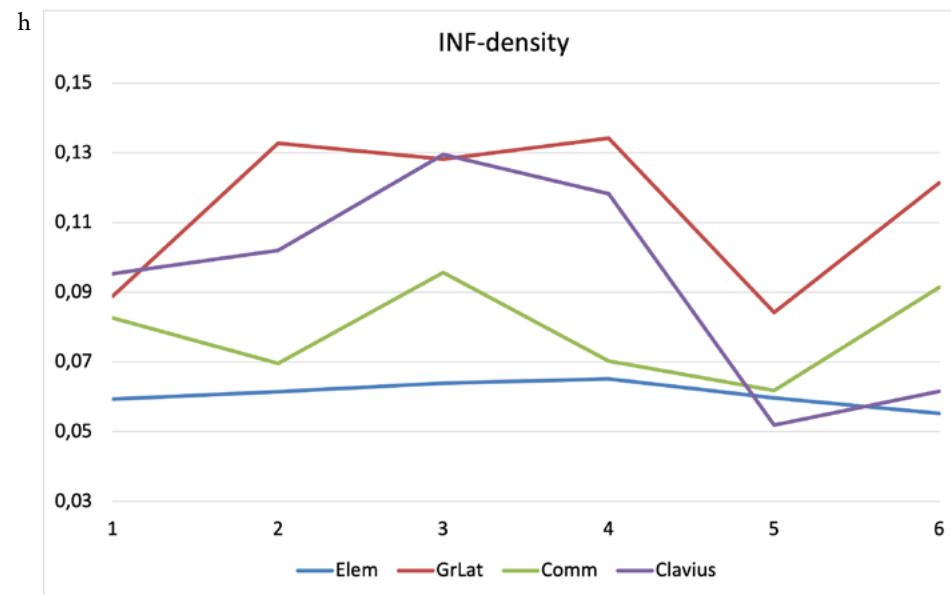
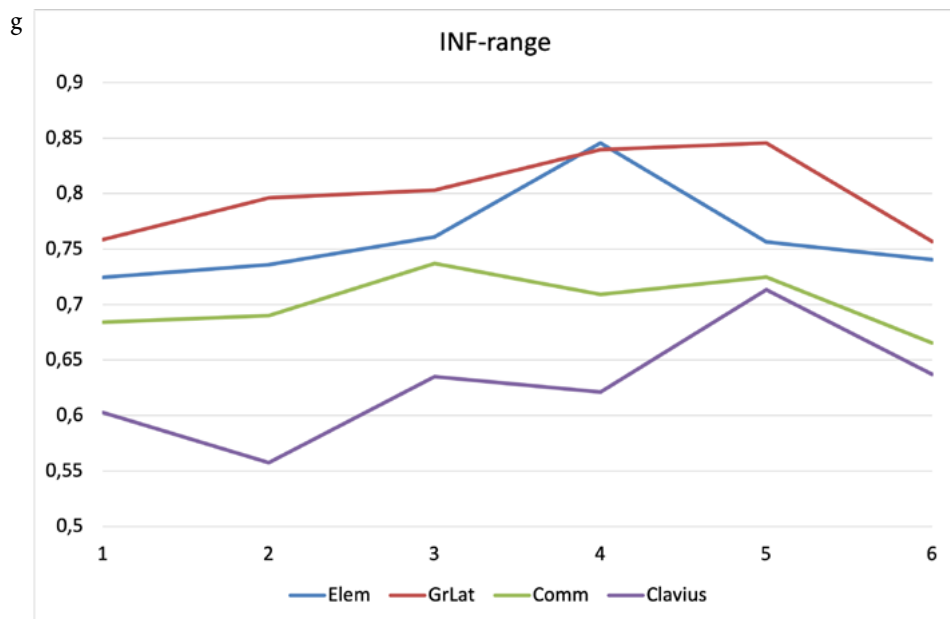
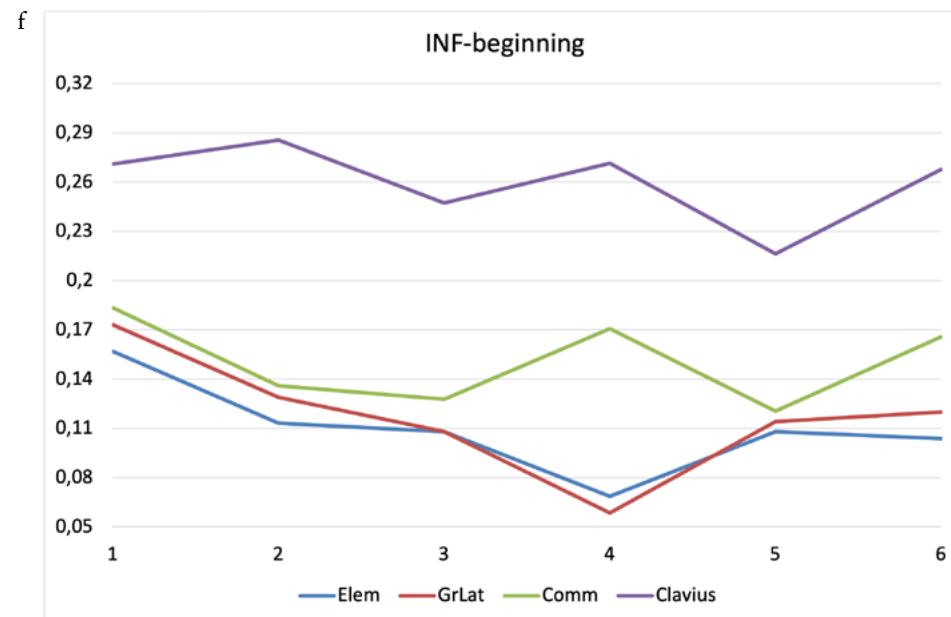
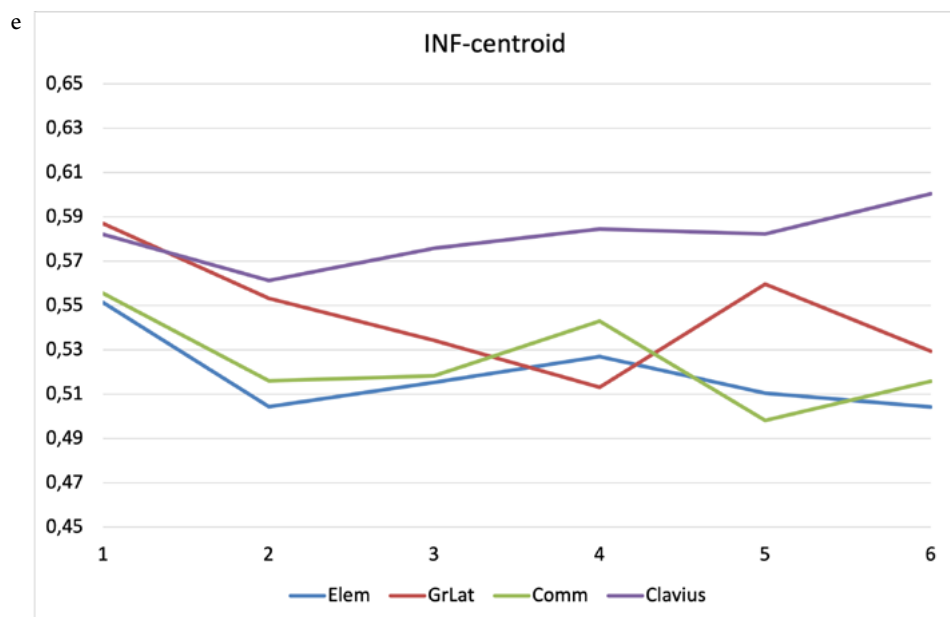


Fig. 7 (i-l) – The bookwise behaviour of the four X-cloud indicators in the four works for three selected categorial types.

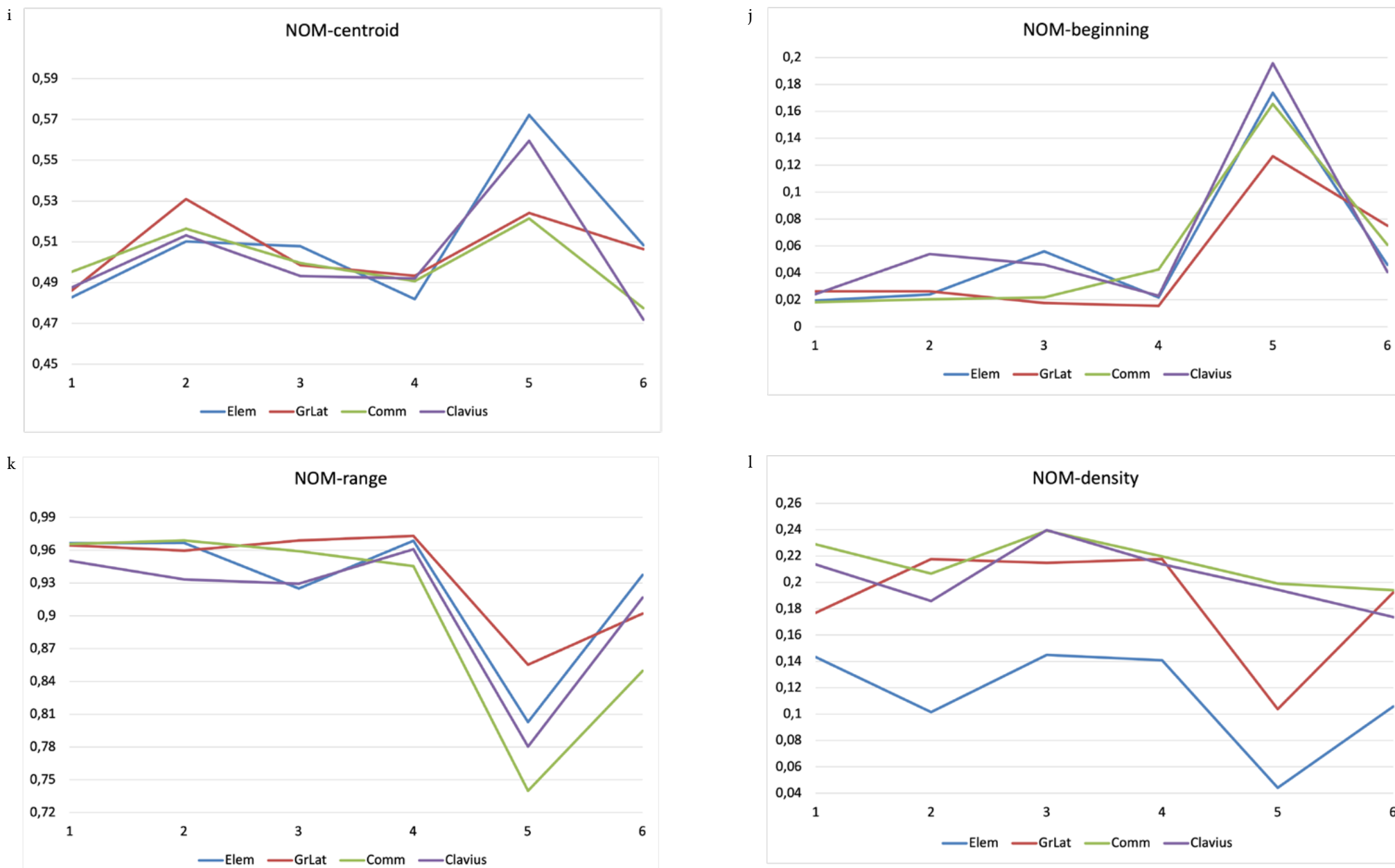


Fig. 8 (a-c) – The propositionwise behaviour of three X-cloud indicators in the four works for two selected categorical types.

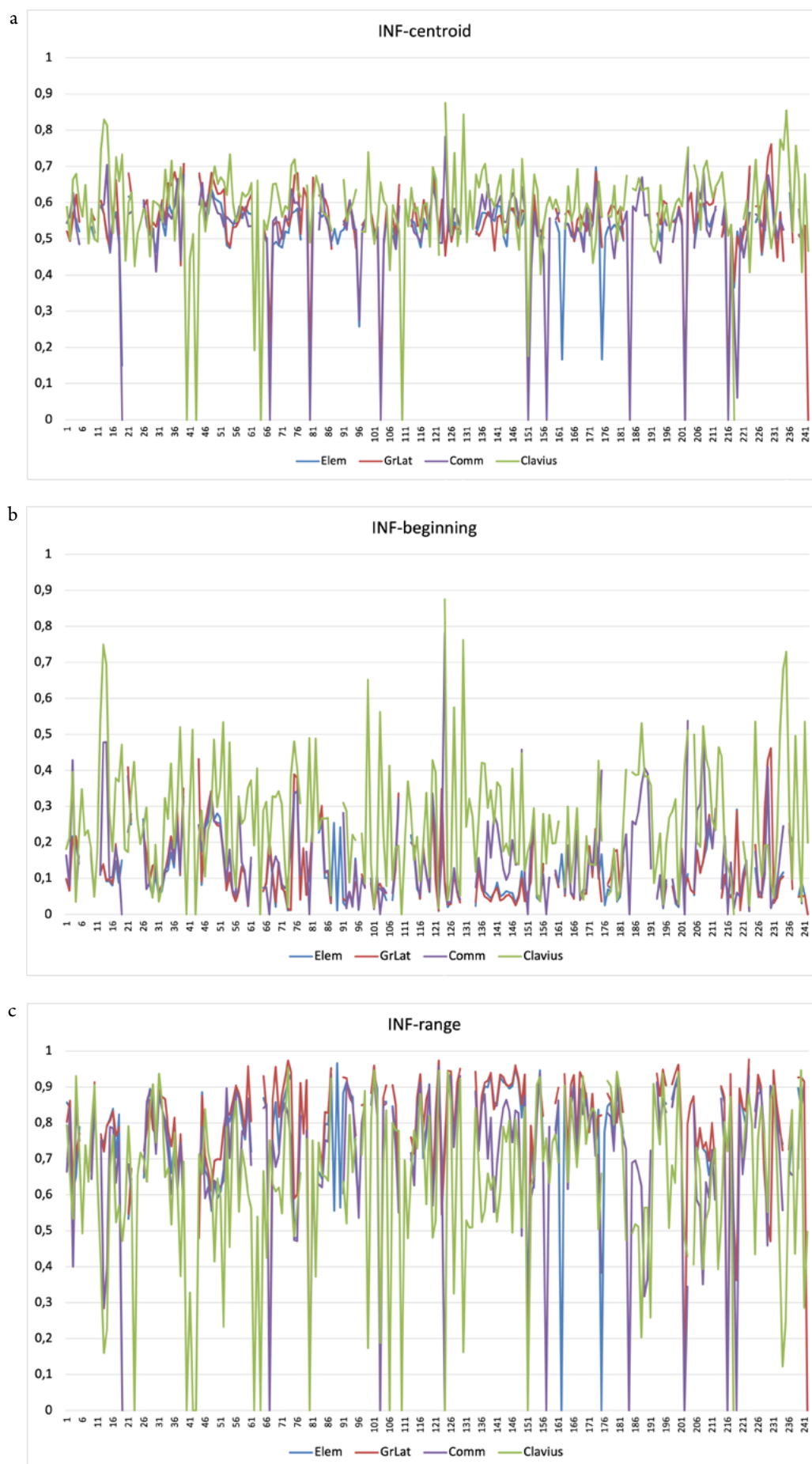


Fig. 8 (d-f) – The propositionwise behaviour of three X-cloud indicators in the four works for two selected categorical types.

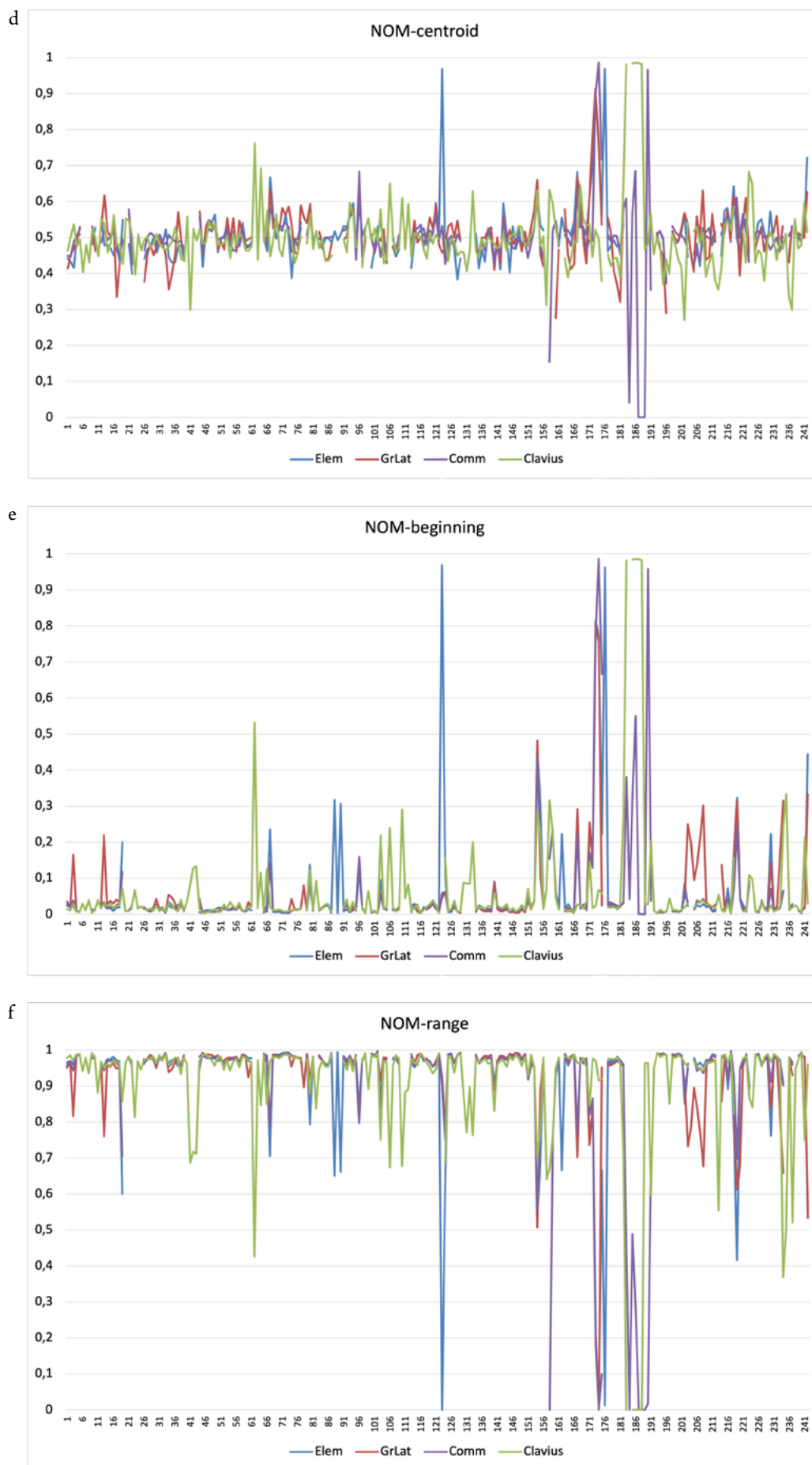


Fig. 9 – The number of Latin lemmas are ranked in a given position in the sequences that translate one and the same Greek lemma.

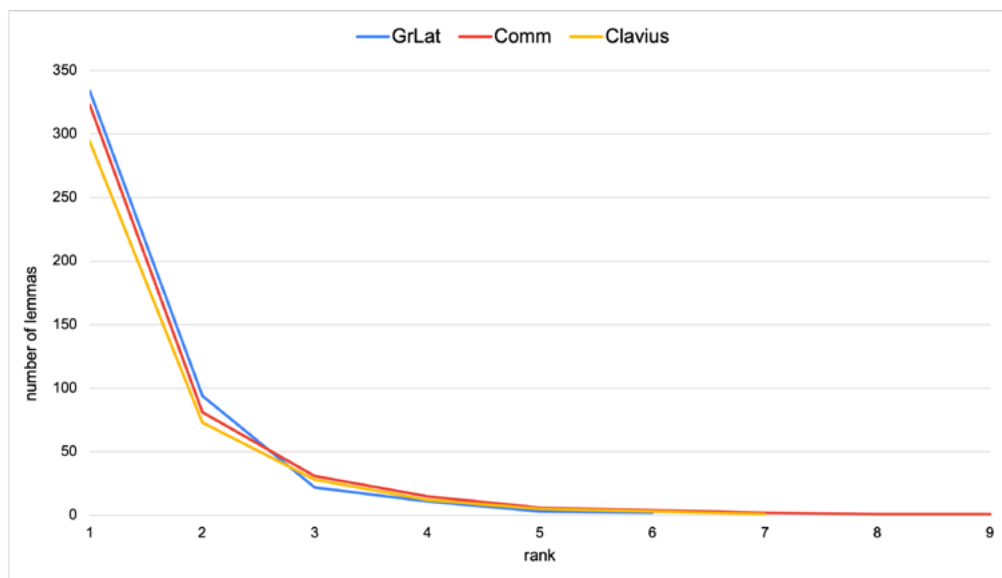


Fig. 10 – The dispersion matrices of the three Latin translations.

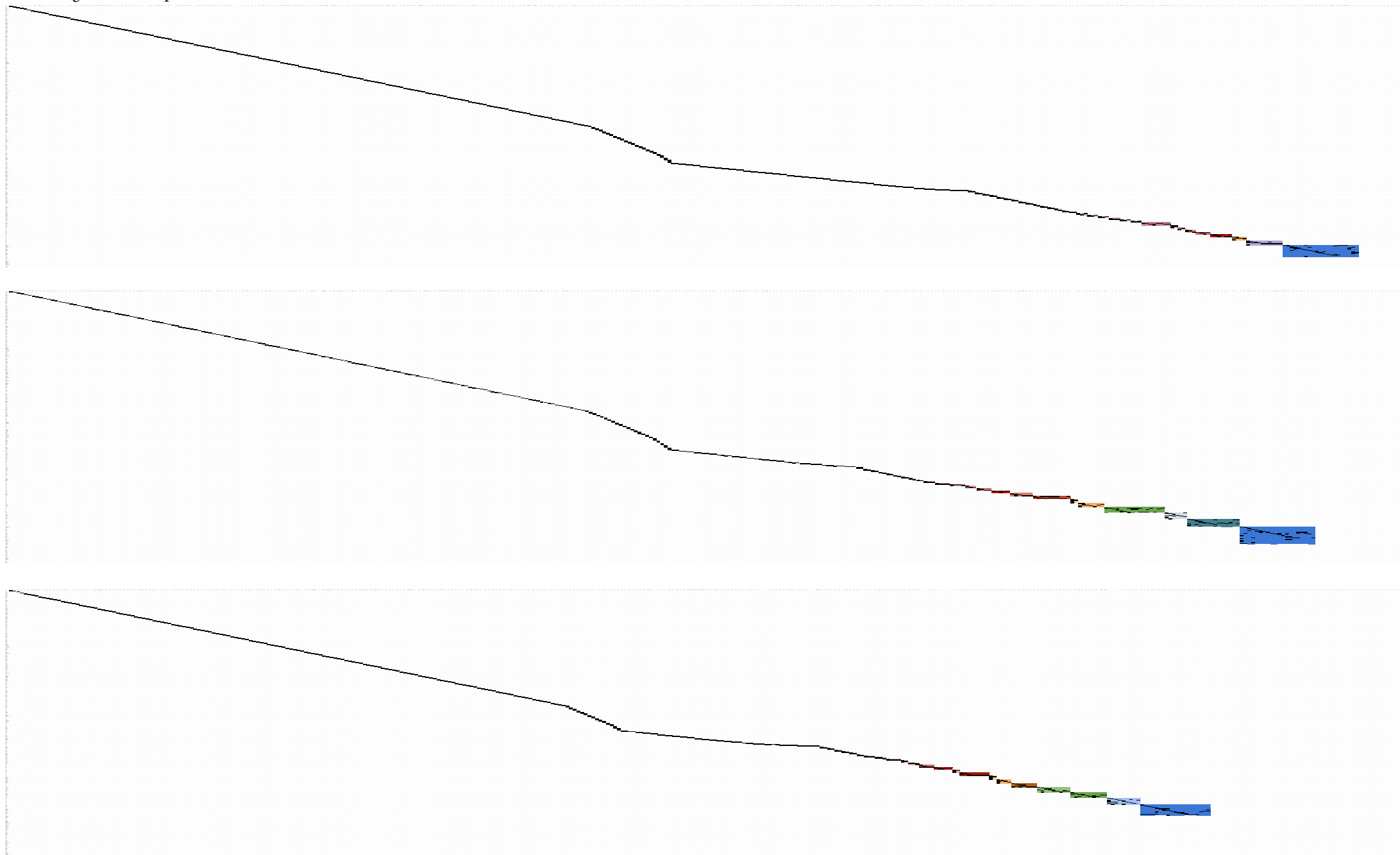


Fig. 11 – Dendrograms resulting from the cluster analysis.

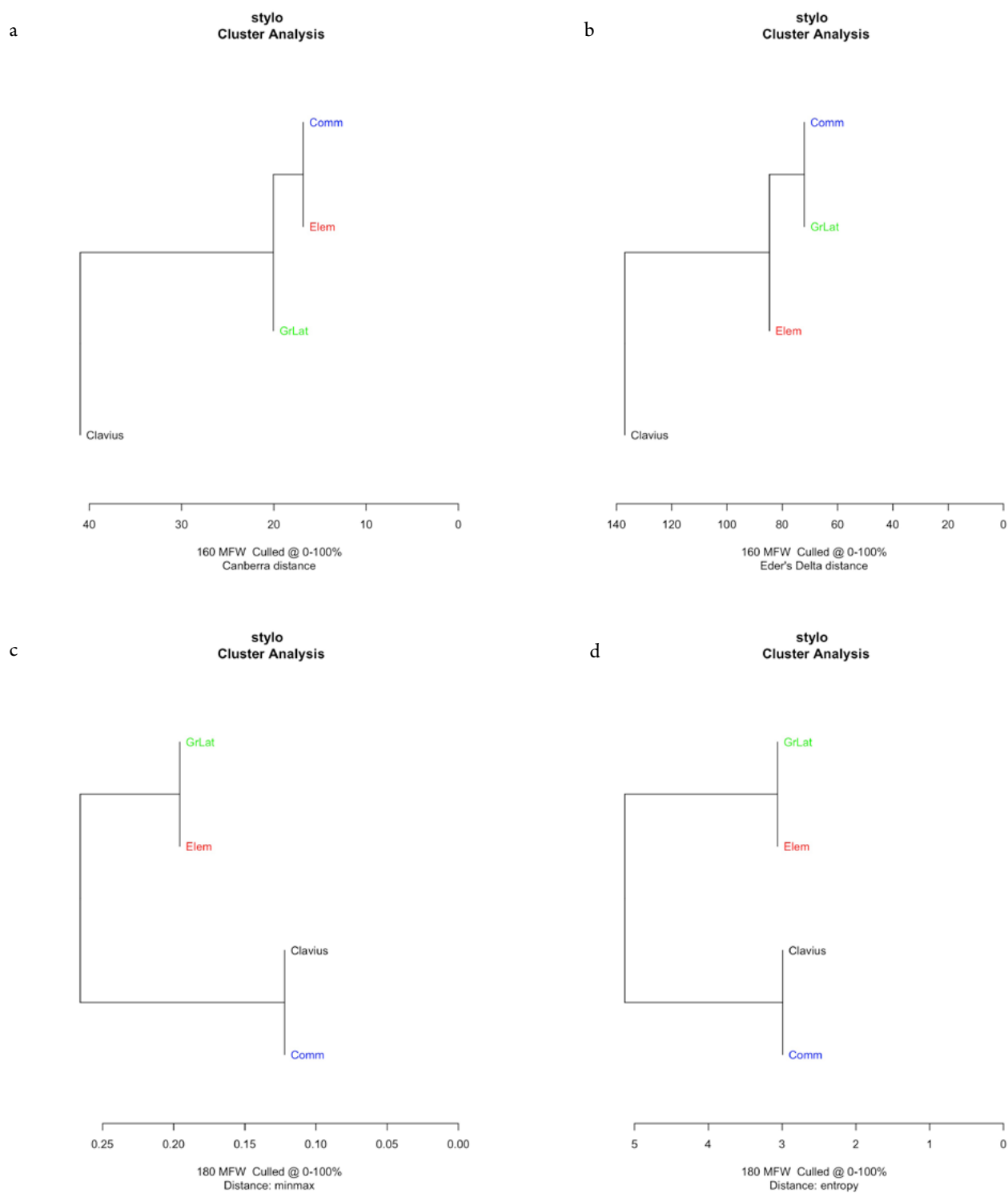


Fig. 12 – Consensus trees resulting from the cluster analysis.

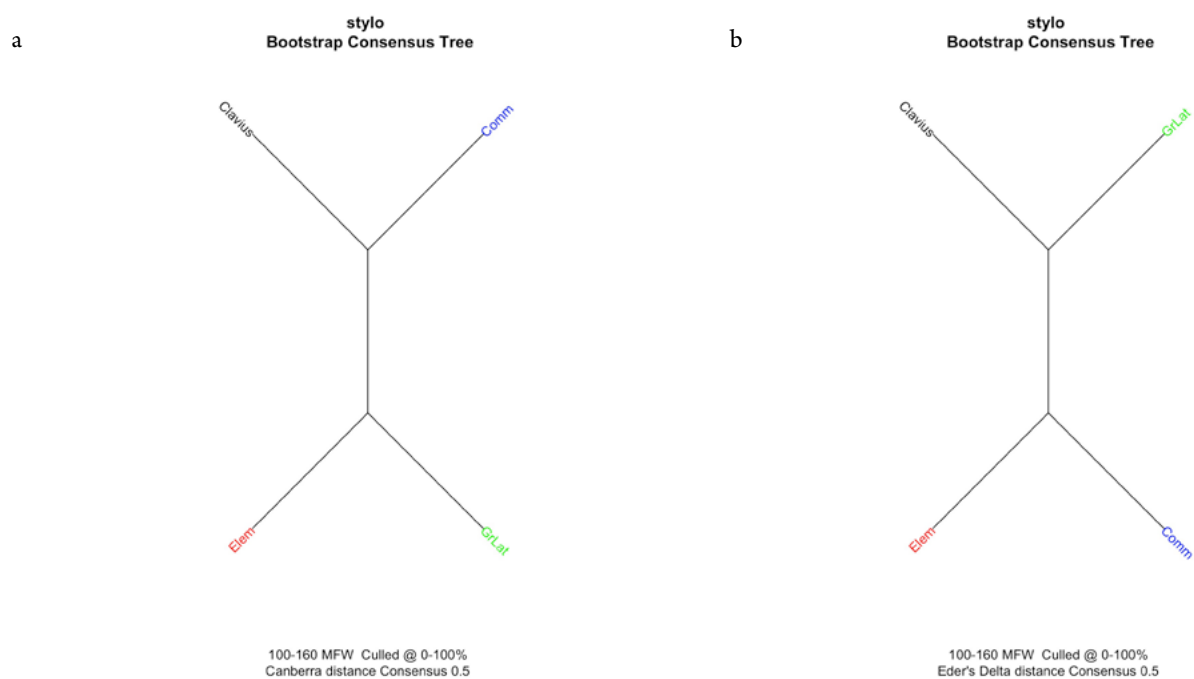


Fig. 13 – Weighted networks resulting from the cluster analysis.

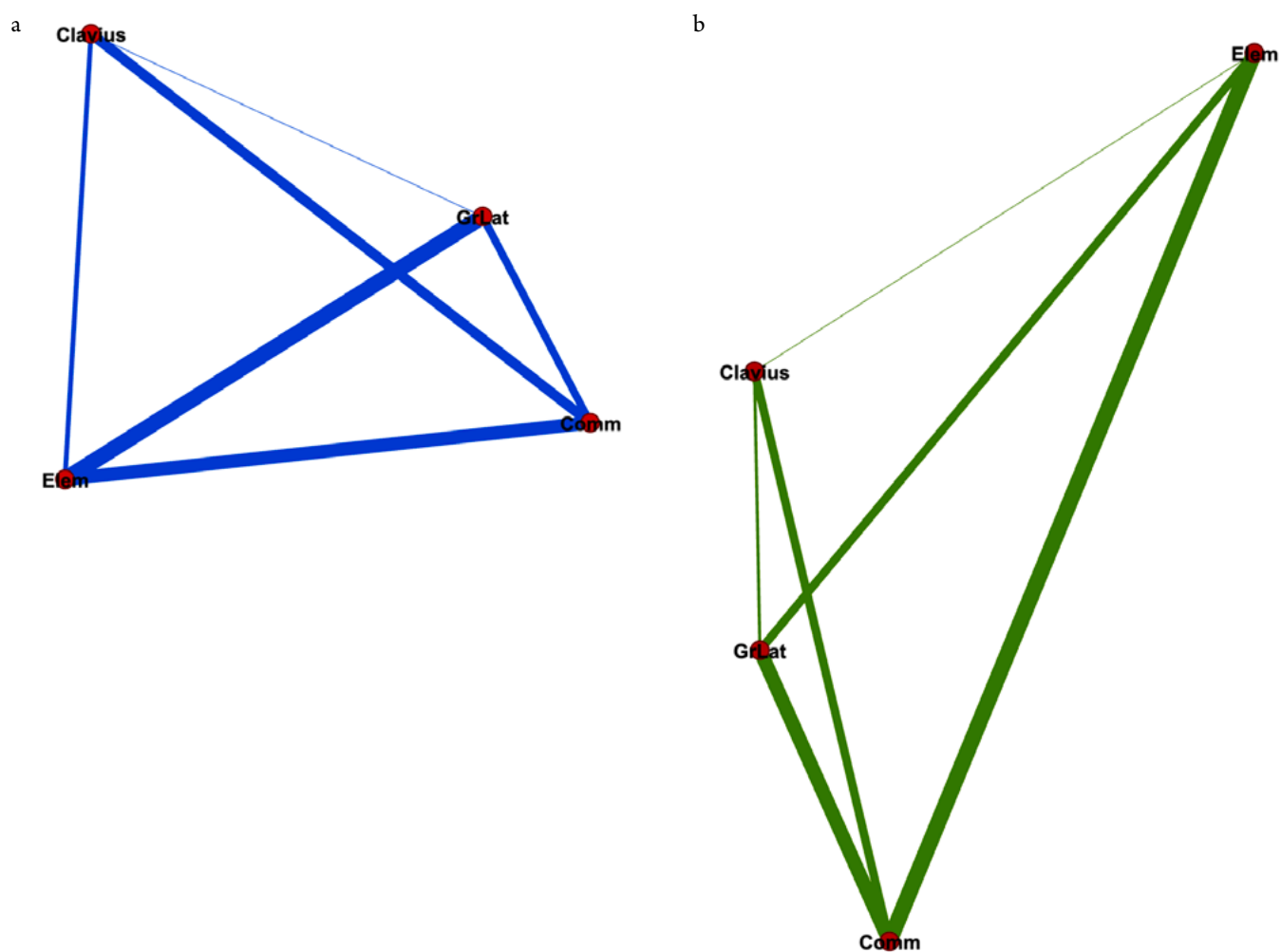


Fig. 14 – Principal component analysis, covariance matrix.

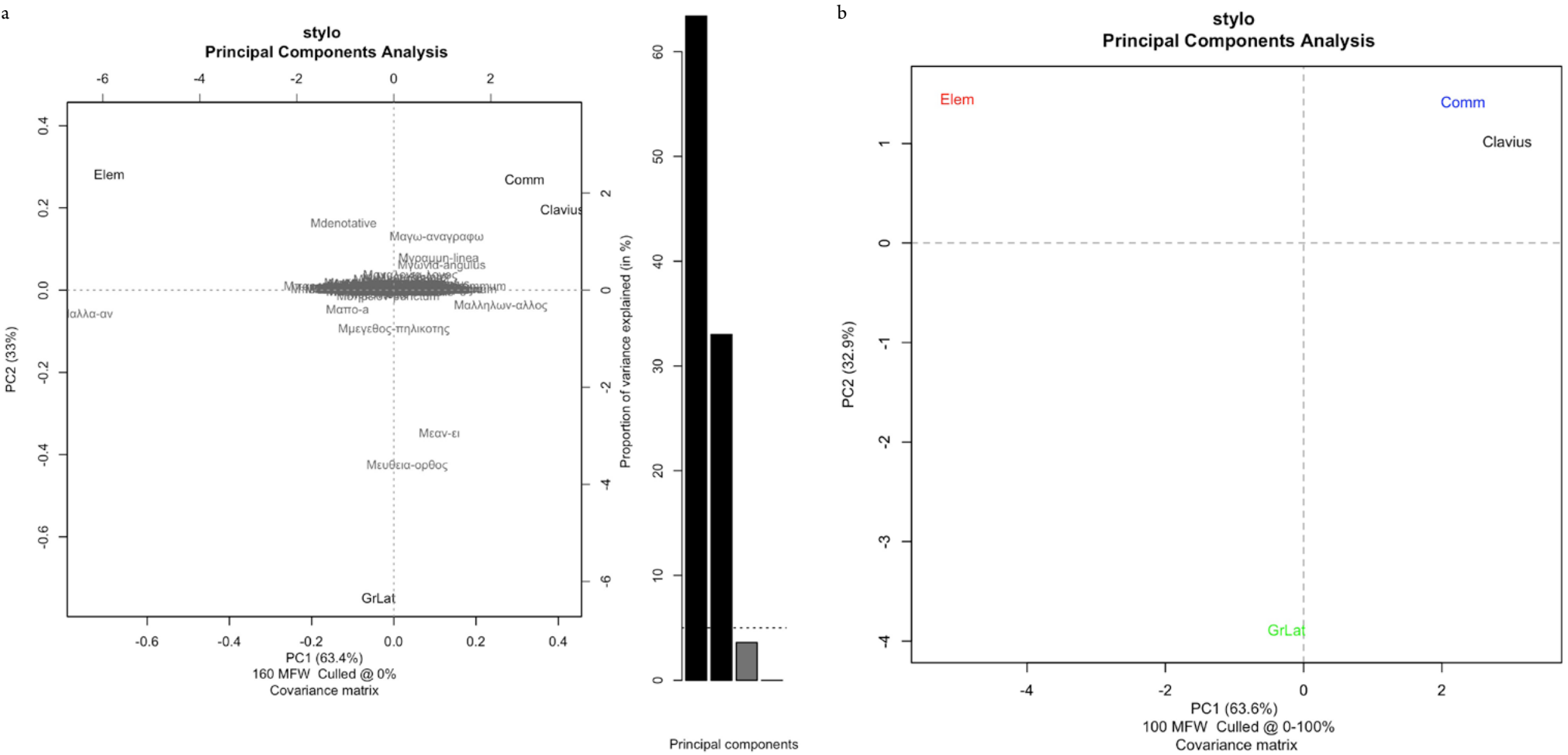


Fig. 15 – Principal component analysis, correlation matrix.

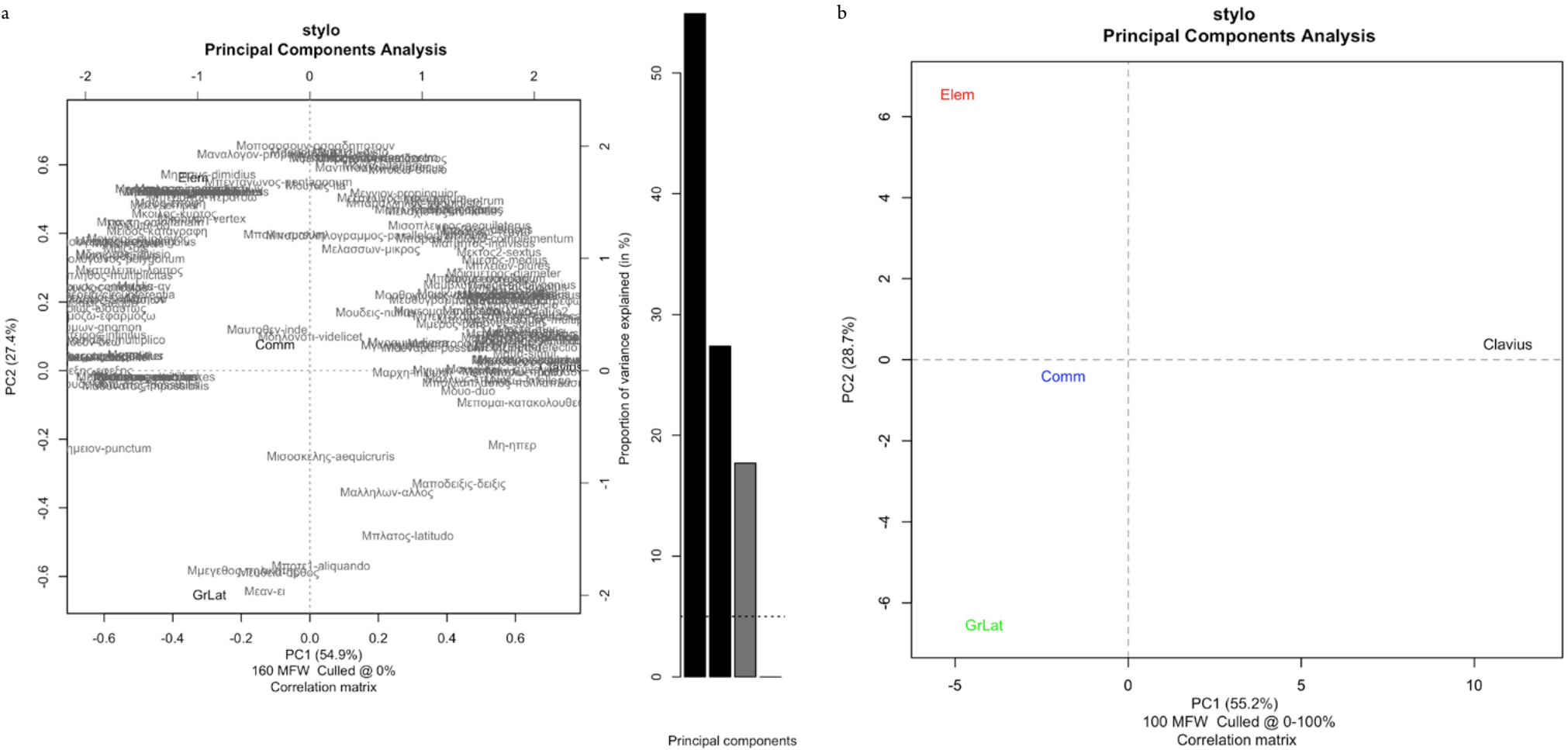


Fig. 16 (a-b) – The subordination diagrams of the four works, with and without the general conclusions.

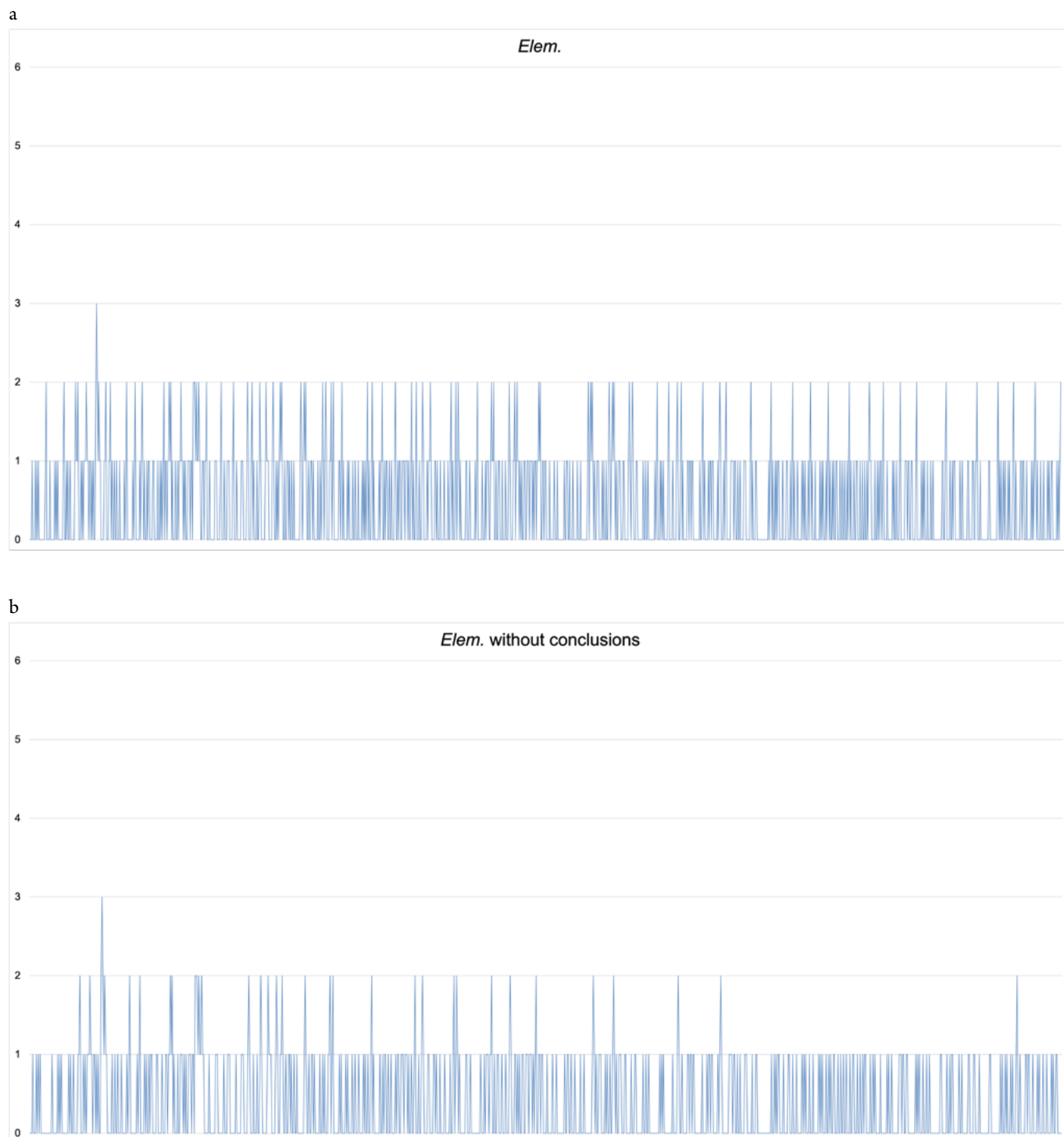


Fig. 16 (c-d) – The subordination diagrams of the four works, with and without the general conclusions.

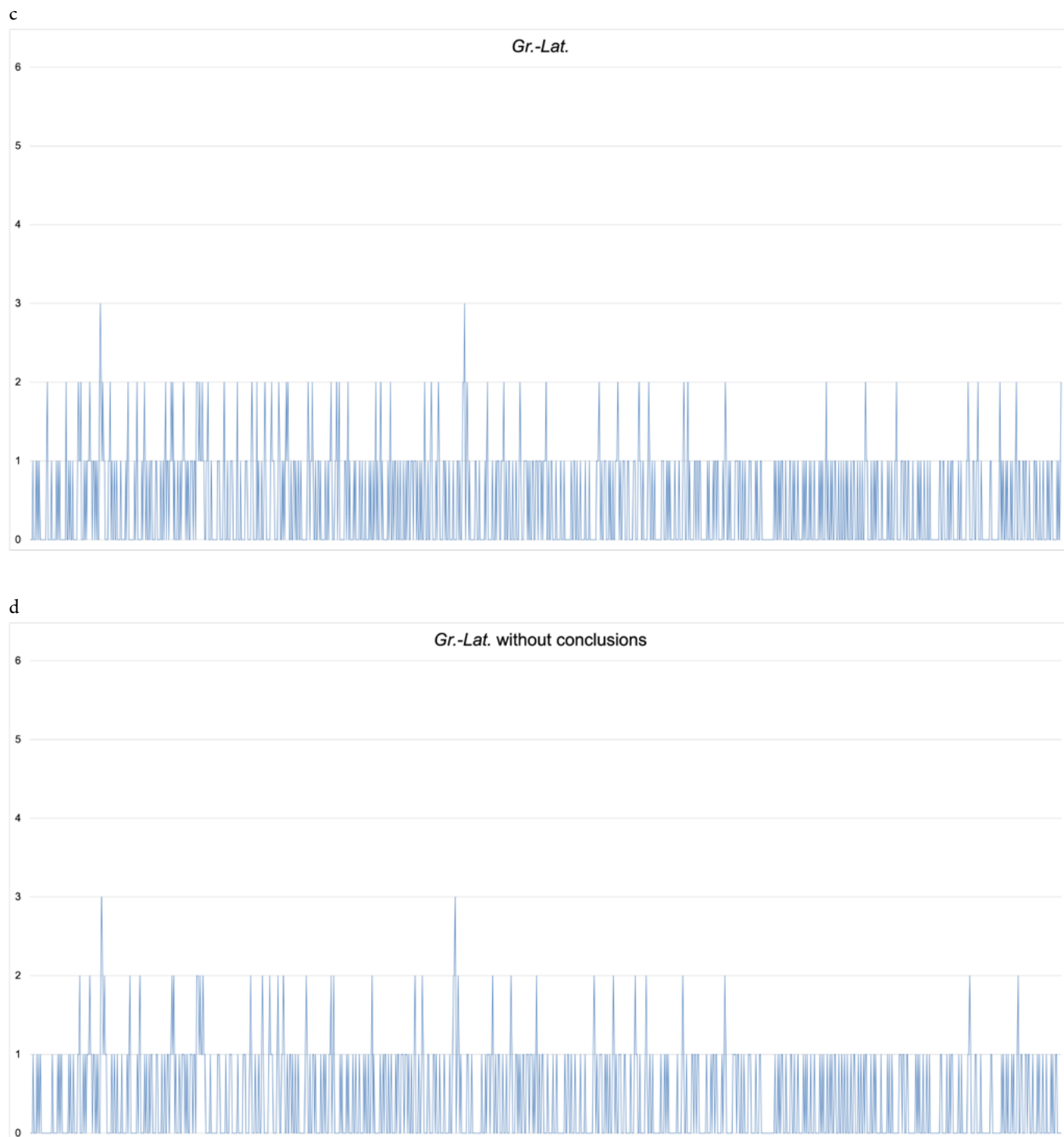


Fig. 16 (e-f) – The subordination diagrams of the four works, with and without the general conclusions.

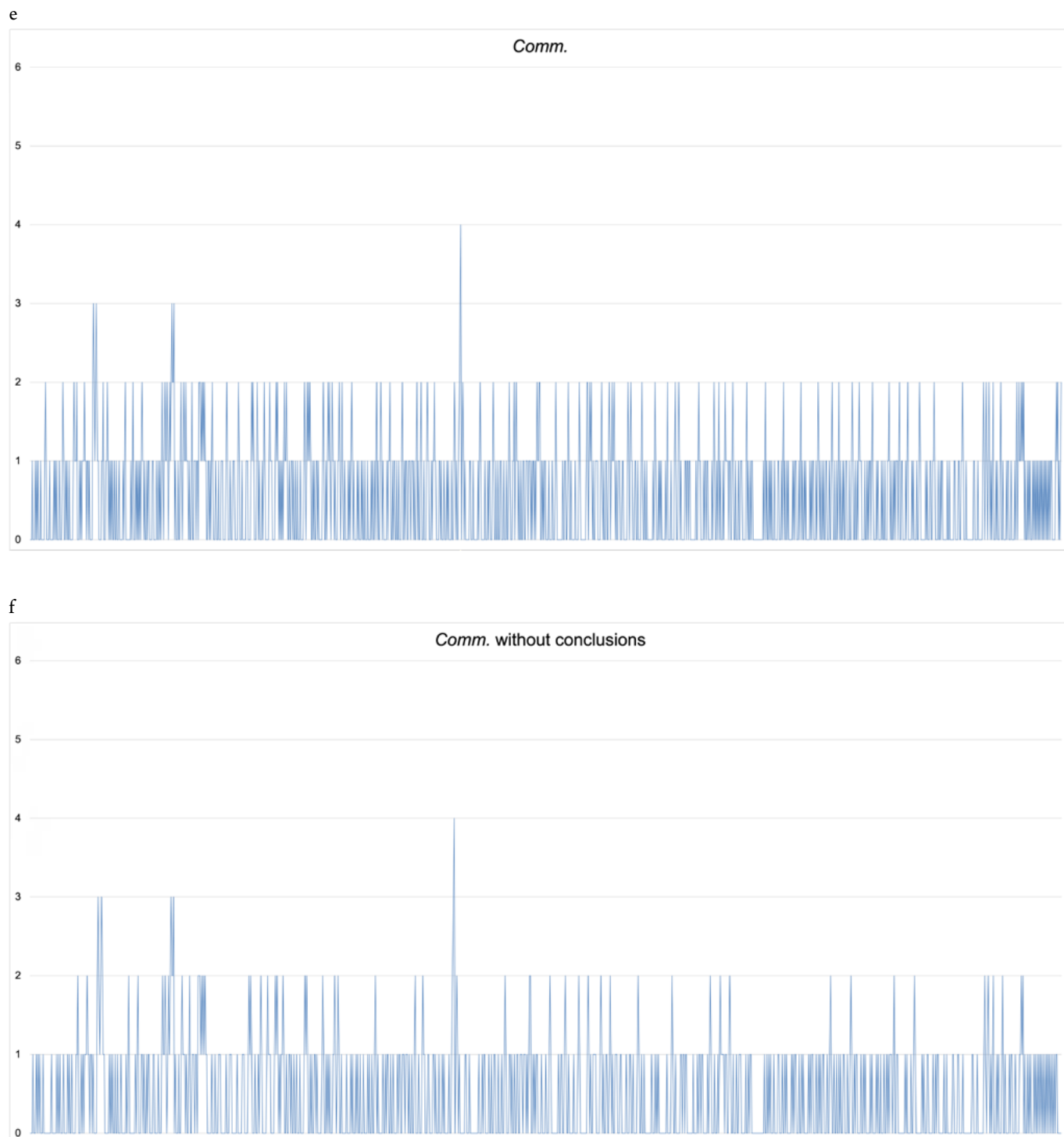
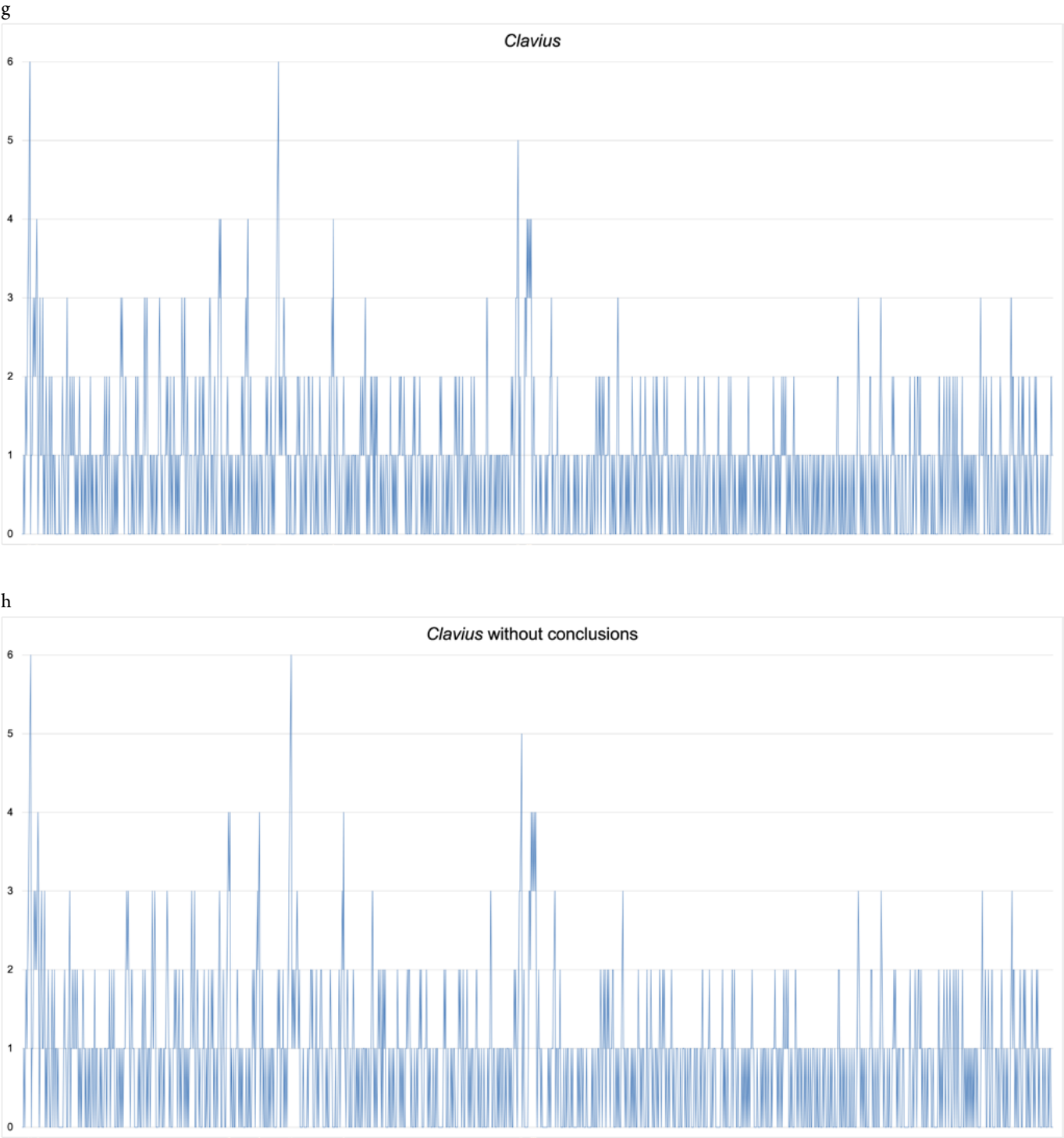


Fig. 16 (g-h) – The subordination diagrams of the four works, with and without the general conclusions.





Johann Scheubel, Wilhelm Xylander and the numerical treatment of Euclidean geometry in early modern Europe

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Abstract

The Latin and German commentaries on the first six books of the *Elements* by Johann Scheubel (1494–1570), professor at the University of Tübingen, and Wilhelm Xylander (1532–1576), professor at the University of Heidelberg, stand out within the sixteenth-century Euclidean tradition by their extensive use of arithmetic and cosmic algebra (in the tradition of Christoff Rudolff and Michael Stifel) in their respective exposition of Euclid's geometrical propositions, which was primarily connected with their Protestant background and pedagogical context. By analysing Scheubel and Xylander's commentaries and their numerical approach to Euclid's geometrical propositions, this article aims to offer an insight into the evolution of the arithmetization of Euclidean geometry in early modern Europe.

Keywords

Euclid's *Elements*, arithmetization of geometry, Johann Scheubel, Wilhelm Xylander, Protestant universities

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Introduction

Euclid's *Elements*, an ancient Greek treatise of geometry and arithmetic, was often regarded in the early modern period as the standard source for the teaching of mathematics (and geometry in particular) at an elementary level, for which reason it was increasingly edited, translated and commented from its first printed edition in 1482 (i.e. Erhardt Ratdolt's edition of Campanus of Novara's thirteenth-century commentary).¹

Since the first Latin translations of the *Elements* in the twelfth century, it was acknowledged that, in this text, Euclid's *modus operandi* was properly demonstrative, deriving the truth of propositions from universally admitted first principles (axioms), and dealing with numbers and magnitudes in a purely abstract manner, without any reference to concrete objects, instrumental procedures or applications.² In the *Elements*, magnitudes were also kept separate from numbers, as the quantitative properties or relations of magnitudes were not expressed in numerical terms, by attributing specific measures to lines and figures according to a given unit of length, area and volume.³ These remained abstract and quantitatively unspecified, their equality or proportion being determined through rational deduction only. A similar approach was adopted for numbers in the arithmetical books of the *Elements*, since these were then represented by non-quantified lines of different lengths rather than by specific numerical examples.⁴

Moreover, although the principles of Eudoxian theory of ratios and proportions taught in the *Elements* were applicable to both numbers and magnitudes, Euclid had chosen to propose a separate treatment of this doctrine for magnitudes (in Book V) and numbers (in Books VII–IX).⁵ Although the reason for this separation was not made explicit by Euclid,⁶ it is most likely linked with the opposition between discrete and continuous quantity, that is, between a type of quantity composed of indivisible units,

¹ Campanus de Novara, *Preclarissimum opus elementorum Euclidis megarensis una cum commentis Campani perspicacissimi in artem geometriam* (Venezia: Erhard Ratdolt, 1482).

² This acknowledgement appears for instance in Dominic Gundisalvi's *De divisione philosophiae* in which the nature of theoretical geometry is mainly illustrated through examples drawn from the *Elements*. Gundisalvi, *De divisione philosophiae*, in Ludwig Baur, *Dominicus Gundisalvi. De divisione philosophiae* (Münster: Aschendorff, 1903), 106–107.

³ This was generally displayed in the sixteenth century in editions presented as faithful to its Greek tradition, such as those of Bartolomeo Zamberti, *Euclidis megarensis [...] elementorum libros XIII cum expositione Theonis insignis mathematici* (Venezia: Johannes Tacuinus, 1505) or Conrad Dasypodius, *Euclidis quindecim elementorum geometriae primum: ex Theonis commentariis Graecè, & Latine* (Strasbourg: Christianus Mylius, 1564).

⁴ Bernard Vitrac, *Euclide. Les Éléments. Livres V–VI: Proportions et similitude. Livres VII–IX: Arithmétique* (Paris: Presses universitaires de France, 1994), 280–282.

⁵ Vitrac, *Les Éléments. Livres V–VI*, 15–19.

⁶ Thomas L. Heath, *The Thirteen books of Euclid's Elements* (New York: Dover, 1956), vol. 1, 112–113.

as numbers were thought to be in ancient Greek arithmetic (and thus also in the *Elements*⁷), and a type of quantity that is continuous and therefore infinitely divisible, as geometrical lines and figures (plane and solid). According to this opposition, which gave rise to the distinction between arithmetic and geometry in ancient classifications of mathematics,⁸ if numbers are always commensurable (due to the fact that they may all be resolved into the same indivisible unit) magnitudes may be either commensurable or incommensurable, as is the diagonal of the square with respect to its side, which may not be expressed as a whole number or as a ratio of numbers, but only as a radical number (i.e. $\sqrt{2}$).

In spite of the essentially abstract, rational and non-numerical treatment of magnitudes in Euclid's *Elements*, a number of editions of this treatise published from the mid-sixteenth century presented geometrical propositions in a hands-on and empirical rather than purely rational and demonstrative manner, prioritising utility, efficiency and user-friendliness over logical rigour and scientific universality.⁹ These editions aimed at offering a version of Euclid's treatise that would be more accessible to young students and readers (including artisans and merchants) who were less familiar with the style and content of ancient mathematics, but who might see in the *Elements* a source of information on procedures useful to their art as well as a knowledge that could give them access to a higher status.¹⁰ In this context, the numerical treatment of magnitudes played a key role, as it enabled a first-hand verification of theorems through specific examples, thus reducing the importance of logic-based demonstrations, and showed how certain Euclidean propositions may be applied in handling concrete quantities. This numerical approach to geometry, which was at the time characteristic of the treatises on practical geometry that focused on measuring procedures (instrumental and computational),¹¹

⁷ Heath, *The Thirteen Books*, vol. 2, 277, Df. VII.2: "A number is a multitude composed of units."

⁸ Vitrac, *Les Éléments. Livres V–VI*, 19–32.

⁹ Antoni Malet, "Euclid's swan song: Euclid's *Elements* in early modern Europe," in *Greek Science in the Long Run: Essays on the Greek Scientific Tradition (4th c. BCE–17th c. CE)*, ed. by Paula Olmos (Cambridge: Cambridge Scholars Publishing, 2012), 205–234; Marta Menghini, "From practical geometry to the laboratory method: The search for an alternative to Euclid in the history of teaching geometry," in *Selected Regular Lectures from the 12th International Congress on Mathematical Education*, ed. by Sung Je Cho (Cham: Springer, 2015); Angela Axworthy, "The hybridization of practical and theoretical geometry in the sixteenth-century Euclidean tradition," *Journal of Interdisciplinary History of Ideas* 11/22 (2022): 1–104.

¹⁰ Thomas Morel, "Bringing Euclid into the mines: Classical sources and vernacular knowledge in the development of subterranean geometry," in *Translating Early Modern Science*, ed. by Sietske Fransen et al. (Leiden: Brill, 2017), 154–181.

¹¹ On the nature of pre- and early modern practical geometry, see for instance Hervé L'Huillier, "Practical Geometry in the Middle Ages and the Renaissance," in *Companion Encyclopedia of the History and Philosophy of the Mathematical Sciences*, ed. by Ivor Grattan-Guinness (London:

diverged from the abstract and non-numerical Euclidean treatment of magnitudes.¹²

In this framework, the numerical expression of magnitudes also went beyond the use of whole numbers by including fractions¹³ and irrationals. These were then generally called ‘surd numbers’ due to the fact that they cannot be numerically expressed with precision or in a direct manner, some being at least expressible as square or cubic roots, as the ratio of the diagonal to the side of the square, and others only as approximations, as the ratio of the diameter to the circumference of the circle. As would be expressed by Henry Billingsley, who published the first English commentary on Euclid’s *Elements* in 1570 and adopted within it many numerical examples: “There are certaine lines magnitudes or quantities which can not be named and expressed by number, and therefore commonly are called Surd lines or magnitudes. [...] the line BC shall be $\sqrt{32}$, which is a surd number, and can not be expressed by any determinate and certaine number, but only by this maner of circumlocution Roote square of 32. [...] the proportion betwene them is irrational, confused, unknowen, uncertaine, and surd.”¹⁴

The numerical expression of Book X, which deals with incommensurable magnitudes, thus did not only overthrow Euclid’s separation between numbers and magnitudes in the *Elements*, but also raised the issue of the ontological status of irrationals (whether or not these may be considered as numbers in their own right) and of their admissibility in the context of a commentary on a Greek mathematical treatise as Euclid’s *Elements*, which only admitted whole numbers and which only confined numbers to the arithmetical books.¹⁵ In extreme cases, as that of the numerical treatment of Book X in the practice-oriented *Arithmetique* of the engineer and mathematician Simon Stevin (1585), this approach accompanied a full reassessment of the distinction between numbers and magnitudes founded on the rejection of the notion of number as discrete quantity and on the explicit admission of irrationals (at least as radical numbers) as proper numbers.¹⁶

Routledge, 1994), 185–191 and Eberhard Knobloch, “Géométrie pratique, géométrie savante,” *Albertiana* 8 (2005): 27–56.

¹² Henk Bos, *Redefining Geometrical Exactness Descartes’ Transformation of the Early Modern Concept of Construction* (New York: Springer, 2001), 126, 128 and 135–136; Antoni Malet, “Renaissance notions of number and magnitude,” *Historia mathematica* 33 (2006): 63–81.

¹³ Katherine Neal, *From Discrete to Continuous. The Broadening of Number Concepts in Early Modern England* (Dordrecht: Springer Science & Business Media, 2002), 16. Neal considers (ibid., 18) that, under certain conditions, Euclid may be considered as having admitted fractions. However, no concept of rational number appears in this framework, including to characterise ratios, unlike how it would be in the middle ages and early modern era through the concept of denomination, as will be presented later.

¹⁴ Henry Billingsley, *The Elements of Geometrie of the most auncient Philosopher Euclide of Megara* (London: John Daye, 1570), 127r–v. See Malet, “Renaissance notions.”

¹⁵ Neal, *From Discrete to Continuous*, 14–22; Bos, *Redefining Geometrical Exactness*, 130–131.

¹⁶ Simon Stevin, *L’Arithmetique* [...] *Avec l’Explication du Dixiesme livre d’Euclide* (Leiden: Chris-

This approach thus departed from the properly Euclidean approach to magnitude, which was associated with the rigour, universality and geometrical certainty that was often attributed to the *Elements* as a model of scientific knowledge by many ancient and modern philosophers and mathematicians.¹⁷ It was, on the other hand, fully in line with the practice transmitted in treatises of practical geometry, practical arithmetic and algebra, notably in the handbooks of Italian *Maestri d'abbaco* and German cossic *Rechenmeister*.¹⁸ The fact of attributing specific dimensions to magnitudes in the diagrams that accompany Euclid's proofs and using these dimensions to perform computational procedures, or even replacing magnitudes by numbers when dealing with ratios in Book V, thus connected the *Elements* with the computational practice of surveyors, merchants and bankers. This would also be the case in Stevin's *Arithmetique*, where content relating to commercial and financial arithmetic is proposed and where the author explicitly addressed a public of practitioners.¹⁹

In this regard, putting aside the special (and later) case of Stevin's treatment of Book X, the editions of Euclid's first six books by Johann Scheubel²⁰ and Wilhelm Xylander,²¹

tophe Plantin, 1585), 4v–5r and 32–33. On Stevin's approach to numbers, see Bos, *Redefining Geometrical Exactness*, 138–141; Neal, *From Discrete to Continuous*, 34–36; Malet, “Renaissance notions;” Jürgen Naets, “How to Define a Number? A General Epistemological Account of Simon Stevin's Art of Defining,” *Topoi* 29 (2010): 77–86.

¹⁷ Angela Axworthy, *Le Mathématicien renaissant et son savoir. Le Statut des mathématiques selon Oronce Fine* (Paris: Vrin, 2005), 83–123.

¹⁸ The influence of practical mathematics on the transformation of the classical approach to numbers and magnitudes in the early modern era is shown in Neal, *From Discrete to Continuous*. On these Italian and German handbooks of practical arithmetic and algebra, see Albrecht Heeffer, “The Genesis of the Algebra Textbook: From Pacioli to Euler,” *Almagest* 3/1 (2012): 26–61; Jens Høyrup, *The World of the Abbaco. Abacus Mathematics Analyzed and Situated Historically between Fibonacci and Stifel* (Cham: Birkhäuser, 2024).

¹⁹ See, for instance, the title page and the address of the preface of *La Disme* (also published in Dutch in 1585 as *De Thiende*), in Stevin, *L'Arithmetique*, 132–133. On the practical orientation of Stevin's *La Disme*, and of his mathematical work more generally, see Neal, *From Discrete to Continuous*, 2.

²⁰ Johann Scheubel, *Euclidis Megarensis, Philosophi & Mathematici excellentissimi, Sex libri priores, de Geometricis principiis, Graeci & Latini, unà cum demonstrationibus propositionum, absque literarum notis, veris ac proprijs, & alijs quibusdam, usum earum concernentibus, non citra maximum huius artis studiosorum emolumentum adiectis. Algebrae porro regulae, propter numerorum exempla, passim propositionibus adiecta, his libris praemissae sunt, eademque demonstratae* (Basel: Hervagius, 1550).

²¹ Wilhelm Xylander, *Die sechs erste Bücher Euclidis, Vom anfang oder grund der Geometri, in welchen der rechte grund, nitt allain der Geometri (versteh alles kunstlichen, gwisen, und vortailigen gebrauchs des Zirckels, Linials oder Richtscheittes und andrer werckzeüge, so zu allerlai abmessen dienstlich) sonder auch der fürnemsten stuck und vortail der Rechenkunst, furgeschriben und dargethon ist, auß Griechischer sprach in die Teütsch gebracht, aigentlich erklärt, auch mit verstentlichen Exempeln, gründlichen Figurñ geziert, dermassen vormal in Teütscher sprach nie gesehen worden* (Basel: Oporinus, 1562).

published in Basel in 1550 and 1562 respectively, stand out among the sixteenth-century editions²² of the *Elements* that adopted a numerical treatment of geometrical principles and propositions by their extensive use of practical arithmetic and cosmic algebra (in the tradition of Christoph Rudolff and Michael Stifel). As I will attempt to show, this was connected with their social, cultural and institutional background.

Although quite exceptional at the time, this approach became more frequent in the seventeenth-century Euclidean tradition across Europe, as the pedagogical agendas of the growing number of professors of mathematics evolved and as new mathematical developments took place.²³

After presenting the lives and mathematical work of Scheubel and Xylander, this paper will show how these two authors, in their respective commentaries on the *Elements*, adapted and transmitted Euclid's geometrical propositions in a numerical manner through the integration of arithmetical and algebraic content. After that, a few words will be said on the practical character of this approach, before concluding on the role played by the context in which Scheubel and Xylander evolved, namely that of German Protestant universities, in fostering their numerical approach to Euclid.

The lives and works of Scheubel and Xylander

Johann Scheubel (or Scheybl) was born in 1494 in Kirchheim unter Teck and died in 1570 in Tübingen.²⁴ After early studies in the Latin school of Kirchheim, he enrolled in 1513 at the Faculty of Arts of Vienna. From 1532, he studied at the University of Leipzig and, in 1535, he enrolled at the University of Tübingen. After obtaining the title of *Magister* in 1540, he started giving lectures in mathematics (arithmetic and Euclid's geometry) in Tübingen in 1544 and became 'professor ordinarius' by 1550.²⁵ Evidence shows that he

²² For the sake of convenience, the term 'edition' includes here all the works designed to transmit the content of Euclid's *Elements*, thus including translations and commentaries.

²³ Malet, "Renaissance notions"; Id., "Euclid's swan song."

²⁴ On Scheubel's life and works, see Hermann Staigmüller, "Johannes Scheubel, ein deutscher Algebraiker des XVI. Jahrhunderts," *Abhandlungen zur Geschichte der Mathematik* 9 (1899): 429–469; Mary S. Day, *Scheubel as an Algebraist, Being a Study of Algebra in the Middle of the Sixteenth Century, Together with a Translation of and a Commentary upon an Unpublished Manuscript of Scheubel's Now in the Library of Columbia University* (New York: AMS Press, 1926); Ulrich Reich, *Schriftenreihe des Stadtarchivs Kirchheim unter Teck. 500 Jahre Johann Scheubel* (Kirchheim unter Teck: Gottlieb & Osswald, 1994); Id., "Johann Scheubel (1494–1570): Geometer, Algebraiker und Kartograph," in *Der „mathematicus“: zur Entwicklung und Bedeutung einer neuen Berufsgruppe in der Zeit Gerhard Mercators*, ed. by Irmgard Hantsche (Bochum: Brockmeyer, 1996), 151–182; Id., "Scheubel, Johann," in *Neue Deutsche Biographie* 22 (2005): 709–710.

²⁵ Rudolph von Roth, *Urkunden zur Geschichte der Universität Tübingen aus den Jahren 1476 bis 1550* (Tübingen: H. Laupp, 1877), 236–237 and 658. The title of *professor ordinarius* appears

was still teaching mathematics in Tübingen in 1562²⁶ and that he bequeathed his instruments and library to the University at his death in 1570.²⁷

Beside his 1550 Latin edition of the *Elements* (Books I–VI) and a German edition of the arithmetical books (Books VII–IX) published in Augsburg in 1555,²⁸ Scheubel wrote commentaries on the *De Numeris Datis* of Jordanus Nemorarius²⁹ and on Robert of Chester's Latin translation of al-Khwārizmī's *Algebra*,³⁰ as well as two treatises of practical arithmetic published in Leipzig in 1545 and in Basel in 1549.³¹ He also published in Paris in 1551 and 1552 a treatise of algebra, *Algebrae compendiosa facilisque descriptio*,³² which was first published in 1550 as an introduction to his Latin edition of Euclid.³³ The latter represented one of the earliest treatises of algebra published in France. He also published in 1553 an edition of Jacques Lefèvre d'Étaples's introduction to Boethius' *De institutione arithmetica*.³⁴ As shown by the inventory of his library and manuscripts, Scheubel also worked on a Latin version of his exposition of Euclid's arithmetical books, as well as on Book X (in Latin and German) and Books XI–XV of the *Elements* (in Latin).³⁵ He is also

notably in the title page of his Latin edition of Euclid: Scheubel, *Euclidis elementa*, title page: “in inclitya Academia Tubingensi Euclidis professore ordinario.”

²⁶ Staigmüller (“Johannes Scheubel,” 437) notes that Scheubel requested an increase of salary from the Senate in 1562 (as in 1551).

²⁷ Barnabas Hughes, “The private library of Johann Scheubel, sixteenth-century mathematician,” *Viator* 3 (1972): 417–432.

²⁸ Johann Scheubel, *Euclidis elementa* and Id., *Das sibend, acht und neünt Büch, des hochberümbten Mathematici Euclidis Megarensis, in welchen der operationen und regulen aller gemainer rechnung, ursach grund und fundament, angezeigt wirt, zû gefallen allen den, so die kunst der Rechnung liebhaben, durch Magistrum Johann Scheybl, der löblichen universitet zû Tübingen des Euclidis und Arithmetice Ordinarien, auß dem latein ins teütsch gebracht, unnd mit gemainen exempeln also illustriert unnd an tag geben, das sy ein yeder gemainer Rechner leichtlich verstehn, unnd jene nutz machen kan* (Augsburg: Ottmar, 1555).

²⁹ On this work, which remained unpublished, see Barnabas Hughes, “Johann Scheubel's revision of Jordanus de Nemore's *De numeris datis*: An analysis of an unpublished manuscript,” *Isis* 63/2 (1972): 221–234.

³⁰ Louis Karpinski, *Robert of Chester's Latin translation of the Algebra of al-Khowarizmi* (New York: Macmillan, 1915).

³¹ Johann Scheubel, *De numeris et diversis rationibus seu regulis computationum opusculum* (Leipzig: Michael Blum, 1545) and Id., *Compendium arithmeticae artis* (Basel: Johannes Oporinus, 1549).

³² Scheubel, *Algebrae compendiosa facilisque descriptio* (Paris: Guillaume Cavellat, 1551).

³³ Scheubel, *Euclidis elementa*, 1–76.

³⁴ Johann Scheubel, *Iacobi Fabri Stapulensis in Arithmetica Boëthi epitome, unà cum difficiliorum locorum explicationibus & figuris (quibus antea carebat) nunc per Ioannem Scheubelium adornatis & adiectis. Accessit Christierni Morssiani Arithmetica practica* (Basel: Henri Estienne, 1553).

³⁵ Hughes, “The Private Library,” 425: “1. Commentarius in decem librorum Euclidis teutsch Scheubelii imperfectus et latine bis. 2. Idem Scheubelii in libros Arithmeticos 7, 8, 9 Euclidis latine bis.” Scheubel's commentary on Books XI–XV, written in 1561, is situated in Ms. Biblio-

credited for having produced the oldest map of the Duchy of Nuremberg in 1559.³⁶

Wilhelm Xylander (Holtzman or Holtzmann) was born in 1532 in Augsburg and died in Heidelberg in 1576.³⁷ He came from a modest family, but received the support of the humanist Sixtus Birk (or Birck) for his early education.³⁸ In 1549, he enrolled at the University of Tübingen, where he received the bachelor degree in 1550. Although his biographer Fritz Schöll claims that he was self-taught in mathematics, he may have attended Scheubel's mathematics lectures at Tübingen. In 1555, Xylander started working on his German translation of Euclid's *Elements*, which was published in 1562.³⁹ In 1557–1558, he studied at the university of Basel, where he received a Master's degree. He then went to the University of Heidelberg where he taught Greek (1558–1563), mathematics (1562–1563) and logic (1563–1576). In 1561, he was appointed librarian of Elector Friedrich III's library. He was also dean of the Arts Faculty of Heidelberg in 1563–1564 (and in 1571), as well as rector of the university in 1564–1565.

Beside his 1562 German edition of Euclid and a Latin translation of Diophantus' *Arithmetica* published in 1575,⁴⁰ Xylander translated and edited various Greek and Latin works by Aristotle, Strabo, Diodorus of Sicily, Plutarch and Pausanias, among others, as well as Michael Psellos' *De quattuor mathematicis scientiis*.⁴¹ He also authored a mathematical compendium entitled *Opuscula Mathematica* published in 1577 in Heidelberg,⁴² as well as a versified eulogy of the astronomical clock of the cathedral of Strasburg, dedicated to its

teca Apostolica Vaticana, Pal. lat. 1350, 1–320, under the title *Tertius tomus geometriae Euclidis, de quantitate continua in solidis*.

³⁶ Ulrich Reich, *Johann Scheubel und die älteste Landkarte von Württemberg 1559* (Karlsruhe: Hochschule für Technik, 2000).

³⁷ On Xylander's life and works, see Fritz Schöll, "Xylander, Wilhelm," *Allgemeine Deutsche Biographie* 44 (1898): 582–593; Dorothee Gall, "Xylander, Guilielmus," *Brill's New Pauly Supplements I Online*, vol. 6; Dagmar Drüll, *Heidelberger Gelehrtenlexikon 1386–1651* (Berlin; Heidelberg: Springer, 2002), 562–563.

³⁸ On Sixt, see Alfred Hartmann, "Birk, Sixt (Xystus Betul[e]ius)," *Neue Deutsche Biographie* 2 (1955): 256.

³⁹ As indicated in the preface, which is addressed to the city council of Augsburg, Xylander had produced seven years earlier a translation of the first four books of the *Elements*, which he presented to these Augsburg notables. Xylander, *Euclidis elementa*, a2r: "Ich hab [...] vor sibem Jaren, die Vier erste biecher Euclidis vom Grund der Geometrij, auß Griechischer sprach in die Teutsche gebracht."

⁴⁰ Xylander, *Diophanti Alexandrini Rerum arithmeticarum...* (Basel: Eusebius Episcopus, 1575).

⁴¹ Xylander, *Pselli, doctissimi viri, perspicuus Liber de quattuor mathematicis scientiis* (Basel: Johannes Oporinus, 1556). This work was dedicated to Ulrich Fugger, a humanist and member of the Fugger family of Augsburg.

⁴² Xylander, *Opuscula Mathematica. Aphorismi Cosmographici; De minutiis; De Surdorum Numerorum natura & tractatione; De usu Globi & Planisphaerii tractatus* (Heidelberg: Jacob Müller, 1577).

designer, Conrad Dasypodius, and published in 1575 in Tübingen.⁴³

The numerical treatment of Euclid's geometrical books by Scheubel and Xylander

In the following pages, each case will be analysed separately, starting with that of Scheubel. For each author, I will start with a general presentation of their commentary on Euclid's first six books of the *Elements*, describing its main goal, style, addressed readers and pedagogical apparatus. I will focus in particular on the numerical treatment of magnitudes proposed in each work, its distribution within Euclid's treatise, its place in the proof and/or commentary of specific propositions, the type of numbers and of arithmetical and/or algebraic teaching provided in this framework, and how the adoption of this numerical approach to Euclid's geometry is justified by their authors. I will also indicate how this treatment related to the tradition of practical mathematics, as represented both by practical geometry treatises and by the works of German *Rechenmeister*. For Scheubel in particular, a brief consideration of his 1555 German commentary on Euclid's arithmetical books (VII-IX), and of other mathematical works, will be provided as a means to manifest the way Euclid's *Elements* and the tradition of common *Rechenbücher* were interrelated in Scheubel's mathematical teaching.

Scheubel's numerical treatment of Euclid's *Elements*

Scheubel's 1550 Latin edition of Euclid's first six books of the *Elements*,⁴⁴ which included the Greek text of the principles and enunciations, as well as a pedagogical commentary, most likely reflected the content of his teaching at the University of Tübingen at the time he wrote this work.⁴⁵ In his rendering of Euclid's proofs, Scheubel chose to totally abandon the use of letters to label geometrical diagrams, which is fundamental to the style of Euclidean proofs, as it allows to connect the text with the diagram at each step of the constructive and deductive process.⁴⁶ As Scheubel claimed in the preface, this approach would make Euclid's propositions more accessible to early students, who, in his experience, would get confused by the abundance of letters in geometrical proofs.⁴⁷

Scheubel's desire to make Euclid's *Elements* more accessible to beginners is also marked

⁴³ Nicodemus Frischlin and Wilhelm Xylander, *Carmen de Astronomico Horologio Argentoratensi* (Strasbourg: Nicolas Wyriot, 1575).

⁴⁴ On Scheubel's Euclid, see also Staigmüller, "Johannes Scheubel," 440–446.

⁴⁵ This is confirmed by many passages of the preface, as Scheubel, *Euclidis elementa*, a3v and a4r.

⁴⁶ The abandonment of diagrammatic letters was already announced in the title of Scheubel's edition, manifesting the importance this aspect played for him in this framework. Scheubel, *Euclidis elementa*, title page: "Euclidis [...] Sex libri priores, de Geometricis [...] unà cum demonstrationibus propositionum, absque literarum notis."

⁴⁷ Scheubel, *Euclidis elementa*, a3r–v.

by the inclusion of several pedagogical strategies that were typically found at the time in practical geometry treatises,⁴⁸ such as compass arcs in diagrams to facilitate the comprehension of the steps of the construction (Fig. 1),⁴⁹ dotted lines to help visualise impossibilities,⁵⁰ explicit references to the use of instruments in the text,⁵¹ deictic sentences to invite the reader to draw information from the diagram,⁵² but also the addition of briefer or more practical means of construction or demonstration to supplement Euclid's proof.

To consider compass arcs in Scheubel's diagrams, let us take the example of the diagram accompanying Prop. I.10 ("To bisect a given finite straight line"⁵³) depicted in Fig. 1. While in prior editions of the *Elements* the diagram illustrating this proposition simply consisted in a bisected equilateral triangle,⁵⁴ namely the equilateral triangle that one is required to construct on the given straight line and then to divide equally (as per Prop. I.9) by bisecting the angle subtended by the given line, Scheubel here depicted the intermediary steps of the construction, that is, the intersection of the two circles required to bisect the angle of the triangle subtended by the given line. Yet, instead of drawing the whole circles, he only represented their intersecting arcs. This visual strategy, which represents the construction as performed in practice, as briefer to accomplish than through the tracing of circles, and which was often adopted in practical geometry treatises when teaching geometrical constructions, was also considered easier for students to understand, as the diagram is then less encumbered by a multitude of lines. This motivation for the use of such diagrams when teaching Euclid's *Elements* was brought forth much later by Christoph Clavius, who was one of the most influential sixteenth-century professor of mathe-

⁴⁸ Preliminary research on such features within Scheubel's and Xylander's commentaries on Euclid, including on certain aspects of their numerical treatment of Euclid, may be found in Axworthy, "The Hybridization," 47–62 and 75–83.

⁴⁹ On the function of compass arcs in geometrical diagrams, notably in early modern commentaries on Euclid's *Elements*, see Eunsoo Lee, "Let the Diagram Speak: Compass Arcs and Visual Auxiliaries in Printed Diagrams of Euclid's *Elements*," *Endeavour* 42/2–3 (2018): 78–98.

⁵⁰ On the function of dotted lines, see Scheubel, *Euclidis elementa*, I.4, 86.

⁵¹ *Ibid.*, I.1, 83 or I.9, 90: "Officio igitur circini."

⁵² *Ibid.*, I.12, p. 93: "id quod ex sequenti cuiusque descriptione apparet."

⁵³ Heath, *The Thirteen Books*, vol. 1, 267.

⁵⁴ This is the case in Prop. I.10 of the following editions published before Scheubel's commentary on Euclid: Campanus, *Euclidis elementa*, a4v; Zamberti, *Euclidis elementa*, A4v; Luca Pacioli, *Euclidis Megarensis philosophi acutissimi mathematicorumque omnium sine controversia principis opera* (Venezia: Paganino Paganini, 1509), 7r; Oronce Fine, *In sex priores libros geometricorum elementorum Euclidis Megarensis demonstrationes* (Paris, Simon de Colines, 1536), 17; Niccolò Tartaglia, *Euclide megarense philosopho, solo introduttore delle scientie mathematiche, diligentemente reassetato, et alla integrità ridotto* (Venezia: Venturino Ruffinelli, 1543), 18v; Joachim Camerarius, *Euclidis elementorum geometricorum libri sex conversi in latinum sermonem* (Leipzig: Valentinus Papa, 1549), 18.

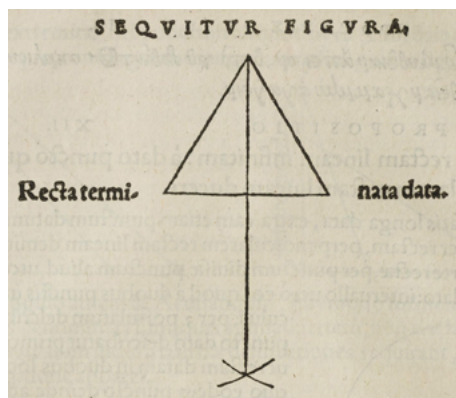


Fig. 1 – Scheubel, *Euclidis*, Prop. I.10, 91. Image source: Max Planck Institute for the History of Science, Berlin (Public Domain Mark 1.0).

matics and commentator of Euclid.⁵⁵ In this work, it is specifically related to a purely practical mode of teaching Euclidean constructions in the problems of Book I within commentary sections entitled *Praxis*.

While in Prop. I.10, Scheubel did not propose a construction different from that of Euclid, but simply displayed its steps in a more practical manner, one may find in Prop. I.3 (“Given two unequal straight lines, to cut off from the greater a straight line equal to the less”⁵⁶) the addition of a different and properly mechanical construction to supplement Euclid’s own construction. In this framework, Scheubel proposed three different constructions, starting from the easiest and most mechanical and ending with Euclid’s construction, which is the most complex and theoretical.⁵⁷ In the first construction, unlike Euclid, Scheubel did not first request that a line equal to the shorter be placed at one extremity of the longer line (as per Prop. I.2), to then cut the longer line according to the length of the shorter by tracing a circle with a corresponding radius around their common extremity.⁵⁸ Rather he requested that the length of the shorter line be measured with a

⁵⁵ Christoph Clavius, *Euclidis elementorum libri XV: Accessit XVI*, in *Opera omnia*, vol. 1 (Mainz: Anton Hierat, 1611–1612), I.9, 35.

⁵⁶ Heath, *The Thirteen Books*, vol. 1, 246.

⁵⁷ Scheubel, *Euclidis elementa*, I.3, 84: “Est huius propositionis triplex operatio, seu fabrica.”

⁵⁸ Heath, *The Thirteen Books*, vol. 2, p. 246: “Given two unequal straight lines, to cut off from the greater a straight line equal to the less. Let AB, C be the two given unequal straight lines, and let AB be the greater of them. Thus it is required to cut off from AB the greater a straight line equal to C the less. At the point A let AD be placed equal to the straight line C; and with centre A and distance AD let the circle DEF be described. Now, since the point A is the centre of the circle DEF, AE is equal to AD. But C is also equal to AD. Therefore each of the straight lines AE, C is

compass and then that the latter be moved onto the given point, having maintained the same opening and one of its feet having been placed on the point, to mark out the length of the shorter line on the longer line with its other foot.⁵⁹ Since it leaves out certain steps of the construction necessary to the demonstration (namely, the fact of first placing a line equal to the shorter at one extremity of the longer line according to the construction demonstrated in Prop. I.2) and because of its entire dependence on the compass, this mode of construction differs from that of Euclid by its mechanical and practical character. Its addition to Euclid's construction (along with a second construction of intermediary level of complexity⁶⁰) allowed Scheubel to propose to students a construction easier to perform in practice, in addition to the less user-friendly construction of Euclid. In the constructions provided for Prop. I.3, the demonstrative part is also reduced to a minimum, consisting mainly in the indication of the principle and/or proposition on which it depends. Nevertheless, Scheubel was generally careful to maintain the classical Euclidean proof, even if he added many elements destined to help students understand them better or learn some of their further uses.

In addition to these pedagogical devices, numerical examples are employed when dealing with geometrical propositions, which was supported by the inclusion of a nearly eighty-page treatise of algebra, entitled *Brevis regularum algebrae descriptio una cum demonstrationibus geometricis*,⁶¹ at the beginning of his edition of Euclid. In this treatise, Scheubel taught the fundamental notions and rules of algebraic computations, using at times concrete examples from trade, money-changing, financial partnerships, the military art and the measurement of physical magnitudes. The inclusion of this treatise of algebra in Scheubel's edition of the *Elements* confirms that the numerical treatment of Euclid's plane geometry and geometrical theory of proportions played a prominent role in his pedagogical agenda.⁶² Scheubel applied indeed a numerical approach in all of Euclid's first six books except Book IV.⁶³ By contrast, most of the sixteenth-century editions that proposed

equal to AD; so that AE is also equal to C. Therefore, given the two straight lines AB, C, from AB the greater AE has been cut off equal to C the less."

⁵⁹ Scheubel, *Euclidis elementa*, I.3, 84: "Prima, ut officio circini quantitas brevioris accipiat: ea deinde in longiore, ab extremitate una incipiendo, puncto aliquo signetur: & factum erit negotium."

⁶⁰ It was left aside here as irrelevant to the present study. For the complete treatment of this proposition by Scheubel, see Axworthy, "The Hybridization."

⁶¹ Scheubel, *Euclidis elementa*, 1–76. On this treatise, see Staigmüller, "Johannes Scheubel," 467; Day, *Scheubel as an Algebraist*.

⁶² The importance of the numerical approach to Euclid in Scheubel's transmission of the *Elements* is also marked by its dedication to members of the Fugger family, a powerful family of bankers of Augsburg.

⁶³ Given that Book IV deals with inscriptions and circumscriptions of polygons within and about a circle, Scheubel may have regarded it as better suited to a fully constructive form of geometrical practice, as was used by painters and architects. On this type of practical geometry, see Lon

numerical examples to illustrate Euclid's geometrical teaching focused on Books II and V, and in rarer cases on Book X (when they went beyond Book VI).

Commentators of Euclid, already in the middle ages, indeed sometimes used numbers (mostly as whole numbers) to illustrate Book V (especially the definitions)⁶⁴ for pedagogical reasons on account of the fact that the Eudoxian theory of proportions taught in this book was considered applicable to all types of quantity (discrete and continuous), as was clearly acknowledged by Scheubel.⁶⁵ Since Book X also dealt with ratios, albeit between incommensurable magnitudes, it was also held acceptable in certain editions to use numerical examples with irrationals or surd numbers.⁶⁶ Book II, which mostly deals with quantitative relations between evenly or unevenly cut lines and the quadrangular areas on them, was sometimes considered as expressing algebraic identities as well as rules for arithmetical and algebraic operations, and was therefore sometimes treated in numerical terms (at least in Prop. II.1-10).⁶⁷ This was notably due to the influence of the arithmetical translation of these ten first propositions by Barlaam of Seminara in the fourteenth century.⁶⁸

Yet, while most sixteenth-century editions that proposed a numerical treatment of Book V only used whole numbers and in some cases included fractions (in accordance with the late medieval notion of denominations of ratios⁶⁹), mostly reserving irrationals

R. Shelby, "The Geometrical Knowledge of Mediaeval Master Masons," *Speculum* 47/3 (1972): 395–421.

⁶⁴ Ratios of magnitudes expressed as whole numbers in Book V appear, mostly in the diagrams illustrating the proofs, in about a third of the published sixteenth-century editions of Euclid.

⁶⁵ Scheubel, *Euclidis elementa*, Df. V.1, 226.

⁶⁶ This was the case in the editions of Pacioli (1509), Tartaglia (1543), Pierre Montdoré (1551), Jean Magnien and Étienne Gracilis (1558), François de Foix-Candale (1566), Billingsley (1570) and Federico Commandino (1572).

⁶⁷ This was the case in the editions of Jacques Peletier (1557), Dasypodius (1564), Pierre Forcadet (1564), Billingsley (1670), Commandino (1572) and Clavius (1574), in addition to Scheubel and Xylander.

⁶⁸ Barlaam's treatment of Book II was taken up by Dasypodius, *Euclidis quindecim elementorum Geometriae secundum* [...]. Item, *Barlaam monachi Arithmetica demonstratio eorum, quae in secundo libro elementorum sunt in lineis & figuris planis demonstrata* (Strasbourg: Christianus Mylius, 1564); Billingsley, *The Elements of Geometrie*, Book II; Federico Commandino, *Euclidis Elementorum libri XV. Unà cum scholijs antiquis* (Pesaro: Camillo Francischino, 1572), Book IX; Clavius, *Euclidis elementa*, Book IX. On Barlaam's arithmetical adaptation of Props. II.1–10, see Leo Corry, "Geometry and arithmetic in the medieval traditions of Euclid's *Elements*: A view from Book II," *Archive for History of Exact Sciences* 67 (2013): 637–705. On the arithmetical and algebraic interpretation of Euclid's Book II more generally, see Vitrac, *Les Éléments. Livres V–VI*, 366–376.

⁶⁹ On this topic, see for example Sabine Rommevaux, "Aperçu sur la notion de dénomination d'un rapport numérique au Moyen Âge et à la Renaissance," *Methodos: Savoirs et textes* 1 (2001): 223–243.

for Book X (if they included Book X and dealt with it numerically), in Scheubel's commentary (as said) all of the commented books (except Book IV) were illustrated through numerical examples, using rational numbers or fractions (notably to express ratios in Book V) and irrational numbers (in Books I–II and VI), using various notations for square and cubic roots derived from the German cossic tradition.⁷⁰ In Book III, which merely includes numerical examples in the last two propositions (III.36–37), only whole numbers are used.⁷¹

Although Scheubel took care to explain that the definitions and propositions of Book V may be applied to numbers as well as to magnitudes⁷² (wherefore he replaced the Euclidean term 'magnitude' by the more general term 'quantity' in this book), he did not only use numerical examples in Book V, as would Xylander for greater convenience,⁷³ but rather proposed, wherever it was possible, two types of examples: geometrical (using lines, as in the classical treatment of this book) and numerical (using integers and fractions).⁷⁴ He may have done so in response to the difficulties raised by earlier commentators of Euclid concerning the use of numbers in Book V, since the latter concerns ratios between both commensurable and incommensurable magnitudes⁷⁵ (some of which, as the ratio between the circumference and diameter of the circle, cannot even be expressed through radical numbers). But he may also have done so to simply manifest the universal scope of Book V, which extends to all types of quantities (discrete and continuous, commensurable and incommensurable).

To look in more details at Scheubel's numerical approach to Euclid's geometrical propositions, we may take the example of Prop. I.34 (Fig. 2–4), which states that "In parallelogrammic areas the opposite sides and angles are equal to one another, and the diameter bisects the areas"⁷⁶ (Fig. 2). It is the very first proposition of Euclid which Scheubel treated numerically,⁷⁷ but it is by no means exceptional in its style and content within his commentary on the *Elements*. As shown by the page exhibited in Fig. 2, while the figures accompanying the proof bear no letters (conforming to Scheubel's intention to free Euclid's

⁷⁰ Scheubel notably took up Rudolff and Stifel's notations for square, cubic and fourth roots: $\mathbf{J}$, $\sqrt{}$, $\sqrt[3]{}$. On this notation, see Florian Cajori, *A History of Mathematical Notations*, vol. 1 (Chicago: The Open Court, 1928), 133–136 and 139–146.

⁷¹ Scheubel, *Euclidis elementa*, III.36–37, 203–205.

⁷² *Ibid.*, Df. V.1, 226.

⁷³ Xylander's approach to Book V is discussed later.

⁷⁴ He also did so for the principles that were acknowledged as common to all branches of mathematics. Scheubel, *Euclidis elementa*, CN1–6, 81–82.

⁷⁵ This problem had been, for instance, raised by Niccolò Tartaglia, in response to Campanus' use of numerical examples in his commentary of Book V. Tartaglia, *Euclide*, Df. V.11, 64v.

⁷⁶ Heath, *The Thirteen Books*, vol. 1, 323–324.

⁷⁷ It is certainly not coincidental, since this proposition was necessary for the demonstration of several propositions of Book II.

diagrams and proofs from the confusion of letters), they are attributed numerical values. This was very different from most other sixteenth-century editions that made use of numerical examples, as numerical diagrams were generally kept separate from the proofs and maintained within the commentary section (except perhaps in the case of Book V).

If the text of the proof itself remains devoid of numbers (except to refer to prior propositions), the commentary section, which is labelled *Appendix* in Prop. I.34, is entirely filled with computations and sometimes extends over several pages (three of which are presented in Figs. 2–4). In Prop. I.34, for that matter, the numerical commentary occupies more than four pages of the book.⁷⁸ Indeed, as we may see on Fig. 2 and 4, the *Appendix* starts just after the proof in the bottom half of p. 110 (Fig. 2) and ends at the bottom of p. 114 (Fig. 4), without interruption or addition of any other type of commentary. It is interesting to note in this regard that such sections were sometimes labelled *Numerorum praxis*,⁷⁹ which marks the operative and practical character of the numerical commentary.

Furthermore, Scheubel did not only attempt to illustrate the Euclidean proposition at hand through numerical examples, as was most often done in other editions that appealed to such pedagogical devices, but also extended the scope of his commentary by teaching computational rules, as well as methods to abbreviate the steps of the procedure, as they came in useful to solve geometrical questions by numerical means. In the framework of Prop. I.34, the second paragraph of the *Appendix* (Fig. 2) thus provides in general terms the method through which one can determine the areas of any triangle from the knowledge of their sides, which follows the method taught by Hero of Alexandria in his *Metrica*.⁸⁰

Let first the sides of the triangles, whose area it is proposed to find, be added together; then let each of the sides of the triangle be subtracted from half of their total. Three numbers will remain, along with half of the added sides as a fourth number, which, if they are multiplied with each other, that is, the first is multiplied with the second, their product with the third, and the latter with the fourth (and the fact that the numbers are said to be the first, the second, the third or the fourth does not refer to the order according to which they should be added), then the square root of this last product will exhibit how much the area of the proposed triangle will be.⁸¹

⁷⁸ Scheubel, *Euclidis elementa*, 110–114.

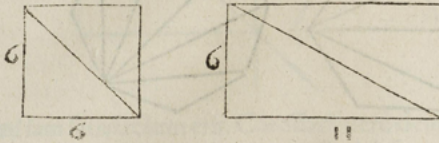
⁷⁹ Ibid., I.36, 116: “Nunc quantum ad praxim numerorum;” I.40, 120: “Sequitur praxis numerorum” or I.41, 121: “Numerorum praxis.”

⁸⁰ Hermann Schöne, *Heronis Alexandrini Opera quae supersunt omnia*, Vol. III (Leipzig: Teubner, 1899), 18–25.

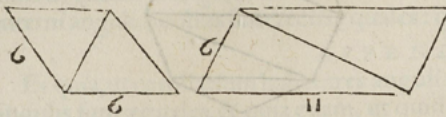
⁸¹ Scheubel, *Euclidis elementa*, I.34, 110.

Parallelogrammorum locorum. & latera & anguli opposita, æqualia inter se sunt. Et diameter ea bifariam secat.

Parallelogrammum, ut uocabuli $\pi\alpha\rho\alpha\lambda\lambda\epsilon\gamma\mu\alpha$ indicat, est figura, sub parallelis rectis lineis comprehensa. Fit autem seu describitur parallelogrammum, per ductam rectam lineam, punctumq; extra eā sumptum, si ex hoc puncto, per propositionem 31 & 3, recte ducte parallela æqualis ducatur, utriusq; rectæ deinde extremitates,



lineæ sint. Talium igitur parallelogrammorum locorum, seu talium figurarum, & latera & anguli opposita, æqualia inter se sunt. Ducatur in figura diameter. Et quoniam anguli alternatim positi, ex prima parte propositionis 29, inter se æquales sunt,



unde sic duo triangula, qualia propositio 26 requirit, apparent, quod latera opposita inter se æqualia sint, angulus item unus suo opposito æqualis, per hanc 26 propositionem inferri potest. Et rursus quoniam, si æqualibus æqualia adiciantur, ex cōmuni quadam noticiā, ipsa tota æqualia sunt: huius sententiæ memor, alterum etiam suū opposito angulo æqualem esse, facile concedet. Pater itaq; prior propositionis pars. Posterior nunc, quod scilicet diameter ipsum parallelogrammum bifariam secet, si quis suspicetur id nondum esse demonstratum, per propositionē quartam id deprehendet. Parallelogrammorum igitur locorum & latera & anguli opposita, æqualia inter se sunt. Et diameter ea bifariam secat, quod demonstrari oportuit.

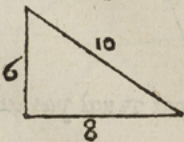
APPENDIX.

Quoniam autem hæc propositio 34, & multæ etiam sequentes, in numeris, quantitate nimirum discreta, non minus atq; in quantitate continua, ueræ esse reperiuntur, quo id offendamus commodius, canonem quandam generalem, per quē omnis generis triangulorum (modo latera eorum nota fuerint) areæ inueniri possent, subiungere necesse fuit, his uerbis.

Trianguli, cuius aream propositum est inuenire, latera primò in unum colligantur, à medietate deinde huius collecti singula trianguli latera subtrahantur. Relinquantur autem tres numeri, qui unā cum medietate collecti ex lateribus, tanquam numero quarto, si inter se multiplicati fuerint, primus scilicet cum secundo, productum hoc cum tertio, quodq; iam producet cum numero quarto (nec refert quo ordine numeri sumantur, qui uel pro primo, secundo, tertio uel quarto reputetur) tū huius ultimi producti radice quadrata, quanta propositi trianguli area fuerit, manifestabitur.

SEQUUNTUR HUIUS CANONIS EXEMPLA.

Triangulū propo.

Latera
trianguliExcessus uniuscuiusque lateris respec-
tu medietatis, aggregati ex lateribus,

10

8

6

24

12

laterum summa,
laterum mediet.

2

4

6

2

4

6

12

Quatuor numeri

Insituantur

Instituantur nunc multiplicationes.

prima	secunda	tertia
12	6	24
2	4	24
24 primum	24 secun.	576 ultimū productū

Quadra, $\frac{12}{4} \cdot \frac{6}{4} = 9$ (Radix 24. Tanta igitur est trianguli, cuius latera sunt 10 8 6, area.

EXEMPLVM IN IRRATIONALIBVS.

Latera	Excessus
$\sqrt{180}$	$9 - \sqrt{45}$
12	$\sqrt{45} - 3$
6	$\sqrt{45} + 3$
Sum. 18 + $\sqrt{180}$	Medietas 9 + $\sqrt{45}$

Quatuor numeri.

$9 - \sqrt{45}$	$\sqrt{45} - 3$	$\sqrt{45} + 3$	$9 + \sqrt{45}$
36 primum	36 secundum productum		
Tertiò multiplicentur cum		36	
producuntur		1296, cuius radix	
quadrata		36. area est trianguli.	

ABBREVIATIO CANONIS PER COMPENDIVM.

Cum tertiæ multiplicationis numeri, qui nimirum ex prima & secunda multiplicatione proueniunt, inter se fuerint æquales, id quod sæpe contingit, in his item duobus exemplis euident est, eadem tertiã multiplicatio negligitur, nec etiã extractione radices quadratæ tum opus erit. Verum statim per alterutrum productorum, primum uel secundum, trianguli area indicabitur.

EXEMPLVM CANONIS ALIVD.

Est autem in hac 34 propositione triangulum figuræ primæ.

Latera	Excessus
$\sqrt{72}$	$6 - \sqrt{18}$
6	$\sqrt{18}$
6	$\sqrt{18}$
Sum. 12 + $\sqrt{72}$	Medietas 6 + $\sqrt{18}$.

Primum productum sunt 18, secundum tantundem, tertium deinde 324. huius postea radix 18, area est trianguli, atq; medietas etiã parallelogrammi uel figure primæ, quod hoc canone ostendere oportuit.

Potuiſſet ex cõpendio iam præſcripto, tertiã multiplicatio negligi, ac statim per 18 uel 18, primum ſcilicet uel ſecundum productum, quæſtioni reſponderi, quod idem fuiſſet.

SEQVITVR TRIANGVLVM FIGVRÆ SECVNDAE.

Latera	Excessus
$\sqrt{157}$	$\frac{12}{2} - \sqrt{\frac{157}{4}}$
11	$\sqrt{\frac{157}{4}} - \frac{5}{2}$
6	$\sqrt{\frac{157}{4}} + \frac{5}{2}$
Sum. 17 + $\sqrt{157}$	Medietas $\frac{12}{2} + \sqrt{\frac{157}{4}}$

Quatuor

Fig. 3 – Scheubel, *Euclidis elementa*, Prop. I.34, 111. Image source: Max Planck Institute for the History of Science, Berlin (Public Domain Mark 1.0).

Lateralum summa 12, plus $\sqrt{60}$ — $\sqrt{12}$

Excessus igitur, atq; deinceps quatuor numeri,

$\sqrt{15} + \sqrt{3}$ $\sqrt{15} + \sqrt{3}$ 6 minus $\sqrt{15} + \sqrt{3}$ 6 plus $\sqrt{15} + \sqrt{3}$

Primum secundum productum

18 + $\sqrt{180}$ 36 minus 18 + $\sqrt{180}$.

Tertium productum.

648 + $\sqrt{233280}$ minus 504 + $\sqrt{233280}$

hoc est, 144. Area igitur trianguli 12, ut prius.

ALIUD ITEM TRIANGVLVM FIGVRAE
quartæ, cuius quidem

Latera sunt

Excessus igitur

Ra. qua. bi. 157 + $\sqrt{9680}$

$8\frac{1}{2}$ minus ra. qua. bi. $39\frac{1}{4}$ + $\sqrt{605}$

11

ra. qua. bi. $39\frac{1}{4}$ + $\sqrt{605}$ minus $2\frac{1}{2}$

6

ra. qua. bi. $39\frac{1}{4}$ + $\sqrt{605}$ plus $2\frac{1}{2}$

17 plus ra. qua. bi. 157 + $\sqrt{9680}$ $8\frac{1}{2}$ plus ra. qua. bi. $39\frac{1}{4}$ + $\sqrt{605}$

Producta,

primum

secundum

$72\frac{1}{4}$ minus $39\frac{1}{4}$ + $\sqrt{605}$

$39\frac{1}{4}$ + $\sqrt{605}$ minus $6\frac{1}{4}$

Inuentio producti tertij.

$72\frac{1}{4}$

minus

$39\frac{1}{4}$ + $\sqrt{605}$

$39\frac{1}{4}$ + $\sqrt{605}$

minus

$6\frac{1}{4}$

$2835\frac{13}{16}$ + $\sqrt{\frac{50510205}{16}}$ item $245\frac{5}{16}$ + $\sqrt{\frac{378125}{16}}$

minus $451\frac{9}{16}$

minus $2145\frac{9}{16}$ + $\sqrt{\frac{59650589}{16}}$

Summa productorum.

plus $3081\frac{1}{8}$ + $\sqrt{\frac{59650589}{16}}$

minus $2597\frac{1}{8}$ + $\sqrt{\frac{59650589}{16}}$.

Hoc est, facta subtractione, 484.

Huius nunc tertij producti radix quadrata, nimirum 22, area est trianguli.

Atq; hac tenus dicta de triangulorum arcibus inuestigandis sufficiant. Sequitur

ΠΡΟΤΑΣΙΣ

ΔΕ.

Τὰ παραμειλόγραμμα τὰ ὑπὸ τῇ αὐτῇ βάσει ὄντα, καὶ ἐν ταῖς αὐταῖς
παραμείλοις ἴσα ἀμείλις ἐσίν.

PROPOSITIO

XXXV.

Parallelogramma super eadem basi constituta, atq; in eisdem paral-
lelis: æqualia inter se sunt.

Potest huius propositionis figura geometrica tripliciter variari. Aut enim paral-
lelogrammis super una & eadem basi, inter easdem item parallelas positis, alterum
unius latus est diameter alterius, aut ea brevius, aut longius. Si primum, cum ex co-
rollario propositionis præcedentis, Omne parallelogrammū a sua ipsius diametro
bifariam

Fig. 4 – Scheubel, *Euclidis elementa*, Prop. I.34, 114. Image source: Max Planck Institute for the History of Science, Berlin (Public Domain Mark 1.0).

The Heronian formula to calculate areas of triangles may be expressed in modern terms as $A = \sqrt{s(s-a)(s-b)(s-c)}$, in which s corresponds to the semiperimeter of the triangle and a, b, c to each of the three sides of the triangle.

After providing the rule in general terms, Scheubel provides a first example with whole numbers (*Sequuntur huius canonis exempla*, bottom of Fig. 2 and top of Fig. 3), in which the considered triangle is depicted (with indication of the length of each side) next to the arithmetical operations required to compute its area. As we can see at the top of p. 111 (Fig. 3), Scheubel appeals in this first example to crossed-out numbers to mark out the steps of the calculation, as was generally done in treatises of practical arithmetic.

The example he proposes immediately after (*Exemplum in irrationalibus*, Fig. 3) involves more complex objects, notably irrational numbers (as the square roots $\sqrt{180}$ and $\sqrt{45}$) and binomials (as $9 - \sqrt{45}$ or $\sqrt{45} + 3$). Following this example, Scheubel provides a method to perform the computation in a briefer, and thus more practical, manner ('Abbreviation of the rule by economy,' or in Latin *Abbreviatio canonis per compendium*, as shown in the bottom half of p. 111, Fig. 3). This method consists in leaving aside the steps corresponding to the third multiplication and the extraction of the square root, when the product of two of the numbers to multiply (as here $\sqrt{18}$ and $\sqrt{18}$, corresponding to $(s-b)(s-c)$ in the modern expression of the formula) equals the product of the other two numbers (here $(6 - \sqrt{18})(6 + \sqrt{18})$ or $s(s-a)$).

In the following three pages of Scheubel's commentary, the same rule is illustrated through a multitude of examples (nine in total) with different types of numbers and numerical expressions, from the simplest to the most complex. Fig. 4, which exhibits the last page of the commentary on Prop. I.34 (p. 114), shows that in the last example proposed by Scheubel (*Aliud item triangulum figurae*), one may find fractions (as $72\frac{1}{4}$), radical numbers (as $\sqrt{605}$), binomials (as $39\frac{1}{4} + \sqrt{605}$), but also square roots of fractions (as $\sqrt{\frac{50530205}{16}}$) and square roots of binomials (as Ra. qua. bi. $157 + \sqrt{9680}$). In this framework, arithmetical operators are presented in both symbolic and rhetorical forms for a better legibility, allowing to distinguish binomials from operations on binomials (as $72\frac{1}{4}$ minus $39\frac{1}{4} + \sqrt{605}$). As we may see, after the first numerical example provided to illustrate the rule (starting on p. 110 in Fig. 2), no geometrical representation of triangles is given for the other examples. At the end of this long series of examples, Scheubel simply concludes: "And what has been said so far on the determination of the areas of triangles will suffice."⁸²

As Scheubel wrote in the introduction of his *Appendix* on this proposition (Fig. 2), the fact of dealing with propositions such as Prop. I.34 numerically is justified by the fact that it is applicable to both numbers and magnitudes: "Since this thirty-fourth proposition, and also many of those that follow, are found to be true for numbers, that is, for discrete quantity, as much as for continuous quantity, it was necessary, in order to show this more

⁸² Ibid., I.34, 114.

appropriately, to provide a general rule by which the areas of any type of triangles may be found (provided their sides are known) in the following terms.”⁸³ As he explains then (and as we saw above), the arithmetical rules that he taught in this framework were not simply applied to numbers, in addition or in parallel to the geometrical demonstration (e.g. to manifest the connections between arithmetic and geometry), but were directly applied to the specific dimensions of particular magnitudes, and more specifically to the areas of triangles (whether visually represented or not), overthrowing thereby the separation that had been set between numbers and magnitudes in the *Elements*.

The properly algebraic content Scheubel introduced in his commentary on Euclid, beyond the *Algebrae descriptio*, is to be found, as expected, in Book II, and notably in Prop. II.4, which states that “If a straight line be cut at random, the square on the whole is equal to the squares on the segments and twice the rectangle contained by the segments,”⁸⁴ and which may thus be taken to express (in modern terms) the following identity: $x^2 = a^2 + b^2 + 2ab$. The *Appendix* to Prop. II.4, which is three and a half pages long, is introduced thus: “Computists (*logistici*) employ this proposition rather frequently in the rules of algebra. For by means of it they usually demonstrate, among other things, the addition of irrational numbers, and then the equation in which, when three quantities are proposed in natural order or in equal means (*naturalis ordinis vel aequalibus medijs*), the two of greater denomination is equal to the third, which has the smallest denomination.”⁸⁵

While the *Algebrae descriptio* is rarely cited in the commentary on Euclid, it is mentioned when explaining the use of Prop. II.4 to teach rules for the resolution of equations,⁸⁶ which confirms that Scheubel did not publish his treatise of algebra and his edition of the *Elements* together out of convenience, but initially intended for them to be read together. As Scheubel mentions in the commentary on Euclid, among the types of equations presented in his *Algebra*, it is the second type of the first form or canon (among the three distinguished canons), i.e. *Prima + radix = Numero* (or in modern terms $ax^2 + bx = c$),⁸⁷ that is founded on Euclid’s Prop. II.4.⁸⁸ Euclid’s proposition is itself cited in the *Algebra* when presenting this type of equation.⁸⁹

As was previously mentioned, Scheubel also published a German adaptation of Books VII–IX of the *Elements*,⁹⁰ which differs from the edition of Books I–VI by its less scholarly

⁸³ Ibid., I.34, 110.

⁸⁴ Heath, *The Thirteen Books*, vol. 1, 379.

⁸⁵ Scheubel, *Euclidis elementa*, II.4, 143.

⁸⁶ Ibid., II.4, 144.

⁸⁷ The other two correspond to $bx + c = ax^2$ and $ax^2 + c = bx$. Scheubel, *Algebrae descriptio*, in *Euclidis elementa*, 26–27 and II.4, 144.

⁸⁸ Ibid., 144.

⁸⁹ Scheubel, *Algebrae descriptio*, in *Euclidis elementa*, 28.

⁹⁰ On this work, see also Staigmüller, “Johannes Scheubel,” 447–450.

rendering (beside the properly arithmetical character of its subject⁹¹), as by its more explicit references to the knowledge of *Rechenmeister*,⁹² and notably to the work of Christoff Rudolff.⁹³ This edition also taught, in addition to Euclid's theory of numbers and the Latin names of the employed German arithmetical terms,⁹⁴ various numerical examples and computational rules, with crossed-out numbers to mark out the steps of the computations, as in the 1550 Latin edition and as in common arithmetic handbooks.

Yet, while in Scheubel's Latin (and partly Greek) edition of Euclid, examples demonstrating concrete applications remained confined to the treatise of algebra, in the German edition, such examples, which mostly relate to the practice of merchants, bankers or book-keepers,⁹⁵ are directly inserted in the commentary of Euclid's propositions, sometimes extending over more than ten pages.⁹⁶ Numbers are then often expressed in concrete counting units, such as currencies (e.g. Gulden, Schilling, Pfennig) and measuring units (as ells), their regional differences being sometimes taken into consideration.⁹⁷

This German edition confirms that Scheubel, in his general approach to Euclid's mathematical work (hence also in the 1550 Latin edition), intended to offer a different teaching of the *Elements* from that of his predecessors (as Oronce Fine, whose 1536 edition of Euclid was part of Scheubel's library⁹⁸), which would be both more accessible and more useful to students and practitioners. In doing so, he also transmitted certain arithmetical and algebraic rules, and some of their concrete applications, through a different and more scholarly approach than through a mere *Rechenbuch*. These two editions of Euclid, whether in Latin or German, additionally allowed Scheubel to exhibit the theoretical foundation

⁹¹ Even if Scheubel, in the commentary on Prop. VII.19 (Scheubel, *Das sibend, acht und neünt Büch Euclidis*, 63–69), also quoted certain propositions from Book V and VI (namely, V.11, V.9, V.7 and VI.13).

⁹² Scheubel, *Das sibend, acht und neünt Büch Euclidis*, Introduction, 1: "In diesen dreyen büchern findet man erstlich etlicher wörter, so bey allen Rechenmaistern in gemainen brauch seind" or 2: "Man findet auch weyter beyleüfftig die ursach, warumben der Rechenmayster im süchen des quadrats wurtzel [...]" (my emphasis).

⁹³ Ibid., VIII.5, 143: "welches der fürnem und kunstreich Rechenmaister, Christoff Rudolff selig, in seinem gemainen Rechenbüchlen."

⁹⁴ Ibid., Df. VII.1–3, 3: "Ains, im latein *Unitas* [...] Die zal, im latein *Numerus* [...] Ain tail, im latein *Pars*."

⁹⁵ Ibid., VII.13, 39–40: "Das der Kauffman oder Handelßman [...]" See also 69: "Ainer kaufft 5 eln tûch zû einem rock, ist brait 1 1/2 eln."

⁹⁶ In Prop. VIII.5 (ibid., 131–146), such examples cover fifteen pages.

⁹⁷ Scheubel, *Das sibend, acht und neünt Büch Euclidis*, 58–61: "Würtenberger müntz [...] Nürnberger müntz [...] Meysnisch müntz [...] Baierisch und schwartz müntz [...] Osterreichisch müntz [...] Oetsch müntz, etc."

⁹⁸ Fine, *Euclidis elementa*; Hughes, "The Private Library," 422: "Orontii Geometrica in Euclidem." Although Fine's Euclid does include numerical examples, it overall remains faithful to the style of classical editions of Euclid, as that of Zamberti.

of a part of his arithmetical and algebraic teaching and thus to raise the status of common *Rechenkunst* in the hierarchisation of mathematical arts while expanding the readership of both traditions. It should be noted in this regard that his 1545 treatise of arithmetic, namely the *De numeris et diversis rationibus seu regulis computationum opusculum*, which was published in Latin in a more scholarly style than his German edition of Euclid's arithmetical books, included examples of computational practices applied to concrete uses⁹⁹ as well as references to propositions of Euclid,¹⁰⁰ marking there too the continuity between scholarly and practical mathematical culture.

Xylander's numerical treatment of Euclid's Elements

Xylander's Euclid corresponds to a German translation from the Greek of the principles and enunciations of the propositions, together with an adaptation of the proofs and a pedagogical commentary, of the first six books of the *Elements*. As Xylander explains in his preface, his aim in producing this translation was to fill a gap in the mathematical knowledge available in German at his time by making Euclid's *Elements* useful and understandable to a German reader who is inexperienced both in geometry and in ancient languages (Greek and Latin).¹⁰¹ This type of reader, according to Xylander, would include painters, goldsmiths and master builders, and more generally all those who need to count, measure and trace figures by instrumental means.¹⁰² Even if he had written the first draft of his Euclid in 1555 and only started teaching mathematics (at least publicly) in 1562 (i.e. the year of publication of this commentary), Xylander may also have been motivated to have his translation printed for university students, since he had already been teaching at the Faculty of Arts of Heidelberg (lecturing in Greek) since 1558.

Conforming to this pedagogical aim, Xylander often reduced or adapted Euclid's proofs to render them more intelligible and attractive to a lay audience (often addressing the 'simple reader,' or in German 'der einfältige Leser'),¹⁰³ notably by using terms from everyday language side by side with the original Greek or Latin terms.¹⁰⁴ He also provided (as Scheubel) many diagrams with compass marks and dotted lines and made the steps of the instrumental construction explicit in many problems of Book I,¹⁰⁵ explain-

⁹⁹ E.g. Scheubel, *De numeris*, C7r or D5v–D6r.

¹⁰⁰ E.g. *ibid.*, B4r–B5r or C7r.

¹⁰¹ Xylander, *Die sechs erste Bücher Euclidis, An den Leser*, b2r–v.

¹⁰² *Ibid.*, b1v: "wie alle künsten [...] der Maler, Goldtschmid, Barmaister, &c.) sich mit zirckel, lineal, bleywag, ziffern und zalen begehnen und behelfen müessen [...]."

¹⁰³ E.g. *ibid.*, *An der Leser*, b1r or I.1, 7.

¹⁰⁴ E.g. *ibid.*, Df. I.1, 1: "Ain punct oder tipfflin" or I.4, 8: "Basis, ist Griechisch, heißt der grund, boden, oder fuß [des triangels]." On this practice, see Angela Axworthy, "Renaissance approaches to the terminology of mathematics," *Le Français Préclassique* 26 (2024): 61–68.

¹⁰⁵ E.g. Xylander, *Die sechs erste Bücher Euclidis*, I.1, 6: "begreiff ich mitt einem zirckel."

ing what is required in practice for the instrumental construction of certain propositions (e.g. the tracing of small intersecting arcs instead of full circles).¹⁰⁶ This was sustained by a reference to geometrical instruments in the title of the book.¹⁰⁷ In some cases, Xylander actually replaced Euclid's construction by a purely mechanical one¹⁰⁸ or suppressed the proof altogether to invite the reader to derive the construction or demonstration from the diagram.¹⁰⁹

To look at an example of mechanical construction in Xylander's Euclid, let us consider Prop. I.2, which teaches how "To place a straight line equal to a given straight line with one end at a given point."¹¹⁰ In his treatment of this proposition, Xylander entirely leaves aside Euclid's construction, which involves the tracing of an equilateral triangle and of two circles (as shown in Fig. 5, from Pacioli's 1509 Euclid¹¹¹) to guarantee that, at each step, a line properly equal to the given line is produced until the given point is reached. In Xylander's Euclid (Fig. 6), it is replaced by the simple act of measuring the length of the given line with a compass and then placing one foot of the compass at the given point and marking out the other extremity of the requested line with the other foot of the compass (which has maintained its opening) at any place around the given point, which are then joined by a line equal to the given one.

II. The Second Proposition. *From a given point, to draw a line which is equal to a given line.* It is done thus.

Although this may be very easily accomplished, that is, if you take the length of the given line with a compass, and then measure it from the given point and draw a straight line of that extent, which is clear and flawless and founded on the nature of the compass, however those who have written demonstrations for Euclid's propositions have provided here another operation, which is indeed ingenious, but difficult. I have left it aside as unnecessary for the lay German reader (whom I chiefly intend to serve). I trust that I shall not be blamed for it, etc. In the figure (Fig. 6), A is the given line, B the point, the lines BC, BD, BE, etc., are all equal to the given line.¹¹²

¹⁰⁶ E.g. *ibid.*, I.1, 7 or I.9, 11.

¹⁰⁷ *Ibid.*, title page: *Die sechs erste Bücher Euclidis, Vom anfang oder grund der Geometri [...]* (versteh alles kunstlichen, gwisen, und vortailigen gebrauchs des Zirckels, Linials oder Richtscheittes und anderer werckzeuge [...]).

¹⁰⁸ This is the case in I.3 (*ibid.*, 7).

¹⁰⁹ In I.29 (*ibid.*, 18), the proof is replaced by an indication that the proposition is the converse of the previous proposition and that it can be understood and proven by the figure of the following proposition.

¹¹⁰ Heath, *The Thirteen Books*, vol. 1, 245.

¹¹¹ Pacioli's commentary on the *Elements* (Pacioli, *Euclidis elementa*) was based on Campanus' Euclid, even if new images and additional comments were added.

¹¹² Xylander, *Die sechs erste Bücher Euclidis*, I.2, 7.

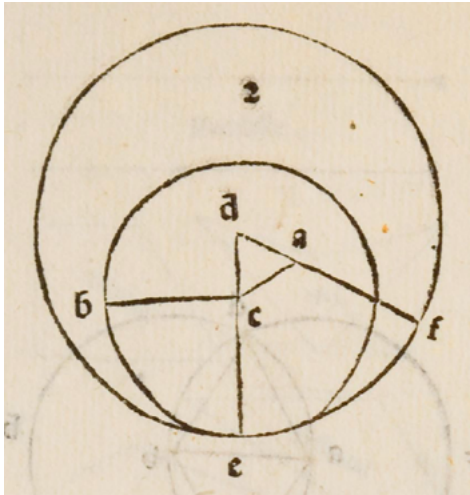


Fig. 5 – Diagram representing the steps of the classical construction of Euclid's Prop. I.2. Pacioli, *Euclidis elementa*, Sv. Image source: Max Planck Institute for the History of Science, Berlin (Public Domain Mark 1.0).

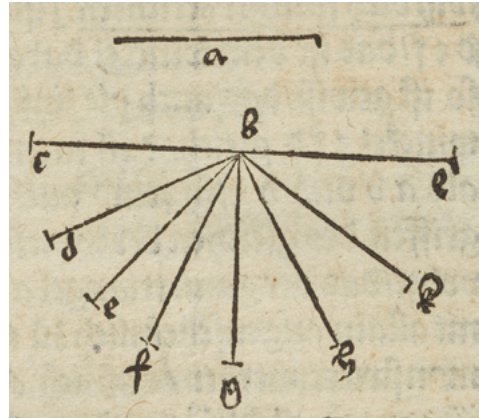


Fig. 6 – Diagram representing the steps of Xylander's construction for Prop. I.2. Xylander, *Die sechs erste Bücher Euclids*, Prop. I.2, 7. Image source and permission: Staatsbibliothek Bamberg.

In this framework, the mechanical character of the construction is not only based on the explicit reference to the instrument (i.e. the compass), which contrasts with the absence of reference to instrumental procedures in the *Elements*, but also on the fact of entirely leaving aside the Euclidean construction and demonstration, which are scientifically grounded in the *Elements* through their dependence on Postulates 1-3, Df. I.15 and Prop. I.1. This also contrasts with Scheubel's attitude in Prop. I.3, who had at least presented his mechanical construction *in addition to* the Euclidean classical construction and at least pointed to the principles and propositions on which it depends.

For Xylander, the fact of circumventing Euclid's construction and demonstration, which adds to its mechanical (as opposed to geometrical) character, is then justified by the too great complexity and lack of necessity of Euclid's construction for the readers he addresses, which are unlearned in both mathematics and the languages of ancient texts and of scholars (Greek and Latin). The accompanying diagram (Fig. 6), which depicts the taught construction as having been repeated a certain number of times, in different directions, adds to the operative character and pedagogical aim of Xylander's discourse, which thus invites the reader to perform the procedure several times to learn and master it in practice.

As Scheubel, Xylander also proposed in his commentary an extensive numerical treatment of Euclid's geometrical propositions. But contrary to Scheubel, he added numeri-

cal examples in all of the books of the *Elements* on which he commented (I–VI), even if this treatment was not evenly distributed throughout the commentary, being as expected more extensive in Books II, V and VI than in Books I, III and IV.

Xylander, as Scheubel, often placed numerical examples in dedicated commentary sections,¹¹³ which sometimes covered several pages and contained several subsections. In these, he either applied a computational process on different figures (as in Prop. I.41¹¹⁴) or taught different arithmetical or algebraic rules (as in Prop. II.4¹¹⁵), sometimes together with a presentation of mathematical or practical uses of the proposition.¹¹⁶ However, contrary to Scheubel, he often included numerical examples in the section that immediately follows the enunciation of propositions,¹¹⁷ which only loosely corresponds to the proof, in a classical Euclidean sense. In certain cases, Xylander specified that he replaced the geometrical proof of Euclid by a numerical illustration due to its too great obscurity or complexity.¹¹⁸

To consider Xylander's numerical approach to Euclid's geometrical propositions, we may take the example of Prop. I.36.¹¹⁹

XXXVI. The Thirty-Sixth Proposition. *All parallelogrammatic figures which stand between two parallel lines and upon equal bases or foundations are equal to one another.*

Explanation (Erklärung). This proposition may be easily understood from the previous one, since, between one line and two equal lines, no difference is apprehended and no alteration is produced. Examples follow.

In the first example, it is also proven by numbers that the rectangle ABCD is equal to the rhomboid EFGH: multiply the sides of the oblong rectangle together, and you have its area. Now, to find the area of the figure EFGH, note that, according to Proposition 34, it is divided by its diagonal FH, which is 6, into two equal triangles; and the sides of each such triangle are known, namely 6, 9, and $\sqrt{117}$. Since the two depicted parallelograms are equal to one another according to this proposition, it necessarily follows that the triangle EFG must contain 27 (that is, $\frac{1}{2}$ of 54).

¹¹³ This appears, for example, in the titles of commentary sections, in I.34 (Xylander, *Die sechs erste Bücher Euclidis*, 21): “*Volgen Exempell, mit ziffern erkleret.*” or I.47 (ibid., 40): “*Demonstration durch rechnung und zalen.*”

¹¹⁴ The various sections of the commentary (ibid., I.41, 27–30) mark the succession of the different figures whose areas are calculated: *Erklärung durch exempel; Rechnung der Triangel, vorge-setzter figuren; Der dritten Figur; Der vierdten Figur; Der fünfften Figur.*

¹¹⁵ Ibid., II.4, 48–54. See *infra*.

¹¹⁶ E.g. ibid., I.41, 30; I.47, 40 or II.1, 45.

¹¹⁷ In theorems, this part is generally called *Erklärung* (i.e. explanation).

¹¹⁸ E.g. *Die sechs erste Bücher Euclidis*, I.38, 26 or I.41, 27.

¹¹⁹ Heath, *The Thirteen Books*, vol. 1, 331: “Parallelograms which are on equal bases and in the same parallels are equal to one another.”

This may be discovered by the given rule with the aid of art of counting (without which all other arts, but especially those who proceed by compass and straightedge, measure and number, remain defective and imperfect). Sum the sides. From half of the sum subtract each side separately, multiply, etc., as represented above, and you will find the area of the triangle to be 27, as indeed it is. This operation, together with its advantages, I have wished to set down for the reader's sake, exhorting him not to neglect the practice of the art of counting, especially concerning surds and binomials, nor to despise it as useless or too difficult; otherwise the art will often bring him nothing. On this (God willing), more in its proper place.¹²⁰

Instead of providing a geometrical proof in the classical Euclidean style, Xylander started by simply pointing out its evidence on the basis of the previous proposition (I.35), which states that “Parallelograms which are on the same base and in the same parallels are equal to one another”¹²¹ and which is the first proposition in which Xylander adopted a numerical treatment in his commentary on Euclid.¹²² He then directly pursues with a numerical example based on the numbers featured in the provided diagram (as shown in Fig. 7), outlining in the text the computational rules to be applied in order to find the areas of the two compared parallelograms ABCD and EFGH situated under the same parallels, that is, the multiplication of the length (9) and width (6) of the rectangular parallelogram ABCD to obtain its area (54) and the computation of the areas of the two triangles with sides 9, and 6 (6 measuring the diagonal of the parallelogram EFGH), following the Heronian method taught in the previous proposition (as Scheubel had done in Prop. I.34). The detailed steps of the second computation are laid out in columns below the text (as shown in the bottom half of the page, in Fig. 7), according to the style of German *Rechenkunst* handbooks. This is the first proposition in which a vertical computations is employed in Xylander's commentary.

Interestingly, we may note that the computational procedure applied to the numerical example provided by the diagram is justified by its foundation on a prior proposition (Prop. I.34), as was done in classical Euclidean propositions, and in other geometrical propositions (as Prop. I.1), in which Xylander remained more faithful to Euclid.¹²³ This suggests that, for Xylander, an arithmetical computation, for certain propositions, should

¹²⁰ Xylander, *Die sechs erste Bücher Euclidis*, I.36, 24.

¹²¹ Heath, *The Thirteen Books*, vol. 1, 326. Xylander, *Die sechs erste Bücher Euclidis*, I.35, 22.

¹²² In Prop. I.35, the only geometrical proof that is proposed corresponds to a statement of the given through a general description of the diagram. It is then followed by a long appendix (*Anhang*) teaching the Heronian method of computation of the areas of triangles with specific numerical examples, also in abbreviated form, as in Scheubel's Euclid, to which Xylander added a numerical proof of a converse of the proposition (*ibid.*, 23: *Erklärung gegenwertiger Proposition mitt zalen*).

¹²³ This was the case, for example, in Prop. I.1, *ibid.*, 6–7.

Erklärung.

Diser proposition verstand ist leichtlich zûhaben auß der vorgehenden/angesehen das ain linj/vnd zwo gleich linj khain vndercheid dem verstand nach haben / vnd khain verenderung bringen volgen exempel. Im ersten exempel/auch durch die zalen zûbeweysen/das die vierung a b c d dem rhomboide e f g h gleich sey / multiplicier die seitten der vberlengten vierung mit einander/hastu jren inhalt. Nun den begreiff der figur e f g h zûfinden/merck das sy laut der 34 prop. durch iren diameter f h/welcher ist 6/in zwen gleichetriangel zertailt ist/ vnd die seitten jedes sollichen triangels bekant/als nemlich 6.9 vñ 117. So nun gemelte zway parallelogram/ainander nach vermög diser prop. gleich/muß von not der triangel e f h/27 (das ist $\frac{1}{2}$ von 54) inhalten. Sollichs durch gegebne regell mit hilff der Rechenkunst (on welche alle andre künsten/sonderlich aber die/so mit dem zirkel vnd lineal maß vnd zû sich begehrt/brechthafft vnd vnuolckommen) zûerfinden. Summier die seitten/vom halben tail der sumi nim jede seitten insonderheit/multiplicier/72. wie oben gemelt/sindst den begreiff des triangels 27/wie dan recht. Dife operation mit iren vortailen/hab ich dem leser zû lieb wöllen verzeichnen in vermanend/wölle in der Rechenkunst/der Surdischen vñnd Binomischen ybung nit vergessen/noch die als vnus oder zûschwer verachten/anderst im wirt offi khunst zerinne/darvon (wils Gott) an seinem ort.

Operation zûfinden den inhalt
gemeints triangels.

Die seitten

$$\begin{array}{r} 6 \\ 9 \\ \hline 117 \\ \text{Summa } 15 + 117 \end{array}$$

Der halbtail
 $7\frac{1}{2} + 129\frac{1}{2}$

Erst rest
 $129\frac{1}{2} + 1\frac{1}{2}$
Multiplirer mit
seinem residuo
 $129\frac{1}{2} - 1\frac{1}{2}$

Ander Rest
 $129\frac{1}{2} - 1\frac{1}{2}$

Dritt Rest
 $7\frac{1}{2} - 129\frac{1}{2}$
Multiplirer mit
seinem Binomio
 $7\frac{1}{2} - 129\frac{1}{2}$

Product $129\frac{1}{2} - 2\frac{1}{2}$ das ist
27

Product $56\frac{1}{2} - 129\frac{1}{2}$ das ist
27

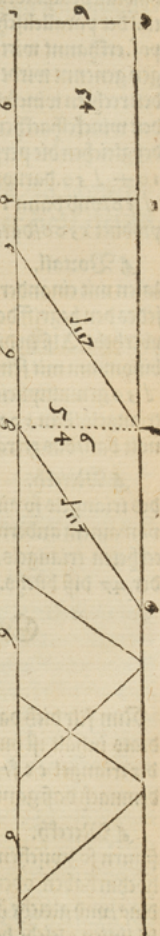
Merck den vortail das man binomia mit residuts multiplicier. Die zway product gleich/derhalben 27 die seldung ob inhalt des triangels/wie oben gemeldt.

XXXVII Die sibenvnddreyßigst

Proposition.

Alle triangel so auff einer basi/zwischen zwayen parallel linien stehn/seind ainander gleich.

Erklä



be regarded as having the same degree of validity as a geometrical demonstration, at least for the readership he was aiming.

In his *Explanation* (*Erklärung*), Xylander justified the arithmetical treatment of this theorem, and with this the numerical treatment of magnitudes in general in his commentary on Euclid, by asserting the necessity of arithmetic for geometry, especially in its practical form (i.e. the type of geometry resorting to instruments and measurements), as for all arts in general. With this, Xylander took care to insist on the importance of arithmetical operations on surd numbers and binomials, in spite of their difficulty or apparent uselessness for certain readers. By stating this, Xylander marked the connection between his interpretation of Euclid's *Elements* and the measurement procedures taught in practical geometry treatises, which often involved irrational numbers. This also legitimized the rather frequent and sometimes lengthy explanations of arithmetical and algebraic rules Xylander provided in the commentary on some of Euclid's propositions.

In this regard, in Prop. II.4, which is also practically only treated numerically,¹²⁴ with no consideration for the classical Euclidean proof (contrary to how it was treated in Scheubel's Euclid),¹²⁵ Xylander provided in the appendix an explanation of the uses and applications of the proposition in the art of counting¹²⁶ with various numerical examples. The rules and operations taught in this framework explain 1) how to extract square roots (featuring examples with crossed-out numbers),¹²⁷ 2) how to find the next larger or smaller square number,¹²⁸ 3) how to find squares roots of binomials,¹²⁹ 4) how to add surd numbers,¹³⁰ and how to solve the equation corresponding to 'a square + a root = a number' (or in modern terms $ax^2 + bx = c$),¹³¹ which (as was already shown by Scheubel) is founded on the geometrical relation expressed in Prop. II.4. In this framework, Xylander refers to Christoff Rudolff's *Coß*, notably in the section on equations, where the cossic notations

¹²⁴ Ibid., II.4, 48–49.

¹²⁵ Interestingly, it contains a *Demonstration* section, but this one merely points to an alternative way of considering the dimensioned parts of the diagram (ibid., 49). Scheubel, on the contrary, generally maintained the classical geometrical proof before providing the additional arithmetical or algebraic content.

¹²⁶ Ibid., II.4, 50: "I. Anhang, darinn der nutz und gebrauch diser prop. In der rechenkunst ettlicher massen erklertt."

¹²⁷ Xylander, *Die sechs erste Bücher Euclidis*, II.4, 50: [in margine] "Wie man radicem quadratam ainer zal finde."

¹²⁸ Ibid., 51: "II. Anhang das quadrat zûmehren und mindern."

¹²⁹ Ibid., 52: "III. Von Binomischen quadraten und wurzeln."

¹³⁰ Ibid., 53: "IV. Surdisch addition."

¹³¹ Ibid.: "Demonstration. Der vergleichung in der Coß, oder regel Algebre, da under dreyen quantiteten natürllicher ordnung, die zwo grössern der klainern vergleicht werden, als $z + \mathfrak{z}$ gleich ainer ledigen zal."

for roots and powers derived from this tradition (such as $\mathfrak{z}$, $\mathfrak{e}$, $\mathfrak{z}$, $\mathfrak{j}$, $\mathfrak{w}$) appear.¹³² Rudolff's *Coß* was actually the work which Xylander cited the most in his commentary on Euclid.¹³³ In addition to Rudolff, he also quoted Stifel's *Arithmetica integra*,¹³⁴ Scheubel's German edition of Euclid's arithmetical books¹³⁵ and the *Rechenbücher* of Peter Apian, Adam Ries and Simon Jacob.¹³⁶

In line with this tradition, Xylander's numerical examples did not only include whole numbers, but also fractions and surd numbers or, as he also called the latter, 'unnatural numbers' (*unnaturliche Zahlen*).¹³⁷ Although Xylander, in his commentary on Prop. II.11,¹³⁸ wrote that surd numbers are not to be considered as numbers in their own right, at least in the context of the *Elements*,¹³⁹ he nevertheless asserted their usefulness for dealing with propositions (such as Prop. II.11) which involve incommensurable magnitudes,¹⁴⁰ providing a numerical example in which he applied a rule derived from Rudolff's *Coß*.¹⁴¹ This contrasted with the commentators of the *Elements* (such as Campanus and Clavius¹⁴²) who, while proposing a numerical expression of Prop. II.1–10, presented Prop. II.11 as inexpressible through numbers (i.e. through whole numbers) and therefore entirely left aside its arithmetical translation. Xylander's attitude in this regard is comparable to that

¹³² Ibid.: "Diese Equation oder vergleichung, ist die fünfft bei dem wolgedachten Christoff Rudolffen in seinem Coß büch."

¹³³ E.g., *ibid.*, I.47, 40: "durch rechnung der Coß, wie das Christoff Rüdolff;" II.5: "als du in den exempeln Rudolffs sechen magst" or introd. Book V, 118: "magst sy finden in Christoff Rüdolff Coß."

¹³⁴ Ibid., II.4, 52: "wie dan sollichs der hochgelert herr Michael Stiffel in seiner herrlichen volkhommen Lateinischen Arithmetic fein angezaigt."

¹³⁵ Ibid., I.41, 30: "Die weil aber der Hochgelert und kunstreich M. Joann Scheübel, diser künsten professor zû Tübingen, solliches zû end des 9 büchs Euclidis (das er sambt dem 7 und 8 verteütscht in druckh gegeben) mit schönen bericht und exempelen gethon, wil ich dem Leser da selbst solliches zûerholen befelchen, unnd die 42 proposition für die hand nehmen."

¹³⁶ Ibid., VI.14, 168: "Gib ich dir auß den exempeln, so du inn den Teütschen rechenbüechlin Christoff Rudolffs, Apiani, Adam Risen, Simon Jacobs, unnd andern hast."

¹³⁷ Ibid., Df. V.3, 121.

¹³⁸ Heath, *The Thirteen Books*, vol. 1, 402: "To cut a given straight line so that the rectangle contained by the whole and one of the segments is equal to the square on the remaining segment."

¹³⁹ Ibid., II.11, 64: "die Surdisch oder Irrational quantiteten kains wegs zalen mögen sein noch genennt werden [...] *Euclides selbst* imm zehenden büch vilmals zûverstehen gibt."

¹⁴⁰ Ibid.: "Aber in surdischen, als Residuis und Irrational zalen (also zûsprechen) mag das wolgeschehen."

¹⁴¹ Ibid.: "du dessen in Christoff Rüdolffs Coß, bei der sechßten regel das 7 exempel, hast."

¹⁴² This is also the case of many later commentators who followed Clavius' commentary, such as Pierre Le Mardelé (1620), André Tacquet (1654), John Leake and George Serle (1661) or Heinrich Coets (1691).

of Jacques Peletier (1557), who accepted Campanus' statement of the impossibility of expressing Prop. II.11 numerically (again, through whole numbers),¹⁴³ while nevertheless teaching how to deal with it by means of irrationals, referring to rules set out in his 1554 treatise on algebra.¹⁴⁴ By contrast with all these authors, Scheubel directly dealt with Prop. II.11 numerically without discussing the status of irrationals. The case of Prop. II.11 thus shows that, despite persisting conceptual limits with respect to the notion of number as defined by Euclid, the introduction of methods drawn from practical arithmetic and cosmic algebra in commentaries on the *Elements* contributed to the transformation and expansion of the concept of number in the sixteenth century.

It is noteworthy in this regard that, in Book V, Xylander did not hesitate to express ratios as fractions.¹⁴⁵ Through this treatment, ratios are no longer merely conceived as a relation between quantities but as numbers in their own right,¹⁴⁶ which may themselves be operated on, even if Xylander admitted that it was not proper to Euclid's approach to ratios.¹⁴⁷ This conception is presented as particularly useful for explaining Euclid's Df. V.7,¹⁴⁸ which, together with Df. V.5, defined the equality and inequality of ratios through the concept of equimultiples and raised many difficulties in the early modern period due to its complexity, obscurity and indirectness, given that it relies on comparisons between multiples.¹⁴⁹

In the context of these definitions, which, in Xylander's Euclid, are placed later in the order of Book V's definitions (most likely due to their complexity¹⁵⁰), and in which the term 'magnitude' is replaced by that of 'number,'¹⁵¹ the use of numerical examples enabled

¹⁴³ Jacques Peletier, *In Euclidis Elementa geometrica demonstrationum libri sex* (Lyon: Jean de Tournes and Guillaume Gazeau, 1557), II.11, 60.

¹⁴⁴ Peletier, *L'Algèbre* (Lyon: Jean de Tournes, 1554).

¹⁴⁵ Xylander, *Die sechs erste Bücher Euclidis*, Df. V.3, 122.

¹⁴⁶ Rommevaux, "Aperçu sur la notion de dénomination," Malet, "Euclid's swan song."

¹⁴⁷ Xylander, *Die sechs erste Bücher Euclidis*, Df. V.9, 127: "Darbey du dich alwegen der regel und vortailen der bruchrechnung hast zübehelffen, aber Euclides hatt es nit mögen durch das dividieren beschreiben [...]."

¹⁴⁸ Ibid.

¹⁴⁹ See Df. V.5, in Heath, *The Thirteen Books*, vol. 2, 114: "Magnitudes are said to be in the same ratio, the first to the second and the third to the fourth, when, if any equimultiples whatever be taken of the first and third, and any equimultiples whatever of the second and fourth, the former equimultiples alike exceed, are alike equal to, or alike fall short of, the latter equimultiples respectively taken in corresponding order." On the difficulties raised by these two definitions, and by Euclid's theory of ratios and proportions more generally, in the early modern period, see Enrico Giusti, *Euclides Reformatus. La teoria delle proporzioni nella scuola galileiana* (Turin: Bollati Boringhieri, 1993).

¹⁵⁰ Dfs. V.5 and 7 correspond respectively to Dfs. V.8 and 9 in Xylander, *Die sechs erste Bücher Euclidis*, 125–127.

¹⁵¹ Ibid.

Xylander to explain these two definitions more easily by means of operations on numbers. This is visible in the very formulation he used for Df. V.5, in which Euclid's comparison of equimultiples is directly expressed as a demonstration through numerical operations, before being illustrated by different numerical examples with varying equimultiples:

That four numbers are proportional to one another, and namely, that the first is to the second as the third to the fourth, is demonstrated as follows: if one takes the first and the third an equal number of times (that is, multiplies them by a given number) and afterwards also takes the second and the fourth an equal number of times, then the products of the first and third are either equal to the products of the second and fourth, or they are greater or smaller in the same proportion.¹⁵²

The numerical interpretation of ratios in Book V, through which (as said) ratios are not only taken as relations between whole numbers (e.g. 3 to 4 or 3:4), but also as quotients (e.g. $\frac{3}{4}$),¹⁵³ is justified in the commentary on the definition of ratio (Df. V.3), a definition Xylander expanded to include both numbers and magnitudes (*Zahl oder Größe*),¹⁵⁴ by stating that “number underlies all things, and therefore all of their comparisons may, and indeed must, be expressed through numbers.”¹⁵⁵ In the commentary on Df. V.4, where Euclid's term ‘magnitude’ is then rather replaced by the more general term ‘quantity’ (as in Scheubel's Euclid),¹⁵⁶ Xylander also expanded the objects of Book V to include ratios involving irrational numbers, stating that “although the proportion between these two lines cannot be expressed except by irrational numbers such as 9 and $\sqrt{162}$, they nevertheless bear a proportion to one another.”¹⁵⁷

These examples again confirm that Xylander, although cautious about the status of

¹⁵² Ibid., Df. V.8, 125.

¹⁵³ The term ‘quotient’ appears in the commentary on Df. V.9 (= Df. V.7), *ibid.*, 127: “sind die *quotienten* gleich.”

¹⁵⁴ Ibid., Df. V.3, 120: “When two things of the same kind are compared with one another, or estimated, by their number or magnitude (or what will be taken or understood as number and magnitude), such a comparison or confrontation is called a ratio.” Cf. Heath, *The Thirteen Books*, vol. 2, 114: “A ratio is a sort of relation in respect of size between two magnitudes of the same kind.”

¹⁵⁵ Ibid., Df. V.3, 121: “die zal ann allen dingen hanget, auch derwegen alle deren vergleichungen durch zalen mögen, ja müssen außgesprachen werden.”

¹⁵⁶ Ibid., Df. V.4, 123: “Quantities have a proportion with one another when one of them, taken repeatedly or multiplied, can exceed the other.” Cf. Heath, *The Thirteen Books*, vol. 2, 114: “Magnitudes are said to have a ratio to one another which are capable, when multiplied, of exceeding one another.” On comparable adaptations of Euclid's definition of ratio, see Malet, “Renaissance notions.”

¹⁵⁷ Ibid.: “Wiewol diser zwo linien proportion nit mag außgesprachen werden, dann allein mit unnatürlichen zalen, als 9 und $\sqrt{162}$. Dannocht sind sy gepportioniert.”

numbers and their relation to magnitudes in the *Elements*, contributed (like Scheubel) to the transformation of the concept of number and ratio from within the Euclidean tradition.

With regard to Xylander's motivations for his numerical approach to Euclid's geometrical propositions, Book V shows that this treatment first had a pedagogical aim, for after asserting that "number underlies all things," he wrote that "anyone can easily understand that in explaining these books and its whole subject, nothing is more convenient than presenting and clarifying examples by means of numbers."¹⁵⁸ For this reason, Xylander made no effort, unlike Scheubel, to provide both geometrical and numerical examples in this framework, as he considered it a useless repetition: "It would often be unnecessary to deal with such things through lines or figures, since they must in any case be tested and explained by numbers."¹⁵⁹

Within the numerical examples used in his commentary, in which Xylander constantly addresses the reader in the second person as a master would address his pupil¹⁶⁰ (a rhetorical strategy often employed in early modern practical mathematical treatises), the numerical example is also presented as a means for the reader to experience the truth of the proposition first-hand, justifying the fact of leaving aside the classical Euclidean demonstrative reasoning.¹⁶¹

In the above-mentioned Prop. I.35, this approach is furthermore presented as useful and recreative: "*An appendix*. If you want to experience the truth and certainty of this proposition and of those that follow through numerical computation (*which is very useful, beside being pleasant and entertaining*), it would be necessary for you to know how to find the area, content or magnitude of each figure, provided that the lengths of the lines that enclose them are known to you."¹⁶²

In this context, the recreativeness of the numerical translation of the geometrical proposition was most likely related to the practical or operative character of this approach, notably marked in Xylander's Euclid by the designation of arithmetical or

¹⁵⁸ Ibid., Df. V.3, 121: "Dieweil nun dem also, kain ain yetlicher leichtlich verstehn, das zûerklären dise bücher unnd den gantzen handel nichts bequemer ist, dan das die exempel alle durch zalen fürgeschriben, und erleüttert werden."

¹⁵⁹ Ibid.: "Dann es zum wenigsten unvonnöten, solches durch linien oder figuren zûverrichten, welche doch mit zalen probiert und erklärt müesten werden."

¹⁶⁰ E.g., ibid., I.45, 34: "*Magstu* durch rechnung beweren" or II.4, 50: "wirt *dich* das aussprechen der zalen lehren."

¹⁶¹ E.g., ibid., I.35, 22; I.41, 30 or I.47, 38.

¹⁶² Ibid., I.35, 22: "*Ain anhang*. So du die warhait und gwiß diser und volgender Prop. durch rechnung in zalen erfahren wölltest (*wellichs seer nutzlich zû dem das es kurtzweilig und lustig ist*) thett nott das du einer yeden figure feldung oder inhalt oder begriff wißtest zefinden, so dir die lenge der linien, darmit sy beschlossn, bekhannt wer."

algebraic rules as ‘practices’ (*Practick* or *practica*).¹⁶³ For in early modern treatises of practical arithmetic or geometry, the fact of performing computations on specific numerical data or manipulating measuring instruments (even if merely in the imagination) to solve hypothetical concrete problems was regularly presented as enjoyable and entertaining, and in some cases as a pleasant, easier, and more meaningful way for the lay reader to understand the theoretical principles of the teaching.¹⁶⁴ The recreativeness of such examples was underlined in works held as belonging to ‘mathematical recreations,’ as Leon-Battista Alberti’s *Ludi matematici*,¹⁶⁵ in which measuring practices and computations represented a means to solve intellectual puzzles, in addition to offering useful solutions to concrete problems.

In Xylander’s version of Prop. I.35, the usefulness of the numerical treatment of the proposition is related to its uses for measuring areas, notably in the context of utilitarian activities such as the tiling of the floor of a room, as marked by the example provided in this context.¹⁶⁶ In other propositions, most of all in Books II and V, this numerical approach to geometrical propositions is presented as useful to understand certain rules of practical arithmetic and algebra¹⁶⁷ and also, through the application of the latter to concrete problems, it is presented as useful to the practice of trade, banking or book-keeping.¹⁶⁸

Unlike Scheubel, Xylander did not however publish a treatise of arithmetic or algebra together with his edition of Euclid and thus referred to the concrete uses of the taught computational procedures (relating to commerce, currency conversions, financial partnerships and measuring practices) directly within his commentary on certain propositions. This is notably the case in the commentary on Prop. V.16,¹⁶⁹ in which several practical examples are brought forward to illustrate the rule of three,¹⁷⁰ which was a fundamental procedure for commercial arithmetic. Hence, the whole commentary on this proposition is committed to the teaching of the rule of three, as is indicated by its subtitle: “Use of this proposition. Foundation and practice of the rule of three.”¹⁷¹ In this framework, Xylander

¹⁶³ E.g., *ibid.*, II.1, 46; Book V, 118 or V.16, 146.

¹⁶⁴ See, for example, Oronce Fine, *Geometria libri duo*, in *Protomathesis* (Paris: Gerard Morrhe and Jean Pierre, 1532), 64r: “practicis geometricarum subtilitatum exercitamentis, novimus plerumque delectari” or Leonard and Thomas Digges, *A geometrical practise, named Pantometria* (London: Henry Bynneman, 1571), A2r: “the reader shall not a little delighte himselfe with the finenesse and subtiltie of their inventions.”

¹⁶⁵ Kim Williams *et al.*, *The Mathematical Works of Leon Battista Alberti* (Basel: Springer, 2010).

¹⁶⁶ Xylander, *Die sechs erste Bücher Euclidis*, I.35, 22.

¹⁶⁷ E.g., *ibid.*, II.1, 46; V.15, 142 or V.16, 143.

¹⁶⁸ E.g., *ibid.*, Df. V.3, 122 or VI.4, 158.

¹⁶⁹ *Ibid.*, 143. Heath, *The Thirteen Books*, vol. 2, 164: “If four magnitudes be proportional, they will also be proportional alternately.”

¹⁷⁰ Xylander, *Die sechs erste Bücher Euclidis*, V.16, 143–146.

¹⁷¹ *Ibid.*, 143: “Nutz diser proposition. Grund und practick der regel De tri.”

referred to specific currencies and measurement units (as *Ellen, Pfund, Gulden, Kronen, Heller...*), using the symbols commonly employed in *Rechenbücher*, as fl. and ß to represent Guldens (or Florins) and Schillings.¹⁷²

Xylander's Euclid thus came closer, in its style (in addition to its language), to Scheubel's 1555 edition of Euclid's arithmetical books and to vernacular handbooks of practical mathematics than to Scheubel's Latin Euclid.¹⁷³ It is notable in this respect that Xylander's commentary on Euclid was quoted by Jan Pieterszoon Dou¹⁷⁴ as one of the main sources of his Dutch translation of Euclid, first published in 1606,¹⁷⁵ along with Dou's own treatises of practical geometry (*Practijck des Lantmetens* and *Van het gebruyck der Geometrische instrumenten*), which he wrote with the surveyor, cartographer and engineer Johann Sems.¹⁷⁶

The practical character of Scheubel's and Xylander's numerical treatment of Euclid

The numerical approach to magnitudes proposed in Scheubel's and Xylander's commentaries on Euclid conferred a practical character on these works, whereby they diverged from classical expositions of the *Elements*. Through their teaching of rules for the computation of the areas of figures, they related to practical geometry treatises that dealt with the measurement of surfaces, notably in concrete situations (e.g. the measurement of fields or the division of estates).¹⁷⁷ They also related to handbooks of practical arithmetic and cosmic algebra through their inclusion of various arithmetical and algebraic rules (particularly in the tradition of Rudolff's *Cosß*) in the commentary on certain propositions, such as Prop. II.4. Their link to these traditions is marked in particular by the references made by Scheubel and especially Xylander to the usefulness of Euclid's geometry for practitioners such as surveyors and merchants.

In early modern treatises of practical mathematics, the practical character of the teaching resides not only in its attention to the concrete uses of geometry and arithmetic, but also in its focus on the operations (computational or instrumental) required to perform measurements or constructive procedures, as opposed to the demonstration of their sci-

¹⁷² Ibid., 145–146.

¹⁷³ It is interesting to note that, even if Xylander's Euclid was published after Scheubel's two printed editions of Euclid, Scheubel owned a copy of this work. See Hughes, "The Private Library," 422: "Xylander in sex priores libros Euclidis, deutsch."

¹⁷⁴ Jan Pieterszoon Dou, *De ses eerste Boucken Euclidis, Van de beginselen ende fundamenten der Geometri* (Leiden: Orlers, 1607), A4r–v.

¹⁷⁵ Dou's other source was the French translation of Euclid by the architect and engineer Jean Errard de Bar-le-Duc (1598). Ibid., A4r.

¹⁷⁶ Ibid., A2r–v.

¹⁷⁷ Scheubel's library actually counted practical geometry treatises. Hughes, "The Private Library."

entific validity (let alone the consideration of the properties of the measured or constructed objects). Now, it is primarily in this operative sense that Scheubel and Xylander used the expressions *Numerorum praxis* and *Practick* to designate commentary sections that teach computational rules or propose a numerical translation of Euclid's propositions, these sections teaching the different steps of the computational processes required for the determination of the areas of given or sought magnitudes.

The practical character of Scheubel's and Xylander's approach to Euclid's geometry is also reflected in their use of a multitude of different numerical examples to teach arithmetical or algebraic rules or to express certain propositions numerically. The application of a given computational procedure to multiple examples, involving different values and types of quantities (integers, fractions, irrationals, binomials), would allow the reader to empirically grasp its mode of operation and memorise its different steps through repetition, verifying all the while its validity for all the cases considered, as was often done in abacus handbooks and German *Rechenbücher*.¹⁷⁸ When used to 'demonstrate' a Euclidean proposition numerically, as Xylander chose to do for Prop. I.41,¹⁷⁹ multiple numerical examples would thus enable one to inductively derive its truth or validity from the repeated confirmation that the stated quantitative relation (e.g. the equality of any triangle to half of the parallelogram set on the same base and between the same parallels in Prop. I.41) holds for any of the proposed cases.

The empirical character of the numerical translation of Euclidean definitions or propositions was actually acknowledged by Niccolò Tartaglia, a mathematician and engineer who taught in a *scuola d'abbaco* and who published in 1543 the first vernacular edition of the *Elements*,¹⁸⁰ primarily addressing (like Xylander) a lay audience.¹⁸¹ As he observed in his commentary on Book V, "often the student who sees the provided proposition verified by the experience of numbers does not trouble himself to understand it by demonstration."¹⁸² Although Tartaglia then intended to warn the reader that reliance on numerical examples may lead to errors if not sustained by geometrical reasoning,¹⁸³ he acknowledged thereby the value of numerical examples to facilitate the comprehension of propositions

¹⁷⁸ Høyrup, *The World of the Abbaco*.

¹⁷⁹ Xylander, *Die sechs erste Bücher Euclidis*, 27–30. See *supra*, n. 114, 118 and 161.

¹⁸⁰ Tartaglia, *Euclide*. On Tartaglia's life and work, see Arnaldo Masotti, "Tartaglia (also Tartalea or Tartaia) Niccolò," *Dictionary of Scientific Biography*, vol. 13, ed. Charles Coulston Gillispie (New York: Charles Scribner's Sons, 1976), 258–262.

¹⁸¹ The fact that vernacular translations of Euclid such as those of Tartaglia and Xylander were addressed to early students, laymen or practitioners is indicated in the title of their works. For Tartaglia, see *Euclide*, title page: "ogni mediocre ingegno, senza la notitia, over suffragio di alcun'altra scientia con facilità, sera capace à poterlo intendere."

¹⁸² Tartaglia, *Euclide*, Df. V.11, 64v: "spesse volte il studente che vede con la esperienza de numeri verificarse la propositione preposta, non si cura di intendere quella per demonstratione."

¹⁸³ *Ibid.*, Df. V.11, 64v.

often too complex for beginners, a benefit acknowledged by later commentators of Euclid, as Didier (or Denis) Henrion, who defended numerical examples as means of abbreviating lengthy and obscure demonstrations.¹⁸⁴ Now, in Tartaglia's commentary on Euclid, the use of numerical examples, even accompanied by geometrical demonstrations, was explicitly associated with the approach of practitioners.¹⁸⁵

Yet, despite their connections with treatises of practical mathematics, Scheubel's and Xylander's commentaries on Euclid represented expositions of the *Elements* in their own right. This is evident in their general observance of the list and order of Euclid's principles and propositions found in the classical editions of the *Elements*, even if minor changes were sometimes introduced for pedagogical purposes.¹⁸⁶ The Euclidean origin of the principles and enunciations of propositions, although often reformulated or expanded by Xylander by the addition of synonyms or complementary information to facilitate their comprehension, remains clearly recognisable. In Scheubel's case, the classical geometrical proofs are generally maintained, though sometimes abbreviated or supplemented by more mechanical or empirical reasonings, as in Prop. I.3. Similar tendencies appear, though less frequently, in Xylander.¹⁸⁷ Xylander's and Scheubel's commentaries on Euclid, therefore, cannot be reduced to treatises of practical geometry or to *Rechenbücher*, whose traditions, contrary to that of the *Elements*, were not built around a unique and common source.¹⁸⁸ Nevertheless, these works show how, in the sixteenth century, certain commentaries on Euclid's brought together features traditionally associated with both theoretical and practical mathematical traditions.¹⁸⁹

Scheubel and Xylander's Euclid and the Protestant universities of Southwest Germany

What does Scheubel and Xylander's approach to Euclid's *Elements* say about the type of mathematical teaching and practice that was promoted in the cultural and institutional context in which they were active? Indeed, if Scheubel and Xylander present compara-

¹⁸⁴ Didier (or Denis) Henrion, *Les quinze livres des Elements geometriques d'Euclide* (Paris: Isaac Dedin, 1632), II.1, 83.

¹⁸⁵ Tartaglia, *Euclide*, X.47, 154r–v: “*da pratici se descrivera in questa forma 12 piu 63.*”

¹⁸⁶ E.g. the reduction of the postulates to Euclid's first three postulates (in both Scheubel and Xylander) or the change in the order of the definitions of Book V in Xylander. The former was often found in other editions of Euclid published at the time.

¹⁸⁷ Propositions that remain overall conform in their structure to the classical Euclidean proofs, such as Prop. I.1, were not considered here as they are less relevant to the object of this study.

¹⁸⁸ This was already made clear by Hugh of St Victor at the beginning of his twelfth-century *Practica geometriae*. Frederick A. Homann, *Hugh of St Victor. Practical geometry = Practica geometriae* (Milwaukee: Marquette Univ. Press, 1991), 33.

¹⁸⁹ On this phenomenon, see Axworthy, “The Hybridization.”

ble treatments of Euclid's *Elements*, these authors are also comparable in their geographical and cultural origins, as well as in their professional and institutional status. They were indeed both born in the southwestern region of present-day Germany (Scheubel in Kirchheim unter Teck and Xylander in Augsburg) and both ultimately held a professorship in mathematics at a university situated relatively near their birth places.¹⁹⁰ Furthermore, at the time, each of these institutions adopted comparable pedagogical, institutional and religious principles. For although the University of Heidelberg was founded by Pope Urban VI prior to the Reformation and was initially hesitant to adopt humanist culture and pedagogical practices, unlike the University of Tübingen, established in 1477 by promoters of Humanism,¹⁹¹ both institutions had, by the mid-sixteenth century, become active centres for the development of humanist learning and for the promotion of Protestantism.¹⁹²

As was asserted by Paul Grendler, there was a strong connection between the birth and spread of Protestantism and the humanist intellectual life of reformed German universities.¹⁹³ The University of Heidelberg, though oscillating at the time between Lutheran and Calvinist doctrines, was the second most important university for the Protestant Reformation after Martin Luther's University of Wittenberg. And the University of Tübingen, where both Scheubel and Xylander studied and/or taught, as did many renowned sixteenth- and seventeenth-century German mathematicians such as Johannes Stöffler, Michael Mästlin, Johannes Kepler and Wilhelm Schickard, also played a considerable role in this regard.¹⁹⁴ Gerard Betsch has, for that matter, noted the frequent correlation between the interest in mathematics (and especially in practical mathematics) from these Tübingen students or professors and their involvement in

¹⁹⁰ There is about 40 km between Kirchheim and Tübingen and about 250 km between Augsburg and Heidelberg.

¹⁹¹ I.e. Count Eberhard im Bart (later first Duke of Württemberg) and his mother Mechthild von der Pfalz.

¹⁹² James H. Overfield, *Humanism and Scholasticism in Late Medieval Germany* (Princeton: Princeton University Press, 1984); Thomas A. Howard, *Protestant Theology and the Making of the Modern German University* (Oxford: Oxford University Press, 2006), Chap. 2; Ulrich Köpf, *Die Universität Tübingen und ihre Theologen* (Tübingen: Mohr Siebeck, 2020).

¹⁹³ Paul Grendler, "The Universities of the Renaissance and Reformation," *Renaissance Quarterly* 57, 1 (2004): 1–42.

¹⁹⁴ Gerard Betsch, "Praxis geometrica und Kartographie and der Universität Tübingen um 16. und frühen 17. Jahrhundert," in *Zum 400. Geburtstag von Wilhelm Schickard: zweites Tübinger Schickard-Symposium*, ed. by Friedrich Seck (Thorbecke: Sigmaringen, 1995), 185–226; Id., "Südwestdeutsche Mathematici aus dem Kreis um Michael Mästlin," in *Der „mathematicus“: zur Entwicklung und Bedeutung einer neuen Berufsgruppe in der Zeit Gerhard Mercators*, ed. by Irmgard Hantsche (Bochum: Brockmeyer, 1996), 121–150.

religious offices.¹⁹⁵ This could be partly explained by the predominant importance and interrelation (demonstrated again by Grendler¹⁹⁶) of the Faculty of Arts and the Faculty of Theology in Protestant universities in the sixteenth century, as well as by the important social cohesion that existed in this framework between students and professors, as between the universities and the local communities. For in reformed German universities, unlike Italian universities, not only were professors living among students, which fostered a form of solidarity between the former and the latter, but German students most often only aimed to obtain a bachelor, in order to teach in local schools or work in local administrations, which contributed to bind the needs of the university with those of the city. This situation did not so much exist in Italian universities, at least in the first half of the sixteenth century, where intellectual competition was much stronger and where the various faculties, their members, and the local communities, did not share the same social connection. Also, contrary to the students of reformed German universities, a bachelor's degree was generally not offered in Italian universities. These rather prepared students for the doctorate (especially in law and medicine), or at least for the Master's degree.

In this regard, Scheubel's biographers surmised that it is very likely due to his Lutheran convictions that he left the University of Leipzig to pursue his studies in Tübingen in 1535,¹⁹⁷ as the Duchy of Württemberg (and with it the University of Tübingen¹⁹⁸) had officially turned to Protestantism in 1534, while Leipzig did so only in 1539. It should be noted for that matter that the paratext of his *De numeris et diversis rationibus seu regulis computationum opusculum* from 1545 counts a short versified poem by Philipp Melanchton,¹⁹⁹ who played a key role in the reformation of the University of Tübingen.²⁰⁰

Xylander, who was himself in contact with Melanchton, Ulrich Zwingli and Thomas Erastus (among other important Protestant thinkers), studied philosophy at Tübingen under the Lutheran Aristotelian philosopher Jakob Schegk.²⁰¹ Furthermore, Xylander held in 1564 the function of secretary for the assemblies called at the monastery of Maulbronn to discuss the various points debated by the Lutheran and Calvinist doctrines and again in 1571 at the colloquium with the Anabaptists in Frankenthal. It may also be noted that the Protestant Parisian royal lecturer in mathematics Petrus Ramus

¹⁹⁵ Betsch, "Südwestdeutsche Mathematici."

¹⁹⁶ Grendler, "The Universities."

¹⁹⁷ Staigmüller, "Johannes Scheubel," 435; Day, *Scheubel as an Algebraist*, 15; Reich, *500 Jahre Johann Scheubel*, 66.

¹⁹⁸ Richard L. Harrison Jr., "Melanchthon's Role in the Reformation of the University of Tübingen," *Church History* 47/3 (1978): 270–278.

¹⁹⁹ Scheubel, *De numeris*, i6v.

²⁰⁰ Harrison, "Melanchthon's Role."

²⁰¹ Gall, "Xylander" and Schöll, "Xylander."

recommended Xylander for the chair of mathematics at Heidelberg in 1561.

Moreover, even if Xylander had started working on his Euclid in 1555, and thus before he started teaching at the University of Heidelberg, it was finalised and published during his lectureship at this institution. As a former student of the University of Tübingen, who possibly studied under Scheubel in 1549–1550, he may have been inspired to undertake this translation of the *Elements* by the latter's lectures, as by the publication of his German edition of Euclid's arithmetical books in 1555, in addition to the desire to support the artisans, merchants and bankers of Augsburg, among other German cities.

Scheubel and Xylander therefore fully instantiate the intertwinement proper to the universities of Southwest Germany between the adoption of Protestantism, the promotion of humanist culture, and the efforts to cater to the needs of the local communities. Their interest in practical and applied mathematical knowledge, notably in the practices of German *Rechenmeister*, is coherent with the observations made by Pietro D. Omodeo concerning the mathematical curricula of the Protestant University of Helmstedt, where an important place was attributed to practical mathematical works, in particular the practical arithmetic of the Dutch professor of mathematics Gemma Frisius, which was published in Wittenberg, among other places.²⁰² We can also find a similar orientation in Johannes Sthen's pedagogically-oriented adaptation of Book VII of the *Elements* published in Wittenberg in 1564, which is presented as offering a foundation for the teaching of the rules of practical arithmetic.²⁰³ Even if it only proposes a very limited and superficial numerical treatment of the *Elements* in comparison with the editions of Scheubel and Xylander, it is one of the few sixteenth-century commentary on Euclid that compares to these in its explicit desire to connect the content of the *Elements* with the practices of common arithmetic.

To this must be added that Scheubel and Xylander's approach to Euclidean geometry is also coherent with the pedagogical model promoted by Ramus for mathematics, since the latter defended a definition of arithmetic and geometry as the arts of counting and measuring well (*doctrina bene numerandi* and *ars bene metiendi*)²⁰⁴ and transmitted theoretical principles of Euclidean geometry by combining them with a teaching on the

²⁰² Pietro D. Omodeo, "The German and European network of the professors of mathematics at Helmstedt in the sixteenth century," in *The Circulation of Science and Technology: Proceedings of the 4th International Conference of the ESHS*, ed. by Antoni Roca-Rosell (Barcelona: Societat Catalana d'Història de la Ciència i de la Tècnica, 2012), 294–301.

²⁰³ Johannes Sthen, *Arithmetices Euclideae*, title page: "*vera principia ac solidiora fundamenta Logistices, id est, ut vocant, Arithmetices Practicae*."

²⁰⁴ Petrus Ramus, *Arithmeticae libri duo; Geometriae septem et viginti* (Paris: André Wechel, 1569). It should be noted that this book was part of Scheubel's library. Hughes, "The Private Library," 422: "Petri Ramus arithmetica et geometria."

procedures of practical geometry.²⁰⁵ It is interesting to note for that matter that Ramus' *Algebra* (1560),²⁰⁶ which was taught and further developed by professors of German Protestant schools that followed Ramist pedagogical methods in the 1580s, drew much from Scheubel's *Algebrae descriptio*.²⁰⁷

Hence, the editions of Euclid's *Elements* by Scheubel and Xylander may be regarded as instantiating an approach to mathematics that was representative of sixteenth-century Protestant pedagogical reforms. This approach is not only defined by the fact of giving an important place to practical and applied mathematics (commercial arithmetic, surveying, instrument-making and the like) in the Arts curriculum, but also by the fact of mixing the geometrical principles and propositions of Euclid with the procedures of practical mathematics, making them thereby more accessible, meaningful and useful to students, conceived as members of their social community.

Conclusion

Scheubel and Xylander's numerical approach to the *Elements* holds a unique place in the sixteenth-century Euclidean tradition by the strong connection it made between Euclid's geometry and the practice of German *Rechenmeister*. Nevertheless, their editions of Euclid remained the works of scholars, who studied and taught at the university and were versed in ancient Greek and Latin sources. In their work, one thus finds a pedagogical model defended by Protestant humanists, which aimed to make ancient texts both accessible and useful to their readers, as members of their community.

Such an approach to Euclid would become much more frequent in the seventeenth century, notably among editions of the *Elements* published in Northern Europe, in areas which had adopted the doctrine of the Reformed Church, such as those of Lucas Brunn, Tobias Beutel, Heinrich Meißner in Germany,²⁰⁸ but also Christoff Dybvad in Denmark,²⁰⁹ Martin

²⁰⁵ Menghini, "From Practical Geometry."

²⁰⁶ Petrus Ramus, *Algebra* (Paris: André Wechel, 1560).

²⁰⁷ François Loget, "De l'algèbre comme art à l'algèbre pour l'enseignement: Les manuels de Pierre de La Ramée, Bernard Salignac et Lazare Schoner," *Revue de Synthèse* 132/4 (2011): 495–527. On Scheubel's influence on sixteenth-century algebra, see also Day, *Scheubel as an Algebraist*, 19–24 and Reich, *500 Jahre Johann Scheubel*, 79–82.

²⁰⁸ Lucas Brunn, *Euclidis Elementa Practica, Oder Außzug aller Problematum und Handarbeiten auß den 15. Büchern Euclidis* (Nürnberg: Simon Halbmayer, 1625); Tobias Beutel, *Geometrischer Lust-Garten. Darinnen die edele und höchstnützliche schöne Kunst Geometria, Aus den Euclide gepflantzet* (Leipzig: Christian Michael, 1660); Heinrich Meißner, *Des Gantzen, In 15 Büchern bestehenden, Teutschen Euclidis* (Hamburg: Wieringen, 1690).

²⁰⁹ Christoffer Dybvad, *In Geometriam Euclidis prioribus sex Elementorum libris comprehensam Demonstratio Numeralis* (Arnhem; Leiden: Christopher Guyot, 1603).

Gestrinius in Sweden,²¹⁰ Frans Van Schooten in the Dutch Republic²¹¹ or William Alingham in England,²¹² who proposed arithmetical and algebraic treatments of the geometrical books of the *Elements* combined with a practical approach to Euclid's propositions.²¹³ In comparison, very few seventeenth-century Italian commentators of the *Elements* followed a similar approach (with the exception of Pietro Antonio Cataldi²¹⁴) in spite of the great number of different editions published in Italy at this time.²¹⁵ This also concerns the expositions of the *Elements* of two disciples of Galileo, namely Giovanni Alfonso Borelli and Vincenzo Viviani, who maintained in this respect a rather theoretical approach to geometry, in spite of their attempts to revise or transform Euclid's *Elements*, particularly the theory of ratios contained in Book V, to make it more applicable to the apprehension of the physical world, in line with Galileo's own project of revision of Euclidean geometry.²¹⁶

To which extent the above-mentioned numerical Northern European editions of the *Elements* were determined by the Protestant background of their authors and may be placed in continuity with the editions of Scheubel and Xylander in this respect, as with regard to the specificities of their numerical approach to Euclid's geometrical books, would require a separate inquiry, particularly because many other factors need to be considered for the numerical treatment of Euclid's *Elements* in this period, such as the emergence of modern algebra and analytical geometry, together with the greater focus placed by mathematicians on problem-solving techniques (over the demonstration of theorems), the in-

²¹⁰ Martin Gestrinius, *In geometriam Euclidis demonstrationum libri sex* (Uppsala: Aeschillus Matthiae, 1637). On Gestrinius' algebraic approach to Euclid, see Johanna Pejlar and Staffan Rodhe, "On the relations between geometry and algebra in Gestrinius' edition of Euclid's *Elements*," in *History and Pedagogy of Mathematics. Proceedings of the 2016 ICME Satellite Meeting* (Montpellier: IREM de Montpellier, 2016), ed. by Luis Radford et al., 513–523.

²¹¹ Frans Van Schooten, *De propositiōen van de XV boucken der elementen Euclidis demonstratis* (Leiden: Govert Basson, 1617).

²¹² William Alingham, *Geometry epitomiz'd: being a compendious collection of the most useful propositions in the first, third, fourth, fifth and sixth books of Euclid. Together with their uses, in several practical parts of the mathematicks. Also, Euclid's second book and doctrine of proportion algebraically demonstrated. With some of the most useful problems required in practise* (London: J. Moxon and B. Beardwell, 1684).

²¹³ It is unfortunately not possible to propose an analysis of these sources in the framework of this study. An examination of this later tradition is forthcoming.

²¹⁴ Pietro Antonio Cataldi, *I primi sei libri de gl'Elementi d'Euclide ridotti alla Prattica, dove si mostrano le Inventioni delle Regole Geometriche, & Algebratiche necessarie, & di continuo uso* (Bologna: Sebastiano Bonomi, 1620) [First ed. 1613].

²¹⁵ There were more than twenty different editions of Euclid published in Italy during the seventeenth century.

²¹⁶ Vincenzo De Risi, "Euclid upturned: Borelli on the foundations of geometry," *Physis* 57/2 (2022): 1–23; Angela Axworthy, "Mathematics in the Accademia del Cimento: from a language of Nature to a language of Reason," *Physis* 59/2 (2024): 463–499.

tensification of mercantilism, the changing place and status held by practical mathematics in mathematical culture and university curricula, together with the expansion of mathematical instruction and numeracy beyond the university, as well as the greater ability of printers to cater to new mathematical languages and notations. It remains that the editions of the *Elements* by Scheubel and Xylander may be considered as having inaugurated a phenomenon that would become crucial to the reshaping of the Euclid's treatise, and thereby to the transformation of elementary geometry in early modern Europe, contributing to bring together arithmetic and geometry as well as theoretical and practical mathematics.

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Everything in its right place: Francesco Maurolico and the classification of the sciences

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Abstract

European Renaissance was characterized by an unprecedented growth of information determined by discoveries of ancient texts and distant places, technological advancements, and a new attitude towards human culture and history. This growth urged scholars to reconsider the traditional schemas used to organize knowledge, and in doing so they were more often than not put at odds with a past that they were trying simultaneously to recover and challenge. The case of Francesco Maurolico is a prime example of the tension between tradition and innovation that marked Renaissance thinking: fully committed to the recovery of ancient mathematical knowledge and acutely aware of his own original contributions to the discipline, throughout his career Maurolico sought to hammer out a model of classification that could accommodate new mathematical content in a largely traditional structure. By analyzing Maurolico's failed attempts at devising a satisfactory model, the essay aims to show how, during the Renaissance, mathematical research impacted culture at large, thus decisively contributing to the shaping of modernity.

Keywords

Maurolico, Renaissance mathematics, Classification of sciences

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I. Introduction

The European Renaissance, a period defined by the intellectual recovery of classical culture, simultaneously inaugurated a new epoch of cognitive strain. Driven by the dissemination of recovered Greek and Arabic texts, rapid technological advancements, and the unprecedented geographical expansion of the known world, the century witnessed an explosive proliferation of information. This intellectual deluge placed immense pressure on existing structures used to organize knowledge, fundamentally challenging the long-standing encyclopedic ambitions inherited from antiquity.¹

For centuries, the organization of theoretical philosophy remained anchored by an increasingly eclectic Aristotelian scholasticism, which persisted as the dominant framework in university settings. Aristotle's *Categories* provided the hierarchical model, rigorously separating knowledge domains based on their subject matter and/or epistemological status, with mathematics occupying a middle ground. Within this tradition, mathematics was often partitioned into the discrete (arithmetic) and the continuous (geometry) and grouped into the *Quadrivium*.² However, with the flourishing of mathematical practice, scholars across the 16th century increasingly advocated for the mathematization of natural philosophy, and this methodological shift gave rise to *media scientiae*: disciplines like optics, mechanics, and astronomy that defied easy classification by combining the certainty of mathematical demonstration with the mutable reality of sensible, physical matter,

¹ See Ann M. Blair, *Too Much to Know: Managing Scholarly Information Before the Modern Age* (New Haven [Conn.]: Yale University Press, 2010); Ann M. Blair, "Revisiting Renaissance Encyclopaedism," in *Encyclopaedism from Antiquity to the Renaissance*, ed. by Jason König and Greg Woolf (Cambridge: Cambridge University Press, 2013), 379–397; Daniel Andersson, "Philosophy and the Renaissance Encyclopaedia," in *Encyclopaedism from Antiquity to the Renaissance*, ed. by Jason König and Greg Woolf (Cambridge: Cambridge University Press, 2013), 398–413; Christel Meier, "Introduction," in *Die Enzyklopädie im Wandel vom Hochmittelalter bis zur Frühen Neuzeit*, ed. by Christel Meier (München: Fink [Münstersche Mittelalter-Schriften 78], 2002), 1–22; Ead., "Transformationen des Enzyklopädischen Wissensbegriffs," in *Die Enzyklopädie im Wandel vom Hochmittelalter bis zur Frühen Neuzeit*, ed. by Christel Meier (München: Fink [Münstersche Mittelalter-Schriften 78], 2002), 23–44; Matteo Valleriani, "From the Quadrivium to Modern Science," *HoST – Journal of History of Science and Technology* 16, no. 1 (2022): 121–132.

² Charles B. Schmitt, *Aristotle and the Renaissance* (Cambridge, MA: Harvard University Press, 1983), 89–109; Edward Grant, *The Foundations of Modern Science in the Middle Ages: Their Religious, Institutional, and Intellectual Contexts* (Cambridge: Cambridge University Press, 1996), 33–69; Winston Black, "The Quadrivium and Natural Sciences," in *The Oxford History of Classical Reception in English Literature: Volume 1: 800–1558*, ed. by Rita Copeland (Oxford: Oxford University Press, 2016), 77–94; Jens Høyrup, "Mathematics Education in the European Middle Ages," in *Handbook on the History of Mathematics Education*, ed. by Alexander Karp and Gert Schubring (New York: Springer, 2014), 109–124.

and proceeded not only by transforming existing mathematical tools but also creating new mathematical objects.³

The career of the Sicilian mathematician Francesco Maurolico (1494–1575) stands as a prime illustration of this profound epistemological crisis. Educated in the Humanist tradition, Maurolico dedicated his life to the ambitious reconstruction and correction of the Greek mathematical canon, most notably the foundational works of Archimedes, Euclid, and Theodosius. Simultaneously, his own original treatises introduced revolutionary content, such as the first known formal use of mathematical induction in arithmetic and the application of rigorous Archimedean statics to mechanics, effectively establishing a new geometric standard for natural philosophy. These innovations fundamentally blurred the Aristotelian distinction between discrete and continuous quantities, challenging the very metaphysical boundaries that structured the traditional Quadrivium.

This essay will analyze Maurolico's explicit attempts to organize his work, particularly through the presentation of multiple, inconsistent classification schemes in the preface to his 1575 *Opuscula Mathematica*. Maurolico's inability to settle upon a definitive, coherent model suggests that the content of 16th-century mathematical research had structurally surpassed the capacity of the traditional philosophical container. By examining Maurolico's failed ambition to devise a satisfactory classification, this analysis aims to illuminate how the burgeoning practice of mathematical science decisively impacted the broader culture of knowledge organization, thereby contributing to the intellectual conditions necessary for the shaping of modernity.

II. Maurolico, Archimedes, and the architectural ambition

Maurolico stands among the most intellectually ambitious figures of the Italian Renaissance. Educated within the late Humanist tradition, he devoted his career to restoring and expanding the Greek mathematical canon. The extensive editorial projects he undertook concerning the corpus of Greek mathematics were intended not merely as philological

³ Sophie Roux, "Forms of Mathematization (14th–17th Centuries)," *Early Science and Medicine* 15, no. 4 (2010): 319–337, 327. See also Pier Daniele Napolitani, "La géométrisation des qualités physiques: les modèles de la théorie des proportions," in *Between Mechanics and Architecture*, ed. by P. Radelet de Grave and E. Benvenuto (Basel–Boston–Berlin: Birkhäuser, 1995), 69–88; Maarten van Dyck, "Applying Mathematics to Nature," in *The Cambridge History of Philosophy of the Scientific Revolution*, ed. by David Marshall Miller and Dana Jalobeanu (Cambridge: Cambridge University Press, 2022), 254–273; Peter Damerow and Jürgen Renn, "The Transformation of Ancient Mechanics into a Mechanistic World View," in *Transformationen antiker Wissenschaften*, ed. by Georg Toepfer and Hartmut Böhme (Berlin; New York: De Gruyter, 2010), 243–268; Maurizio Crosilla, ed., *Guidobaldo del Monte (1545–1607): Mechanica, Statica, and the Emergence of Mathematical Physics* (Berlin: De Gruyter, 2015).

recoveries but as systematic reconstructions of mathematical knowledge according to rigorous demonstrative standards. His method, often designated *ex traditione Maurolyci*, involved providing new interpretations and integrating original classical materials with contributions accrued throughout the centuries.⁴

Since the beginning of his career, Maurolico cultivated a clear encyclopedic ambition: he not only pursued original mathematical research and the recovery of classical texts, but also envisaged a comprehensive, ordered architecture of his own intellectual output. As early as 1528, in the preface to his *Grammaticorum rudimentorum libelli sex*, he included a form of what would become his first *Index Lucubrationum* – a catalogue listing both his published and intended works. This catalogue signals that, from this early stage, he perceived his production as a systematic project rather than a series of isolated treatises.⁵ Over the decades, he repeatedly revised and expanded this index: a version appears in the 1543 *Cosmographia*,⁶ another in the 1558 *Sphaerica* edition,⁷ and still longer ones in the 1568 and 1570 manuscript listings prior to the published *Opuscula Mathematica* of 1575.⁸ These successive versions functioned not simply as bibliographical appendices but as strategic maps of his intellectual programme, showing planned works, compendia, and teaching-texts. Crucially, the *Index Lucubrationum* itself became a site of philosophical reflection; its hierarchical arrangement testified to the persistent struggle to impose an ordered structure upon the increasing diversity of mathematical disciplines. The structure of the index reflects the desired intellectual architecture: it records both editions of classical works and his own original treatises, placing them into a sequential scheme that was meant to embed his mathematical productions within the structure of a grand encyclopedia of sciences.

Maurolico's commitment to mathematics was not merely technical; it was a philosophical statement, articulated most clearly in his August 1556 letter to Juan de Vega, Viceroy

⁴ On Maurolico's life and work, see Rosario Moscheo, *Francesco Maurolico tra Rinascimento e scienza galileiana: materiali e ricerche* (Messina: Società Storia Patria Messina, 1988); Rosario Moscheo, "Francesco Maurolico," *Dizionario Biografico degli Italiani*, vol. 72 (Rome: Istituto della Enciclopedia Italiana, 2009), 404–411.

⁵ Francesco Maurolico, *Grammaticorum rudimentorum libelli sex Francisco Maurolycio auctore* (Messanae in freto Siculo: impressit Petrutius Spira, 1528), 7r–7v.

⁶ Francesco Maurolico, *Cosmographia* (Venetiis: apud haeredes Lucaeantonii Iuntae, 1543), *epistola dedicatoria*. The work was largely completed by 1535. See Giovanni Ciofarelli's introduction to *Cosmographia* in Maurolico Project, "Electronic Edition of the Mathematical Works of Francesco Maurolico," <https://maurolico.it/Maurolico/sezione.html?path=11.A.2> (accessed on 10/11/2025).

⁷ Francesco Maurolico, *Theodosii Sphaericorum elementorum libri III. Ex traditione Francisci Maurolyci* (Messanae: apud Petrum Spira, 1558), f. *3r.

⁸ Cfr. Marshall Clagett, "The Works of Francesco Maurolico," *Physis: Rivista internazionale di storia della scienza* 16 (1974): 148–198.

of Sicily.⁹ This letter serves as a defense of mathematics against the prevailing humanist contempt for the sciences. Maurolico laments the “calamity of our time” where scholars are more driven by *lucrum* and *ambitio* than by true study, resulting in intellectual chaos; the reading of *circulatorum ineptias*, and the practice of *barbaras voces et pueriles cavillationes* in dialectics.¹⁰ Against this decay in the liberal arts, Maurolico champions mathematics claiming that, *hac demonstrativa facultas*, “it secures the highest citadel in demonstrative worth, concludes without any controversy, and if it is neglected, nothing certain remains.”¹¹ For Maurolico, mathematical proof is the foundation of all secure knowledge, providing the absolute certainty that was lacking in the verbose and contentious fields of rhetoric and scholastic dialectics. Moreover, he explicitly positions his work as a direct corrective to contemporary innovation for its own sake. He rejects the practice of those *qui luxui vitisque... dediti, bonas artes parvifaciunt*,¹² and stresses his own humility, stating he would not dare to introduce some new things just for the sake of novelties, which most people do today. Instead, his priority is ensuring that *veterum authorum volumina quam correctissima potuissem habere* before adding his own, carefully vetted *emendationes*.¹³ In this context, Maurolico defines mathematics by its primary subject: quantity (*quantitas* or *magnitudo*), divided into discrete (*arithmetica*) and continuous (*geometria*). He then immediately expands this definition beyond the traditional scope of number and abstract figure. He notes that the principles of continuous quantity extend to *tempus, locus, motus*, and crucially, to *pondus, momentum ac vis*. These new mathematical species are the sub-

⁹ The letter, perhaps written as a dedication for the 1558 edition of the *Sphaerica*, is contained in manuscript *Par. Lat. 7473* (cc. 1r–16v). The letter has been printed several times, most recently in Rosario Moscheo, *I Gesuiti e le matematiche nel secolo XVI: Maurolico, Clavio e l'esperienza siciliana* (Messina: Società Messinese di Storia Patria, 1998), 287–306.

¹⁰ Bibliothèque Nationale de France, Ms. Lat. 7473, f. 1r. “Et deploranda simul nostri temporis calamitas, quando rarissimi sunt literarum cultores, atque ita omnia inversa, ut iam vix ulterius tolerandum sit, pro priscis praestantissimisque authoribus humanitatis, circulatorum ineptias legi, inter dialecticos barbaras voces et pueriles cavillationes frequentari, a caeteris quoque scientiarum professoribus, dum ad lucrum potius aut ambitionem quam ad laudabilem studii finem accelerant, illotis omnia manibus contrectari.”

¹¹ BNF, Ms. Lat. 7473, f. 2r. “Nam, vel ex hoc prae caeteris est magnificienda, quod primam in demonstrandi dignitate sortitur arcem, et citra omnem controversiam concludit, eaque neglecta nil certi superest.”

¹² BNF, Ms. Lat. 7473, f. 1r.

¹³ BNF, Ms. Lat. 7473, f. 1v. “Propterea, in studiis meis hoc praecipue curavi, ut veterum authorum volumina quam correctissima potuissem habere, deinde, si quid in his a tralatoribus bene traditum aut additum fuisset, id non omitterem. Demum, si quid ego quod ad correctionem aut faciliorem demonstrandi viam faceret excogitassem, id cum venia eruditorum traderem. Quibus peractis, aliquid incude nostra elaboratum, iudicioque consummatorum virorum prius expositum emitterem.”

jects of his own works *de momentis aequalibus* and *de mechanicis quaestionibus*.¹⁴

Most of this was driven by Maurolico's encounter, around ten years before, with the *editio princeps* of Archimedes' collected works. Published in Basel in 1544 by Ioannes Herwagen and edited by Thomas Gechauff, this monumental work shifted the focus from earlier, often incomplete or flawed medieval Latin versions to the authentic, rigorous methods of Greek geometry. For a scholar already engaged in reconstructing the geometrical rigor of the ancients, its appearance provided both methodological inspiration and a new point of comparison against which he could better discriminate between original contributions and recoveries of lost wisdom. Archimedes' corpus appeared to Maurolico as an ideal example of unified scientific architecture. His works linked geometry with physical mechanics through rigorous, deductive proofs. This provided a blueprint for Maurolico's encyclopedic project: the ideal classification scheme must be able to house such a unified structure.¹⁵

On the other hand, however, engaging with Archimedes made Maurolico become fully aware of how far he had been distancing himself from the ancients' approach. The case of *De momentis aequalibus* offers perhaps the clearest expression of this. Maurolico started working on it in the 1520s as a reconstruction of Archimedes' treatise on equilibrium; however, after consulting the 1544 edition, he realized that many of the methods and proofs he had formulated did not appear in Archimedes at all. They were original contributions, arising from his own geometric insight and reasoning, and he recognized that his

¹⁴ BNF, Ms. Lat. 7473, ff. 3r–3v. “Sunt etiam aliae quaedam continuae magnitudinis species, ut tempus, locus, motus, quae tamen ad spacia dimensionum referri possunt: metimur enim motum per spacium peractum, et tempus per motum. Item locus, cum sit corporis receptaculum, omnino eosdem quantitatis limites cum corpore suscipit. Item corpus ipsum sequuntur tres aliae continui species, videlicet pondus, momentum ac vis. Nam, corpora aequalia interdum sunt diversi ponderis, et aequalia pondera diversae magnitudinis. Item aequalia pondera nonnunquam sunt inaequalium momentorum, et aequalia momenta diversorum ponderum, ut patet in stateris. Vis autem est impetus incussi mallei aut securis, per quam saepe fit aliquid, quod neque per pondus neque momentum fieri potest. Ex quibus patet, corpus, momentum, pondus ac vim esse diversas species magnitudinum, de quibus agitur in libro de momentis aequalibus et in mechanicis quaestionibus. Item tam discreta quam continua quantitas aliis aut aliis applicata rebus aliam atque aliam generat scientiam, quae arithmeticae, ut calculus, rhythmica, musicae, aut geometriae subiacet, ut astronomia, geographia, chorographia, perspectiva, de quibus postea latius loquemur; quarum authores et prisci et recentiores multi sunt.”

¹⁵ Archimedes, *Archimēdous tou Syrakousiou, Ta mechri nun sōzomena, hapanta* = *Archimedis Syracusani philosophi ac geometrae excellentissimi opera, quae quidem extant, omnia, multis iam seculis desiderata...* (Basileae: Ioannes Hervagius excudi fecit, 1544). See Marshall Clagett, *Archimedes in the Middle Ages*, vol. III, part III (Philadelphia: The American Philosophical Society, 1978); Pier Daniele Napolitani, “Archimedes,” in *The Cambridge History of Science, Volume 3: Early Modern Science*, ed. by Katharine Park and Lorraine Daston (Cambridge: Cambridge University Press, 2006), 87–111; Paul Lawrence Rose, *The Italian Renaissance of Mathematics: Studies on Humanists and Mathematicians from Petrarch to Galileo* (Genève: Librairie Droz, 1975), 159–184.

own investigations exceeded what Archimedes had done.¹⁶ This realization reinforced his encyclopedic ambitions while simultaneously making the traditional classification problem more difficult. The inherited Aristotelian scheme created a strict hierarchy: Metaphysics (highest abstraction), Mathematics (abstraction from sensible matter, but not from quantity), and Physics (study of mobile, sensible matter). Maurolico's most advanced work, however, belonged to the *scientiae mediae* that violated this boundary: they applied the certainty of geometric deduction to the physical, mutable world of natural philosophy. Maurolico's challenge, therefore, was to create a classification system that could grant these *scientiae mediae* a high theoretical status without entirely subordinating them to the uncertain domain of physics, whose subject matter was characterized by *propter materiae fluxum*. His attempt to resolve this tension by devising multiple classification schemes became the central philosophical struggle of his later career.

Maurolico's scattered but recurrent attention to scientific ordering thus illustrates a key feature of Renaissance intellectual life: the tension between the proliferation of knowledge and the need for conceptual containment. As Ann Blair notes, even though "only a few late Renaissance figures openly rejected received classificatory schemes and the curricula to which they were related," there was a widespread acute sense that the surfeit of knowledge, whether driven by the discovery of things unknown to the ancients, the recovery of classical texts, or the proliferation of printed books, required at least a rethinking of the relationships between disciplines.¹⁷ From this perspective, even where Maurolico's formal schemes were ultimately inconsistent or abandoned, they testify to the epistemic

¹⁶ On the genesis of Maurolico's *De momentis aequalibus* see Pier Daniele Napolitani's introduction to Francesco Maurolico, *Francisci Maurolyci Archimedeae. Tomus B. De momentis aequalibus*, ed. by Riccardo Bellé, Pier Daniele Napolitani, and Beatrice Sisana (Pisa: Fabrizio Serra Editore, 2022).

¹⁷ Ann M. Blair, "Organizations of knowledge," in *The Cambridge Companion to Renaissance Philosophy*, ed. by James Hankins (Cambridge: Cambridge University Press, 2007), 291. See also Blair, *Too Much to Know*, 239–245; and, in relation to astronomy, Robert S. Westman, *The Copernican Question: Prognostication, Skepticism and Celestial Order* (Berkeley: University of California Press, 2011), 34–43. The sense of cognitive overload generated by the expansion of the book trade and the recovery of ancient texts is already visible in Petrarch's *De remediis utriusque fortunae*, I.43 where he warns that books have led some to knowledge and others to madness. The intellectual precondition for acknowledging that the ancients left knowledge incomplete is articulated by Coluccio Salutati in *De laboribus Herculis*, I.7.11, where he argues that posterity is both entitled and obliged to supplement inherited knowledge. The topos acquires its full historical charge with geographical discovery and the print revolution: Conrad Gesner, in the preface to his *Bibliotheca universalis* (Zurich, 1545), famously described *confusa et noxia illa librorum multitudo*, while his companion *Pandectarum sive Partitionum universalium libri XXI* (Zurich, 1548) represented the most systematic attempt to convert that anxiety into a classificatory remedy. For the historiographical framework, see Donald R. Kelley, ed., *History and the Disciplines: The Reclassification of Knowledge in Early Modern Europe* (Rochester: University of Rochester Press, 1997).

pressure exerted by the expanding domain of mathematical science, and to the centrality of classification as both a practical and philosophical problem.

III. Maurolico's writings on the classification of sciences

Maurolico addressed the problem of scientific classification throughout his career, producing five known texts over a span of approximately forty years. While relatively small in volume, these writings are historiographically significant, as they illuminate both the genealogy of his intellectual activity and the broader context of his encyclopedic ambitions. For Maurolico, the classification of sciences was never an end in itself. Rather, it was a recurring issue he had to deal with in order to situate his research within a wider, structured framework of knowledge. His engagement with this topic reflects a pragmatic concern: he sought schemes capable of integrating his own discoveries and restitutions of classical science into coherent, teachable systems. All of Maurolico's classification texts show the enduring influence of the Lascarian intellectual milieu. Trained under pupils of Constantine Lascaris – his father Antonio and the priests Francesco Faraone and Giacomo Genovese – and deeply influenced by works such as Giorgio Valla's *De expetendis et fugiendis rebus*, Maurolico consistently relied on models transmitted by this network.¹⁸ Even when he introduced subdivisions to accommodate his own findings, he did not fundamentally depart from these inherited frameworks.¹⁹

The earliest extant treatment of the classification of the sciences appears in a short text appended to the printed edition of the *Grammaticorum rudimentorum libelli sex*, Maurolico's first published work, which was issued in Messina in 1528. The appendix, entitled *Philosophiae divisio*, is an almost verbatim reproduction of Francesco Faraone's scheme

¹⁸ In this connection, Rosario Moscheo observed that the Lascarian 'school' was "not the institution physically established and operating in Messina, but rather the entire network of cultural relations, mutual influences exerted, friendships established and cultivated by Lascaris – including through his teaching – throughout his Italian *peregrinatio*." This cultural universe, which includes not only Lascaris's pupils active in Messina (with whom Maurolico maintained close personal ties) but also those, such as Giorgio Valla, who had met the Byzantine intellectual elsewhere and remained constantly devoted to him, effectively constitutes, according to Moscheo, Maurolico's own cultural milieu. See Rosario Moscheo, "Scienza e cultura a Messina tra '400 e '500: eredità del Lascaris e 'filologia' mauroliciana," *Nuovi Annali della Facoltà di Magistero dell'Università di Messina* 6 (1988): 595–632, 613; and Rosario Moscheo, "Greek heritage and the scientific work of Francesco Maurolico," in *Medieval and Classical Traditions and the Renaissance of Physico-Mathematical Sciences in the 16th Century: Proceedings of the XXth International Congress of History of Science*, ed. by P. Cinti, F. D'Angelo, and C. Vinti (Viterbo: Università degli Studi della Tuscia, 2001), 15–22.

¹⁹ Cfr. Corrado Dollo, *Modelli scientifici e filosofici nella Sicilia spagnola* (Napoli, 1984), 14.

set out in his *Institutiones grammatices* (1500).²⁰ This framework ultimately derives from Isidore of Seville's adaptation of Boethius's fivefold division, later appropriated by humanists such as Pietro Pomponazzi, Agostino Nifo, and Marsilio Ficino, and was widely disseminated through pedagogical and encyclopedic works, notably Gregor Reisch's *Margarita philosophica* (1503), which Maurolico himself was familiar with.²¹ Significantly, Maurolico extends this traditional schema by including, under geometry, the disciplines of optics (*optica*), catoptrics (*catoptrica*), photisms (*photismi*), and the study of transparent media (*diafana*), as well as the theory of isorrhopa (*de aequiponderantibus*) and geography. This extension reflects his effort to subsume his own mathematical investigations within an established philosophical taxonomy and corresponds closely to his earliest scientific writings - the *Photismi de lumine et umbra* and the initial drafts of *De momentis aequalibus* - composed during the same period as the *Philosophiae divisio*.²²

²⁰ Maurolico, *Grammaticorum rudimentorum libelli sex*, c. 123v. The events related to the publication of the first edition of Francesco Faraone's *Institutiones grammatices*, a treatise which would enjoy considerable success in the years to come, have been reconstructed by Alessandra Tramontana and well attest to the cultural influence of Constantine Lascaris in Messina during the first decades of the sixteenth century. The *Institutiones*, in fact, would provoke harsh criticism from another pupil of Lascaris active in Sicily, the humanist Lucio Cristoforo Scobar, who in 1520 published the *Carcinomata pharaonis* as part of his work *De rebus praeclaris syracusanis opuscula*. In these writings, he denounced the alleged errors contained in the *Institutiones* and proposed corrections which were partially accepted in subsequent editions of the *Institutiones*. Beyond the strictly scientific terms of the matter, what is important to note is that the controversy was certainly not unknown to Maurolico, who chose to 'present' himself officially to the Messinese intellectual community precisely with a manual of Latin grammar. This is relevant considering that, as Tramontana emphasizes, there was in Messina, "starting from the 1520s–1530s, and not by chance in conjunction with the resurgence of typographical activity, a composite nucleus of intellectuals who met and discussed systematically, and who, intending to utilize the legacy acquired from the season just past, attempted to define renewed cultural directions according to diversified perspectives, capable of making an impact even beyond the Strait [of Messina]." See Alessandra Tramontana, "L'eredità di Costantino Lascaris nella Messina di Francesco Maurolico e la polemica sulle *Institutiones* di Valla," in *Umanesimo e Università in Sicilia (XV–XVII secolo): Saggi in onore di Daria de Rubertis*, a cura di M. A. Russo (Roma: Aracne, 2013), 133–158, 138.

²¹ Cfr. Boethius, *De Trinitate*, in *Patrologia Latina*, vol. 64, ed. by J.-P. Migne (Paris: Garnier, 1847), col. 1250C; Isidore of Seville, *Etymologiae*, ed. by W. M. Lindsay, vol. I (Oxford: Clarendon Press, 1911), II. 24.2–3; Gregor Reisch, *Margarita philosophica* (Freiburg im Breisgau: Johannes Schott, 1503), I, I, ch. 1.

²² On the *Photismi* and, more generally, on Maurolico's works on optics, see Pier Daniele Napolitani's and Ken'ichi Takahashi's introduction to Francesco Maurolico, *Francisci Maurolyci Optica*, ed. by Riccardo Bellé and Ken'ichi Takahashi (Pisa: Fabrizio Serra Editore, 2017); and Riccardo Bellé, "L'edizione a stampa dei *Photismi de lumine et umbra* (1611) e l'*Index lucubrationum*," *Nuncius: Annali di Storia della Scienza* 29, no. 1 (2014): 101–134.

Main Division (Latin Term)	Secondary Definition	Tertiary/Component Disciplines	Maurolico's Extensions (Sub-Geometry)
Metaphysics (<i>Metaphysica</i>)	Deals with the first cause, first being.	<i>Theologia</i> (discourse on God).	N/A
Physics (<i>Physica</i>)	Deals with the nature of the heavens, elements, and things made from elements.	N/A	N/A
Ethics (<i>Aethica</i>)	Moral law.	<i>Monastica</i> , <i>Oeconomica</i> , <i>Politica</i> (including laws/rights).	N/A
Mathematics (<i>Mathematica</i>)	Deals with quantifiable things.	<i>Arithmetica</i> , <i>Musica</i> , <i>Astrologia</i> .	N/A
Geometry (<i>Geometria</i>)	Continuous quantities, forms, ratios.	N/A	<i>Optica</i> , <i>Photismi</i> , <i>Catoptrica</i> , <i>Diaphana</i> , <i>Isorrhopa</i> (Mechanics), <i>Geographia</i> .
Logic (<i>Logica</i>)	Deals with discourse.	<i>Grammatica</i> , <i>Rhetorica</i> , <i>Dialectica</i> .	N/A

Tab. 1 – The classification of 1528.

The second text is the *Sermo de divisione atrium*, a work preserved in a single autograph manuscript. The small paper codex contains three short discourses dated by Maurolico in June 1554, during his stay in Catania: *De divisione artium*, *De quantitate*, and *De proportione*.²³ Compared with the brief *Philosophiae divisio* appended to the *Grammaticorum libelli* of 1528, the *Sermo de divisione artium* is a markedly longer and more elaborate text, evidently conceived for pedagogical use. It unfolds as a systematic lecture rather than a schematic summary, and reflects Maurolico’s mature concern to present a historically informed and philosophically grounded account of the liberal disciplines. The treatise is divided into two broad parts: the first offers a historical survey of earlier classifications of the sciences, largely derived from Giorgio Valla’s *De expetendis et fugiendis rebus*;²⁴ the second develops a

²³ MS. Molfetta (Bari), Biblioteca del Seminario Minore Vescovile S-7 H 15, ff. 1r–26r. The manuscript was photographed and published by Graziano Bellifemmine in Francesco Maurolico, *Prologi sive sermones quidam de divisione artium, de quantitate, de proportione*, ed. by Graziano Bellifemmine (Molfetta [Bari], 1968).

²⁴ Cfr. Giorgio Valla, *Georgii Vallae Placentini De expetendis et fugiendis rebus opus in quo omnes scientiae & artes. Quas tum veterum tum nostrae aetatis scriptorum monumenta nobis tradiderunt...*

new division which Maurolico explicitly attributes to his teacher, Giacomo Genovese, and which he adapts to his own conception of the mathematical sciences.²⁵ Whereas Faraone had distinguished five principal branches of philosophy, Genovese, according to Maurolico, adopted a more explicitly scholastic framework. His scheme separated *philosophia organica* (or logic), regarded as the instrument of all other sciences, from *philosophia realis*, which he in turn divided into *speculativa* and *practica*.²⁶ The speculative part encompassed metaphysics, physics, and mathematics, while the practical comprised ethics, economics, and politics. This shift toward a more scholastic and pedagogical ordering of knowledge was likely reinforced by the founding of the Jesuit college in Messina in 1548, the *Primum ac Prototypum Collegium Societatis Iesu*. As the prototype of Jesuit education, it promoted a Thomistic framework that valued the hierarchical organization of the sciences, within which Maurolico's reworking of the *divisio scientiarum* naturally found resonance.²⁷

The *Sermo* holds a significant place within Maurolico's *oeuvre*, positioned both chronologically and conceptually between two key programmatic statements: specifically, the dedicatory letter to Pietro Bembo from 1540, which precedes the first printed edition of the *Cosmographia* (1543), and the letter to Juan de Vega discussed above, which was perhaps intended to open the *Spherica* of 1558. In both letters, Maurolico references the classification of the sciences, a topic treated at length in the *Sermo*. Furthermore, in the intervening years between the *Philosophiae divisio* of 1528 and the *Sermo*, Maurolico's scholarly and research agenda had become more comprehensive, ambitious, and systematic than the program initially outlined in the preface to the *Grammaticorum*. He had, in fact, been working proficiently on Euclid, Jordanus Nemorarius, Archimedes, Serenus, Autolycus, Theodosius, Menelaus, Boethius, and Apollonius, in addition to making substantial progress in his studies on *De momentis aequalibus*.²⁸ All these strands of research, which alongside his original investigations constituted a full program for the restitution and reform of mathematical knowledge, are integrated into the classificatory schema derived from Genovese within the *Sermo*.²⁹

(Venetiis: impressit Aldus Manutius, 1501), I, chs. 2–4 and 7–14.

²⁵ Very little is known about Giacomo Genovese apart from what Maurolico himself states in the *Sermo*, namely, that he was a priest of Nolan origin and a pupil of Lascaris.

²⁶ The designation *philosophia organica* (from Aristotle's *Organon*) is found in Jean de Jandun's commentary on Aristotle's *De anima*, which was published several times in Venice starting from 1473, and most notably in 1497, cfr. Jean de Jandun, *Quaestiones super tres libros Aristotelis De Anima* (Venetiis: impressit Simon de Luere, 1497).

²⁷ On the relationships between Maurolico and the Jesuits see Rosario Moscheo, *I Gesuiti e le matematiche nel secolo XVI. Maurolico, Clavio e l'esperienza siciliana* (Messina: Società Messinese di Storia Patria, 1998).

²⁸ Cfr. Clagett, "The Works of Francesco Maurolico," 252–163.

²⁹ Cfr. Antonio Carlo Garibaldi, "I programmi di ricerca mauroliciani," in *Archimede e le sue fortune*, ed. by Vincenzo Fera, Daniela Gionta, and Antonio Rollo (Messina: Centro internazionale di studi umanistici, 2014), 567–588, 570–583.

Primary Distinction	Secondary Division	Tertiary Division	Component Disciplines and Key Features
Instrumental	<i>Philosophia Organica</i>	<i>Logica</i>	<i>Grammatica, Rhetorica, Dialectica.</i>
Real	<i>Philosophia Realis Speculativa</i>	<i>Metaphysica</i>	Theology (Highest Abstraction).
		<i>Physica</i>	Natural Philosophy (Motion, Matter, from <i>Libri octo physicorum</i> to <i>Medicina</i>).
		<i>Mathematica</i>	Pure Mathematics; Applied Mathematics (e.g., <i>Archimedis De aequiponderantibus</i>).
	<i>Philosophia Realis Practica</i>	<i>Activa (Moral)</i>	<i>Ethica, Oeconomica, Politica.</i>
Final Placement of Mixed Sciences	Mathematics/ Applied	<i>Factiva (Productive)</i>	Mechanical Arts, including <i>Medicina</i> .
		N/A	Derived arts (<i>alias artes</i>): <i>Computus, Musica, Astronomia, Doctrinam Momenorum</i> (Mechanics).

Tab 2. – The classification in the *Sermo* (1554).

Another *philosophiae divisio* is introduced by Maurolico as part of the preface to the *Problemata mechanica* (posthumously published 1613), a work he started working on at least during the 1540s as part of a broader programme to provide mathematical foundations for physics and mechanics.³⁰ In the prefatory section, before addressing the mechanical questions, Maurolico briefly discusses a possible classification of sciences, recalling the same scheme outlined in the *Sermo*, suggesting that the two texts were written around the same period. Here, however, the division serves a new purpose: to define the disciplinary place of mechanics within the hierarchy of the sciences. Consequently, Maurolico divides *philosophia realis* as follows:

Realis autem philosophiae pars dividitur in operativam et contemplatricem, hoc est in activam, sive practicam et in theoricam, sive speculativam.³¹

³⁰ Francesco Maurolico, *D. Francisci Maurolyci Abbatis Messanensis Problemata Mechanica cum Appendix et ad Magnetem et ad Pixidem Nauticam pertinentia*, *Omnia nunc primum in lucem edita* (Messanae: Ex Typographia Petri Brae, 1613). On this work, see “*Problemata mechanica*,” in *Edizione Nazionale dell’opera matematica di Francesco Maurolico*, section “12.C.1 *Mechanica et machinae* – 1. *Problemata mechanica*,” ed. by Hervé Barthélemy & Veronica Gavagna, <https://maurolico.it/Maurolico/sezione.html?path=12.C.1>

³¹ Maurolico, *Problemata Mechanica*, 1613, 7.

He further specifies that mathematics, being speculative, is itself divided into arithmetic (*de quantitate discreta*) and geometry (*de quantitate continua*); and under geometry he explicitly places mechanics, along with optics, astronomy, and other *scientiae mediae*. This concise taxonomy closely echoes the *Sermo de divisione artium*, but with an important new nuance: mechanics is now formally integrated into the mathematical sciences as a discipline concerned with the action of natural causes upon quantified bodies – a hybrid of mathematical reasoning and physical reality.³²

Primary Division	Secondary Division	Tertiary Division	Placement and Role of Mathematics and Mechanics
<i>Ars Disserendi</i>	N/A	<i>Logica</i>	Instrument/Mode of Knowing (<i>terminus, propositio, syllogismus</i>).
<i>Realis Operativa</i>	<i>Activa (Moral)</i>	<i>Moralis Doctrina</i>	Ethics, Politics (Morals and administration).
	<i>Factiva (Productive)</i>	<i>Artes Banausicae</i>	Manual/Mechanical Arts (Low status).
<i>Realis Speculativa</i>	<i>Metaphysica</i>	N/A	Separated from matter (God, Intelligences).
	<i>Physica</i>	N/A	Principles and motion of simple and mixed bodies.
	<i>Mathematica</i>	<i>Geometria; Arithmetica.</i>	Pure Mathematics generates <i>Scientiae Mediae</i> . Mechanica is explicitly <i>collocandam esse sub mathematica</i> (must be placed under mathematics).
Methodological Demand	N/A	N/A	Mechanics requires <i>Archimedes' De aequiponderantibus</i> as the necessary philosophical <i>praeambulum</i> (preliminary).

Tab. 3 – The classification in the *Problemata mechanica*.

³² Ibid., 7–8: “Mathematica versatur circa quantitatem et formam a materia separatam. Atque si quantitas sit continua, ea erit geometria, si discreta, erit arithmetica. Metaphysicae pars una naturas ab omni materia separatas, ut sunt Deus et intelligentiae, considerat; altera in communissimis versatur. In quibus et dialectica, arithmetica deinde, atque geometria materiae alicui applicatae generant alias scientias, quae mediae quodammodo sunt inter mathematicas et naturales, quales sunt musica, astronomia, perspectiva, scientiae ponderum, stereometria, cosmographia, geographia, architectura, pictura, sculptura et omnis ratio mechanica. Quae quoniam a mathematica speculatione derivantur, quamquam particularibus rebus applicatae, mathematicae potius, quam naturales appellari solent et debent. Mechanicam igitur scientiam nemini dubium est, sub mathematica, philosophiae parte, collocandam esse.”

Preserved in a unique manuscript dated 1567, the *Compendium de divisione et principiis scientiarum* marks a critical stage in Maurolico's reflection on the order of knowledge.³³ The text opens with the author's statement that, having previously gathered scattered notes on the division, subjects, and principles of the arts, he now intends to rearrange them *in ordinem congruum redacta*.³⁴ The work appears as a master plan for an encyclopedia of knowledge structured in thirteen short books, very likely intended to support the teaching activity Maurolico initiated that year at the Jesuit college in Messina, and is closely connected to Maurolico's project of publishing his own works, which, by that time lacking solid institutional support, he hoped to accomplish through collaboration with the Jesuits.³⁵ Today, only the introductory framework survives in the extant manuscript. In the preface, Maurolico distinguishes two chief operations within the sciences, *distinctio* and *introductio*, and proposes to treat the former in the first book and the latter through the remaining ones. As in the *Sermo*, the work proceeds to divide philosophy into *organica* and *realis*, the latter further subdivided into the speculative (mathematics, physics, astronomy, music) and the practical (ethics, politics, economics, mechanics, medicine). While maintaining the framework inherited from Giacomo Genovese, however, Maurolico explicitly acknowledges two alternative models: an Aristotelian scheme, derived from Gregor Reisch's *Margarita philosophica*, which orders the sciences according to their objects and degrees of abstraction; and a Valla-derived model, based on the capacities of the human intellect and the operations of reason.³⁶

These three alternative classification systems are proposed as equally legitimate in Maurolico's final text: the preface to the 1575 *De sphaera* in the *Opuscula mathematica*, explicitly titled *Philosophiae divisio*.³⁷ The *Opuscula mathematica* collects treatises on mechanics, optics, and astronomy. The preface sought to organize this advanced and methodologically heterogeneous corpus, whose content had already dismantled classical divisions. Here, Maurolico presents the three alternatives, which are further illustrated by three accompanying diagrams, without explicitly subscribing to one of them. This multiplicity of incompatible schemes is highly significant. Rather than arguing for one definitive scheme, Maurolico attempts to reconcile them, offering the following assurance to the reader:

³³ Bibliothèque Nationale de France, MS lat. 7471 [ca. 1567], ff. 14r–23r.

³⁴ BNF, MS lat. 7471, f. 14r.

³⁵ Moscheo, *I Gesuiti e le matematiche nel secolo XVI*, 149–230.

³⁶ On Valla's classification, see Nicolò Magnani, "L'enciclopedismo di Giorgio Valla fra umanesimo e scienze esatte: struttura e fonti del *De expetendis et fugiendis rebus*," in *Letteratura e Scienze: Atti delle sessioni parallele del XXIII Congresso dell'ADI (Associazione degli Italianisti)*, Pisa, 12-14 settembre 2019, ed. by Alberto Casadei, Francesca Fedi, Annalisa Nacinovich, and Andrea Torre (Roma: Adi editore, 2021), 193–203.

³⁷ Francesco Maurolico, *D. Francisci Maurolyci Abbatis Messanensis Opuscula mathematica, nunc primum in lucem aedita cum rerum omnium notatu dignarum indice locupletissimo* (Venetiis: Apud Franciscum Francischium, 1575).

nec te lector ingeniose moveat diversitas positionum: quandoquidem in unaquaque trium divisionum Scientiae, et artes (utcunque disponantur) semper invicem cognatae, et ab eadem radice propagatae consistunt.³⁸

Following this philosophical assurance, however, Maurolico immediately proceeds to justify his choice to focus on Astronomy by explicitly referencing the inherited hierarchy of certainty and nobility:

Quoniam itaque Speculativa pars Philosophiae dividitur in naturalem, Mathematicam, ac Theologiam. Atque Theologia excedit facultatem humanam. Physica vero, propter materiae fluxum, incerta. Idcirco commendatur Mathematica, et praecipue Astronomia, propter certitudinem demonstrationis, et subiecti nobilitatem, ut ait Ptolemaeus. Igitur Astronomiae principia tradituri, praemitemus quaedam geometrica, et necessaria praeambula.³⁹

This excerpt concisely encapsulates Maurolico's dilemma. While he asserts the fundamental unity of all knowledge (*ab eadem radice propagatae*), the need to justify his work on Astronomy compels him to choose and defend a specific hierarchy - one that subordinates both the transcendent (Theology) and the uncertain (Physics) to the demonstrative certainty of his own mathematical practice. This practical selection, following the presentation of multiple conflicting schemes, textually demonstrates that the search for a unifying philosophical framework was, in effect, yielding to the methodological demands of his own advanced mathematical research.

IV. Conclusion

Maurolico's inability to settle on a single classification scheme reflects the structural tension between his groundbreaking contributions and the inherited hierarchies of knowledge. In *Photismi de lumine et umbra* (1521–1554, published 1611), Maurolico applied Euclidean geometry to optical phenomena, rigorously modeling refraction, eclipses, and the pinhole effect, thus implicitly demanding recognition of optics as central to mathematical natural philosophy.⁴⁰ *De momentis aequalibus*, on the other side, marked a methodological breakthrough in mechanics. Drawing on Archimedean statics, Maurolico applied geometric reasoning to determine centers of gravity for pyramids, paraboloids, and other bodies. His use of the method of moments linked volumes with areas, eroding the Aristotelian distinc-

³⁸ Maurolico, *Opuscula mathematica*, 1575, 3.

³⁹ Ibid., 3–4.

⁴⁰ See Napolitani and Takahashi, introduction to *Optica*; Bellé, "L'edizione a stampa dei *Photismi*"; also Giora Hon and Yaakov Zik, "Geometry of Light and Shadow: Francesco Maurolyco (1494–1575) and the Pinhole Camera," *Annals of Science* 64, no. 4 (2007): 549–578.

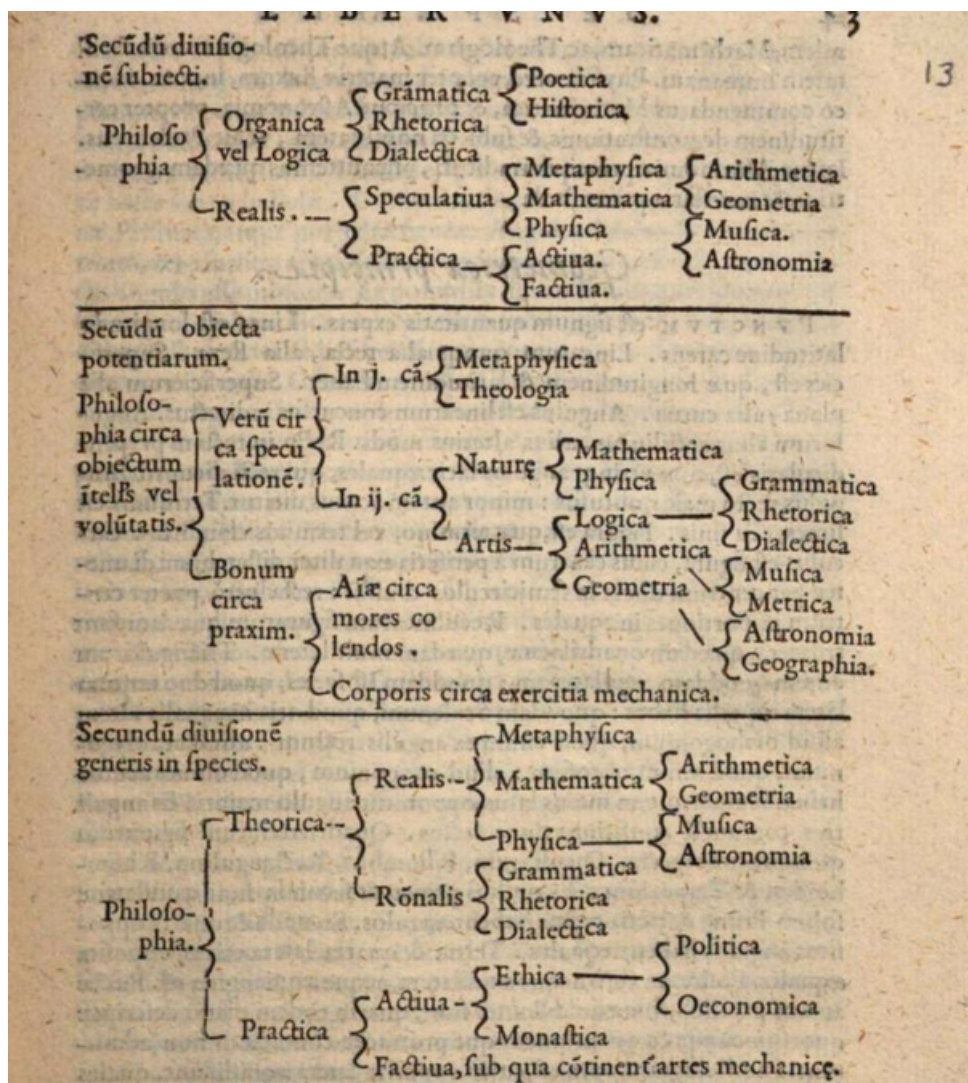


Fig. 1 – The alternatives divisions of sciences in *Opuscula mathematica*.

tion between discrete and continuous quantities.⁴¹ In *Arithmeticonum libri duo* (probably completed by 1557, but published posthumously in 1575), he developed a formal proof by induction, establishing a rigorous recursive structure capable of extending conclusions over infinite sequences. This reconceived arithmetic not merely as the study of discrete quantities but as a field capable of generalized proof, undermining the standard philosophical treatment of numerical reasoning. These innovations, from induction to geometrical mechanics and optics, inherently synthesized domains that traditional classification kept separate, and Maurolico’s struggle to reconcile his methods with classical epistemology is most evident in *De scientiarum divisione* of 1575, as the alternatives presented in this work show a persistent internal ambiguity concerning the hierarchy of the sciences.

Scheme Name	Primary Distinction	Key Structural Detail	Placement Crisis
Scheme 1: <i>Secundum divisionem subiecti</i> (Subject Matter)	<i>Organica</i> / <i>Realis</i>	<i>Musica</i> and <i>Astronomia</i> are listed as species of <i>Physica</i> .	Subordinates mathematical arts (<i>Quadrivium</i>) to Natural Philosophy.
Scheme 2: <i>Secundum obiecta potentiarum</i> (Objects of the Soul's Powers)	<i>Verum</i> (Intellect) / <i>Bonum</i> (Will)	<i>Arithmetica</i> , <i>Geometria</i> , <i>Musica</i> , <i>Astronomia</i> are listed as <i>Artis</i> (Arts) under <i>Verum</i> .	Demotes core mathematics to an instrumental or practical art.
Scheme 3: <i>Secundum divisionem generis in species</i> (Genus into Species)	<i>Theorica</i> / <i>Practica</i>	<i>Musica</i> and <i>Astronomia</i> are again subordinated to <i>Physica</i> within <i>Theorica Realis</i> .	Reinforces the difficulty of granting high theoretical status to <i>scientiae mediae</i> .

Tab. 4 – The three alternative schemes and their shortcomings in the *Opuscola mathematica*.

This first scheme classified disciplines by subject matter, subordinating music and astronomy – traditionally Quadrivial subjects – to physics. The second system arranged the sciences by their aims, linking mathematics and physics as “sciences of nature.” Yet it paradoxically relegated geometry, arithmetic, and related disciplines to *ars*, instrumental to natural philosophy. This reflects a philosophical tension: Maurolico’s innovations demanded theoretical status, yet his schema constrained core mathematics to a practical role, anticipating debates later crystallized by Galileo.⁴² The third classification, based

⁴¹ See, for example, the method employed for determining the centre of gravity of the paraboloid: Pier Daniele Napolitani and JeanPierre Sutto, “Francesco Maurolico et la détermination du centre de gravité du paraboloïde de rotation,” *SCIAMVS* 2 (2001): 187–250.

⁴² See Helen Hattab, “From Mechanics to Mechanism: The *Quaestiones Mechanicae* and Descartes’ Physics,” in *The Science of Nature in the Seventeenth Century: Patterns of Change in Early*

on genus, repeats the first scheme's pattern, again subordinating astronomy and music to physics. The repetition underscores the impossibility of reconciling mathematical certainty with traditional hierarchies: disciplines demonstratively mathematical yet materially physical could not fit the Aristotelian schema.

Maurolico's classification attempts failed because he tried to fit his methodological breakthroughs into a conservative philosophical structure designed to categorize ancient, abstract quantity. Disciplines such as optics, mechanics, and astronomy combined theoretical principles derived from quantity with empirical observations concerning sensible matter. The classical distinction between the theoretical and the practical sciences provided the traditional hierarchical framework. Maurolico's innovations in *Photismi* and *De momentis aequalibus* showed that these sciences were demonstrative, yet inextricably tied to physical reality. The inability to grant them a consistent, high theoretical status without subordinating them entirely to physics proved the existing framework was inadequate to manage disciplines that were, by their nature, hybrid.

Maurolico's intellectual conservatism, evident in his anti-Copernican stance and his adherence to traditional headings,⁴³ compelled him to stretch the established Aristotelian framework beyond its logical breaking point. He was attempting to accommodate radical, modern content - mechanics grounded in Archimedean geometry, proof by induction, and mathematically rigorous optics - within a classification system that was designed for purely abstracted quantity. His struggle was structural: the failure was not a conceptual error in classifying but a demonstrable sign that the content being generated had fundamentally surpassed the capacity of the traditional philosophical container.⁴⁴ Paradoxically, this conservatism ensured the immediate utility and long-term transmission of his work. By attempting to legitimize his new mathematics within the established scholarly apparatus, he compelled scholars like Clavius and subsequent mathematical practitioners to adopt his enhanced rigor and methodological standards. Maurolico's unsuccessful effort to find

Modern Natural Philosophy, ed. by Peter R. Anstey and John A. Schuster (Dordrecht: Springer, 2005), 99–129.

⁴³ Edward Rosen, "Maurolico's Attitude toward Copernicus," *Proceedings of the American Philosophical Society* 101, no. 2 (1957): 177–194.

⁴⁴ A similar struggle can be recognized in Peter Ramus also. In the first three books of the *Scholae mathematicae*, Ramus sought to defend mathematics against charges of its lack of utility and its obscurity, surveying the history of Greek mathematics and insisting on the practical origins of the subject and on the use to which the ancients had put it, both as a theoretical foundation for natural philosophy and as a practical tool in astronomy and mechanics. Recent developments in astronomy and mechanics showed, by contrast, the sterility of a scholastic natural philosophy devoid of mathematics. See, in this regard, Charles H. Lohr, "The Sixteenth-Century Transformation of the Aristotelian Division of the Speculative Sciences," in Donald R. Kelley and Richard H. Popkin, eds., *The Shapes of Knowledge from the Renaissance to the Enlightenment* (Dordrecht: Springer, 1991), 49–58.

“the right place” for his work served as an explicit, high-level textual document detailing the breakdown of Renaissance epistemology.

Francesco Maurolico’s life work encapsulated the central tension of the Renaissance: the obligation to recover ancient knowledge paired with the drive for original discovery. His classification attempts in *De Scientiarum divisione* were an effort to impose an obsolete philosophical order onto a rapidly expanding, methodologically revolutionary corpus of mathematical physics. Maurolico’s unsuccessful effort to coherently categorize his new knowledge within the confines of Aristotelian principles provides evidence of the intellectual chasm that had opened between content and structure. This demonstrable failure forced subsequent generations to acknowledge that mathematical knowledge had transcended its classical definition. Encountering the limitations of older methods, Maurolico pioneered a radically new epistemology, one where mathematics was finally elevated from an abstract study of quantity to the demonstrative language of physical reality.

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Rediscovering Apollonius in the Renaissance: Francesco Maurolico and Federico Commandino

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Abstract

The sixteenth-century revival of Greek mathematics followed multiple – often divergent – paths. This paper explores the distinct yet complementary approaches of Francesco Maurolico and Federico Commandino to Apollonius' *Conics*, focusing on the editorial strategies and innovations each introduced in their Latin editions. Special attention is paid to how each mathematician engaged with both the textual content and the geometrical diagrams, revealing two modes of Renaissance mathematical humanism that would later shape the seventeenth-century reception of Apollonius.

Keywords

Commandino, Maurolico, Apollonius, conic sections

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1. Introduction

The different attitudes of Francesco Maurolico (1494–1575) and Federico Commandino (1509–1575) towards the re-appropriation of the classics are well known, at least from the work of Clagett and Rose in the 1970s.¹ In this contribution, however, we will attempt to make it more concrete and visible by analysing how these two mathematicians approached the subject of conic sections, which was a novelty in the 16th century.

Maurolico and Commandino were leading figures in the sixteenth-century recovery of Greek mathematical knowledge. Both shared the ambition of renewing modern science through the rediscovery of ancient methods – yet their approaches diverged. Commandino engaged with Greek texts first as a philologist and then as a mathematician; Maurolico approached them from the outset as a practicing mathematician. They represent two distinct forms of Renaissance mathematical humanism: Commandino pursued an *imitatio* of the ancients, Maurolico an *æmulatio*. These categories, however, should be used with caution: Commandino was far more than a literal translator of ancient texts, while Maurolico's innovations must also be seen as part of a broader effort to restore the Greek heritage in its entirety.

Commandino's work was guided by a deep reverence for classical models. When confronted with textual lacunae in Euclid, Archimedes, Ptolemy, Apollonius, Serenus, or Pappus, he gathered every available Greek source to restore the original text. When philological tools proved insufficient, he sought to reconstruct the ancients' reasoning – sometimes with brilliant results, as in his work on Archimedes' *On Floating Bodies* or the lost proofs of Serenus.² At the same time, his *Liber de centro gravitatis solidorum* (1565) was the first printed work in which the recovery of Archimedean mathematics became a source of genuine innovation rather than mere restitution.

Maurolico, by contrast, was less deferential. Drawing on fragmentary sources or imperfect Latin versions, he intervened freely whenever he identified errors or gaps – rewriting demonstrations *ex traditione Maurolyci*. On the one hand, he edited or reworked the writings of Euclid, Archimedes, Apollonius, Serenus, Theodosius, Menelaus, and Ptolemy; on the other, he composed original treatises on optics, arithmetic, statics, and astronomy that often went beyond ancient results.³

¹ Marshall Clagett, *A supplement on the medieval Latin traditions of conic sections (1150-1156)* (Philadelphia: The American Philosophical Society, 1980); Paul Lawrence Rose, *The Italian Renaissance of Mathematics* (Genève: Droz, 1975), 159–221.

² See Marshall Clagett, *The Fate of the Medieval Archimedes, 1300 to 1565*, III (Philadelphia: The American Philosophical Society, 1978), 623–635; Argante Ciocchi, *Federico Commandino: Umanesimo matematico e rivoluzione scientifica* (Urbino: Quaderni del Centro Urbino e la prospettiva, Urbino University Press, 2023), 116–140; Id., “Federico Commandino and the Latin Edition of Apollonius's *Conics* (1566),” *Archive for History of Exact Sciences* 77, issue 4 (2023): 393–421.

³ See Pier Daniele Napolitani, “Le edizioni dei Classici: Commandino e Maurolico,” in *Torquato Tasso e l'Università* (Firenze: Olschki, 1997), 119–141.

The two men worked at a distance – Maurolico in Messina, Commandino in Urbino. Commandino, supported by the Farnese and Della Rovere courts, combined philological rigor with privileged access to manuscripts. Maurolico, deeply embedded in the intellectual, civic, and political life of Messina, and later in contact with Jesuit mathematicians, cultivated a more speculative and independent approach.⁴ However, the two categories, *imitatio* and *æmulatio*, used to describe Commandino and Maurolico are ultimately too reductive to fully capture the complexity of their respective bodies of work. Though separated geographically, they were aware of each other's work, and Commandino likely benefited from Maurolico's advice when preparing the 1558 *Archimedis Opera non nulla*.⁵

2. Francesco Maurolico and the reconstruction of Apollonius: a lifelong work in progress

On 24 June 1547, in Palermo, Francesco Maurolico completed his *emendatio* of Giovan Battista Memmo's Latin translation of the first four books of Apollonius' *Conics*, which had been published posthumously a decade earlier.⁶ Maurolico's interest in conic sections, however, long predated this moment.⁷

Before 1537, no complete Latin translation of Apollonius' treatise was available,⁸ with one partial but significant exception: Giorgio Valla's *De expetendis et fugiendis rebus opus* (1501). In book XIII, under the heading *de conica sectione*, Valla included the definitions of the three conic sections according to Apollonius, followed by a section *de cylindrica sectione* containing long excerpts from Serenus' *De sectione cylindrica*.

Relying on such limited materials, Maurolico began his early studies on conics in the late 1520s and early 1530s. Already in the *Photismi de lumine et umbra* (1521), he referred to the circle obtained by the *sectio subcontraria*, one of the few Apollonian propositions transmitted by Valla and entirely unknown to the medieval tradition.⁹

⁴ Pier Daniele Napolitani, "Maurolico e Commandino," in *Il meridione e le scienze. Secoli 16°-19°*, Atti del convegno di Palermo, 14-16 maggio 1985, ed. by Pietro Nastasi (Palermo: Istituto Gramsci Siciliano – Istituto italiano per gli studi filosofici, 1988), 281–316.

⁵ Beatrice Sisana, "Tempi, luoghi e contenuti della corrispondenza tra Federico Commandino e Francesco Maurolico," *Bollettino di Storia delle Scienze matematiche* 51, 2 (2021): 277–314; Ciocci, *Federico Commandino: Umanesimo matematico e rivoluzione scientifica*, 63–75.

⁶ The autograph manuscript is preserved in the Real Biblioteca del Monasterio de San Lorenzo de El Escorial, J. III. 31. Memmo's translation had appeared in Venice in 1537, published by Bernardino Bindoni.

⁷ For a general overview, see Pier Daniele Napolitani, *Introduzione* to vol. 8 of the *Opera mathematica*, <https://maurolico.it/Maurolico/sezione.html?path=8>.

⁸ Clagett, *A Supplement on the Medieval Latin Traditions of Conic Sections (1150–1556)*, vol. IV, 311–316.

⁹ Maurolico, *Optica*, vol. 10 of the *Opera mathematica*, [54–55](#).

In the following decade, while reconstructing Serenus' *De sectione cylindrica* on the basis of Valla's excerpts, Maurolico found himself compelled to rebuild the underlying foundations of conic theory. This work culminated in a now-lost treatise, the *Elementa conicorum*, which offered a systematic account of conic sections and included an early treatment of geometric similarity. The latter likely arose from Maurolico's engagement with Serenus' discussion of cylindrical sections similar or equal to conic sections – specifically ellipses.¹⁰

The reconstruction diverged significantly from Serenus' original – both in its demonstrations and its overall structure – and extended toward a broader theory of similarity in conic sections. Maurolico completed this work on Serenus in 1534 and probably composed his own treatise on conic sections that same year, consisting of at least two, possibly three, books.

He drew on these early investigations in his emendation of Archimedes' *Quadratura parabolae* (1534),¹¹ but his views required revision after 1537, when he encountered Memmo's translation of Apollonius. Maurolico realized that his earlier reconstruction had captured only part of Apollonius' theory: it lacked the concept of the general diameter and was restricted to Conics referring to their axes with perpendicular ordinates. Still, he remained convinced that his work had introduced results absent from Memmo's version – particularly concerning the similarity of conic sections.

This realization marked the beginning of a more ambitious project: the *Emendatio ac restitutio Apollonii Pergaei conicorum elementorum*. In a letter to the Viceroy of Sicily, Juan de Vega (written between 1554 and 1556), Maurolico described the inadequacy of Memmo's edition and articulated the guiding principles behind his own approach:

Giovan Battista Memmo has recently translated the four books of the *Conics*, working from a manuscript so defective that he omitted entire sections where the text was illegible. In translating word for word, he occasionally altered the meaning – which shows he translated even what he did not understand. This is hardly surprising, since Giorgio Valla had done the same when translating certain works of the geometers: so rare are those who truly master this part of philosophy. To Memmo's errors must be added the printer's carelessness, especially in the diagrams, to the point that even the author himself would have struggled to correct the work. From all this, one may judge how much effort we have expended in restoring Apollonius' *Conics*. Learned men will also recognize how we have provided clearer demonstrations and added the necessary propositions and figures.

¹⁰ Roberta Tassora, "I *Sereni Cylindricorum libelli duo* di F. Maurolico e un trattato sconosciuto sulle sezioni coniche"; and in the National Edition, ed. Tassora, <https://maurolico.it/Maurolico/sezione.html?path=8.1>.

¹¹ See Beatrice Sisana's Note on the Text to her edition of the *Quadratura parabolae* in *Francisci Maurolyci, Archimedeae, Edizione Nazionale dell'opera matematica di Francesco Maurolico*, ed. by Riccardo Bellé, Pier Daniele Napolitani, Beatrice Sisana (Pisa-Roma: Fabrizio Serra Editore, 2022), 275–284 (English version, 285–294).

I have not yet seen Apollonius' remaining four books, but – based on his letter to Eudemus and what may be inferred from it – I have composed certain theorems pertaining to the fifth and sixth books myself.¹²

These labours, however, were destined to remain unpublished. While we possess the Escorial autograph for the *emendatio* of the first four books, the *restitutio* survives only in the edition printed in Messina in 1654.

As Ken Saito has demonstrated, the *emendatio* was certainly based on Memmo's translation, and most likely relied on it as its sole source – although it cannot be ruled out that Maurolico may have consulted other Latin or Greek manuscripts.¹³

As for the *restitutio* – Maurolico's own reconstruction, inspired by the brief remarks Apollonius makes in his letter to Eudemus (the fifth book would have treated “maxima and minima,” the sixth, similar and equal conics) – Fabrizio Baccetti has advanced the well-grounded hypothesis that it originated from a reworking of the materials found in the lost *Elementa conicorum* (1534).¹⁴

Already at this stage, a defining trait of Maurolico's engagement with ancient mathematics becomes apparent: he worked with whatever sources were available, approached classical texts with considerable freedom, and reworked and clarified them through his own mathematical imagination. On that basis, he developed original results and new perspectives. He often recast such material into concise compendia or presented it in the form of *quaestiones*, designed to serve as tools for renewed mathematical learning.¹⁵

¹² Transl. from Maurolico's letter to the Viceroy of Sicily, Juan de Vega (1554–1556): National Edition, vol. 2, sect. A3, §§58–64. <https://maurolico.it/Maurolico/sezione.html?path=2>.

¹³ Ken Saito, “Quelques observations sur l'édition des Coniques d'Apollonius de F. Maurolico,” *Bollettino di Storia delle Scienze matematiche* 14, 2 (1994): 239–258; and Id., “Francesco Maurolico's edition of the *Conics*,” in *Medieval and Classical Traditions and the Renaissance of Physico-Mathematical Sciences in the 16th Century*, Proceedings of the XXth International Congress of History of Science (Liège, 20–26 July 1997) (Turnhout: Brepols 2001), 41–46.

¹⁴ Fabrizio Baccetti, *Origine e destino dei Conicorum Elementorum Quintus et Sextus*, Tesi di laurea (rel. Pier Daniele Napolitani, Università di Pisa, A.A. 2005–2006); see also Aldo Brigaglia, “La ricostruzione dei libri V e VI delle Coniche da parte di F. Maurolico,” *Bollettino di storia delle scienze matematiche* 17, 2 (1997): 267–307, and Id., “Maurolico's Reconstruction of the Fifth and Sixth Book of Apollonius' *Conics*,” in *Medieval and Classical Traditions and the Renaissance of Physico-Mathematical Sciences in the XVIth Century*, 47–57.

¹⁵ See this passage from the letter to Juan de Vega (§§ 177–180): “For what harm, I ask by the immortal God, would I do if I were to gather into a single work the definitions, concepts, postulates, problems, and theorems of Euclid's *Elements*, Theodosius' and Menelaus' *Spherics*, Apollonius' *Conics*, Serenus' *Cylindrica*, the works of Archimedes, Jordanus' *Arithmetica*, and the doctrines of music, perspective, astronomy, and mechanical inventions? Nothing at all, certainly; indeed, such a work could even restore the authors themselves to their full integrity, and scholars [*studiosi viri*] would have at hand a library of such great investigations.”

Indeed, the work on Memmo's *Conics* proved fruitful in several directions. After 1544, Maurolico turned to texts newly available through the *editio princeps* of Archimedes' works, especially the *Conoids and Spheroids*. In this treatise, Archimedes assumes many properties of conic sections without demonstration. Building on his earlier studies and, above all, on his continued work on Memmo's translation, Maurolico produced his own version of the text. Divided into two books, his *De conoidibus et sphaeroidibus* devotes the entire first part to rigorously proving what Archimedes had left implicit.¹⁶

Only a few years later, in July 1553, he completed the *De lineis horariis libri tres*, whose third book is an original treatise on conics. There he presented their main properties and offered methods for constructing them graphically, departing in form and method from the Apollonian tradition. The treatise was published posthumously in the *Opuscula mathematica* of 1575 (Venice, Francesco de Franceschi), and it had considerable influence – certainly on Christoph Clavius (who reports having seen the manuscript)¹⁷ and later on Giovanni Alfonso Borelli, who in his *Elementa Conica Apollonii Pergaei et Archimedis Opera Nova et breviori methodo demonstrata* (Rome, Mascardi 1679) wrote: “nostrae propositio- nes tanti viri [scil. Maurolyci] genium imitantur.”¹⁸

This tendency toward reorganisation and clarity naturally led him to take yet another step. As in many of his other writings, Maurolico felt compelled to recast his work on Conics in the form of a compendium, easily accessible to the *studiosi viri*. In his *Ordo congruus compendiorum*, likely dating from the late 1560s, he described a new synthesis:¹⁹

Compendium of Apollonius' *Conics* in three books, with the key and most necessary conclusions, especially those pertaining to the demonstration of the burning mirror, the quadrature of the parabola, and the areas of ellipses, which are of greatest importance for the application of conics to hour lines. Likewise, the *Cylindrica* of Serenus, with superfluities omitted.

¹⁶ See Beatrice Sisana's Note on the Text to her edition of the *De conoidibus et sphaeroidibus* in *Francisci Maurolyci, Archimedeae*, 391–400, especially, 393–394 (English version, 401–410, esp. 403–404).

¹⁷ Cristoph Clavius, *Gnomonices libri octo* (Rome: Francesco Zannetti, 1581), 58; see also pages 8 and 29.

¹⁸ On Maurolico's treatise and its influence on Borelli, see Andrea Del Centina and Alessandra Fiocca, “Gli Elementa conica di G. A. Borelli, un'opera dimenticata tra tradizione e innovazione,” *Bollettino di storia delle scienze matematiche* 43, 1 (2023): 61–104.

¹⁹ The *Ordo congruus compendiorum* – preserved in ms. Par. Lat. 7466, ff. 4r–5r of the Bibliothèque Nationale de France – presents all the mathematical disciplines in the form of compendia. It is found in the same manuscript as one of the versions of the *Index lucubrationum*. This project was closely linked to Maurolico's collaboration with the Jesuits of Messina in developing a mathematical *cursus*, a project only partially realised and never published. See Rosario Moscheo, *I Gesuiti e le matematiche nel secolo XVI. Maurolico, Clavio e l'esperienza siciliana* (Messina: Società Messinese di Storia Patria, 1998).

This compendium, unfortunately, appears to be lost.²⁰ Yet Maurolico's engagement with conic sections did not end there. For him, the task was never complete. A comparison between the 1654 printed edition and the Escorial autograph manuscript shows that he continued to revise and refine the text throughout his life. As Roberta Tassora writes:²¹

The elements just described seem to indicate, with a high degree of probability, that Maurolico may have revised the 1547 draft, introducing changes that are now preserved in the printed edition. These considerations – together with the fact that the 1654 edition is the version that circulated, was read, and was possibly commented upon by contemporary mathematicians – suggest the appropriateness of preparing a critical edition that takes the 1654 printing as the reference text.

The study of conic sections, like much of Maurolico's oeuvre, remained a perpetual *work in progress* – a workshop he never closed.

3. Maurolico's mathematical rewriting of the text

The Madrid manuscript (Escorial J.III.31) shows that Maurolico approaches the *Conics* not as a philologist attempting to reconstruct a damaged text, but as a mathematician intent on restoring the coherence of its demonstrations. Working – as already noted – essentially on Memmo's defective Latin edition, he recasts the material into a clearer and more systematic form. The additions and *scholia* – especially numerous in Book I – do not merely clarify the argumentation but often provide new links, auxiliary propositions, or alternative proofs. Their style closely recalls that of the youthful *Elementa Conicorum*, now lost, to which several of these reconstructions seem to trace back.

Although the overall structure of Apollonius' work is preserved, Maurolico intervenes freely whenever he perceives gaps, obscurities, or inconsistencies in the inherited text. In such cases, he does not attempt to infer the "Apollonian" solution but instead constructs a demonstration that is mathematically rigorous by his own standards. As noted by Saito,²² this occurs, for instance, in Propositions I.2 and III.7, where the lack of a fully preserved diagram or argument leads Maurolico to develop an autonomous reconstruction. These proofs are rigorous,

²⁰ It was still in the hands of Maurolico's nephew, Francesco Jr., Baron della Foresta, at least until the end of the sixteenth century, as attested by letters from the Jesuits Vincenzo Carnava and Vincenzo Reggio (1588–89), who intended to publish it together with other writings. See Christoph Clavius, *Corrispondenza*, ed. Ugo Baldini and Pier Daniele Napolitani (Pisa: Dipartimento di Matematica, 1992), vol. 4, letters 150 and 151.

²¹ Tassora, Note on the Text to the edition of the *Apollonii conica elementa*, National Edition, vol. VIII, sect. 2 (<https://maurolico.it/Maurolico/sezione.html?path=8.2>).

²² Saito, "Quelques observations sur l'édition des *Coniques* d'Apollonius de F. Maurolico," 239–258.

though sometimes more elaborate than what might have emerged from a Greek exemplar.

This attitude is also evident in his handling of the order and composition of the propositions. Maurolico does not hesitate to add what he deems necessary for the flow of the demonstration, or to suppress what he finds superfluous. The resulting exposition is more schematic and formally controlled than that of Memmo: while the underlying demonstrative structure often remains Apollonian, its presentation is recast in a concise and uniform mathematical style.

4. Federico Commandino and the philological restoration of Apollonius Greek sources and working method

Federico Commandino approached Apollonius' *Conics* from a fundamentally philological perspective. Unlike Maurolico, he based his 1566 Latin edition directly on Greek manuscripts. His method combined rigorous philological inquiry, systematic collation, and selective use of late-antique commentaries.

The result was the 1566 edition of Apollonius' *Conics*, entitled *Apollonii Pergaei Conicorum libri quattuor* (Bologna, Alessandro Benacci, 1566),²³ which included his Latin translation of the first four books of the *Conics*, together with Eutocius' commentary, selected lemmas from Book VII of Pappus' *Collection*, and the two treatises of Serenus: *On the Section of a Cylinder* and *On the Section of a Cone*. In putting together this volume, Commandino actively sought out Greek manuscripts, evaluated their textual authority, and integrated them into his editorial work.

From the outset, Eutocius' commentary played a central role in this project. To prepare his Latin translation of the commentary, Commandino certainly had access to Urb. Gr. 73, which contains only Eutocius' commentary on Apollonius' *Conics*.²⁴ His hand can be recognised in marginal additions on the opening folio and, in fact, on almost all the folios of the manuscript; therefore, Urb. Gr. 73 must be counted among the Greek codices underlying the Latin version of Eutocius printed in 1566.

²³ *Apollonii Pergaei Conicorum libri quattuor. Una cum Pappi Alexandrini Lemmatibus, et Commentariis Eutocii Ascalonitae. Sereni Antinsensis philosophi Libri duo nunc primum in lucem editi. Quae omnia nuper Federicus Commandinus Urbinas mendis quamplurimis expurgata a graeco convertit et commentariis illustravit.*

²⁴ On Urb. Gr. 73 and its relationship to Mutinensis A. V. 7.16 (gr. 103), see Decorps-Foulquier, Eutocius d'Ascalon, *Commentaire sur les Livres I-IV des Coniques d'Apollonius de Perge, Fragmenta, Lemmes de Pappus aux Coniques I-IV*. Texte grec, établi, traduit et commenté par Micheline Decorps-Foulquier et Michel Federspiel, (Berlin, New York: Walter de Gruyter, «Scientia Graeco-Arabica», 2014), LXXIV-LXXX, and Heiberg, in *Apollonii Pergaei quae graece exstant cum commentariis antiquis*. Edidit et latine interpretatus est I.L. Heiberg, Lipsiae: In Aedibus G.B. Teubneri 1893, vol. II, LXXXII.

As regards the lemmas from Book VII of Pappus' *Collection*,²⁵ Commandino relied for their Greek text on Ms. 115 of the Newberry Library in Chicago (**k**), which transmits Book VII alone.²⁶

Identifying the Greek sources used for Apollonius' *Conics* and for the two books of Serenus has long been a difficult problem. In his Latin edition, Commandino mentions *codices graeci* – in the plural – only a few times, and Heiberg already noted that these scattered remarks did not suffice to determine his Greek base text.²⁷ Modern work by Micheline Decorps-Foulquier has shown, however, that his Latin translation incorporates the corpus of corrections to Vat. Gr. 203 transmitted by Bodl. Canonicianus Gr. 106; Commandino thus stands within the “Canonicianus” line of the Apollonian tradition.²⁸

Although we know from the loan registers of the Biblioteca Marciana that, on 7 August 1553, during a stay in Venice with Cardinal Ranuccio Farnese, Commandino borrowed Bessarion's Marcianus codex (Z. Gr. 518 = 539), which contains Apollonius' *Conics* together with the works of Serenus,²⁹ the Apollonius-Serenus text on which he actually worked is the later copy *Vindobonensis Suppl. Gr. 9*, prepared in 1557 by Camillo Zanetti – who used Bessarion's Marcianus as his exemplar – and later completed by Manuel Provaris with additional geometrical material.³⁰ This manuscript contains numerous marginal

²⁵ For the distribution of Pappus' lemmas in the 1566 edition, see Argante Ciocci, “Federico Commandino and the Latin Edition of Pappus' *Collection*,” *Archive for History of Exact Sciences* 76, issue 2 (2022): 129–151.

²⁶ On **k** as the source of the Greek text used by Commandino for the lemmas from Pappus VII, see Athanasius P. Treweek, “Pappus of Alexandria. The Manuscript Tradition of the *Collectio mathematica*,” *Scriptorium* 11 (1957): 195–233, and Ciocci, “Federico Commandino and the Latin Edition of Pappus' *Collection*,” 129–151.

²⁷ Heiberg, in *Apollonii Pergaei quae graece exstant cum commentariis antiquis*, II, LXXXII: “in Apollonio vero, quae de codicibus suis dicit, tam pauca sunt, ut inde de eo nihil certi concludi possit.” See Apollonius, *Apollonii Pergaei Conicorum libri quattuor. Vna cum Pappi Alexandrini lemmatibus, et commentariis Eutocii Ascalonitae. Sereni Antinsensis philosophi libri duo nunc primum in lucem editi. Quae omnia nuper Federicus Commandinus mendis quamplurimis expurgata e Graeco conuertit, & commentariis illustravit* (Bononiae: Ex officina Alexandri Benatii, 1566), f. 30v (comment B to I.46), f. 100r (comment A to III.55), f. 109r (comment B to IV.38); and, for Serenus, part II, f. 28v (comment B to *De sectione con*i 43).

²⁸ Micheline Decorps-Foulquier, *Recherches sur les Coniques d'Apollonios de Pergé et leurs commentateurs grecs: histoire de la transmission des livres I-IV* (Paris: Klincksieck, 2000); Ead., “La tradition manuscrite du texte grec des Coniques d'Apollonios de Pergé (livres I-IV),” *Revue d'histoire des textes* 31 (2001): 61–116, especially 98 and LXI–LXII.

²⁹ See Carlo Castellani, “Il Prestito dei codici manoscritti della Biblioteca di San Marco in Venezia ne' suoi primi tempi e le conseguenti perdite de' codici stessi,” *Atti del Reale Istituto Veneto di Scienze, Lettere ed Arti* 55 (1896–1897): 350–351.

³⁰ For the compilation and contents of *Vindobonensis Suppl. Gr. 9* (Apollonius, Serenus, Euclid, Aristarchus, Hypsicles, etc.), and for its relationship to Marc. Gr. 518 and Vat. Gr. 191, see

and interlinear corrections, attributable to two hands: an earlier corrector,³¹ and Commandino himself, who added Greek notes, textual supplements, and various emendations. In this codex he also filled the lacunae at the beginning of the preface and adopted variants drawn from Pappus' *Collection* (VII.32); comparison with his autographs preserved in Urbino confirms the identification.³²

The corrections introduced by Commandino in the *Vindobonensis* – marginal additions, textual integrations from Pappus, and the restoration of lacunae at the beginning of the preface – were incorporated into the 1566 Latin translation and are traceable in several places in the printed edition. Taken together, these elements reflect a remarkably structured philological approach. The 1566 Latin edition does not rely on a single Greek witness, but on a sustained process of collation across several manuscripts: Urb. Gr. 73 (Eutocius); Ms. 115 (**k**) (Pappus); the *Vindobonensis Suppl. Gr. 9*, itself derived from the Marciana codex (Apollonius and Serenus); and readings from Vat. Gr. 203, which tie Commandino's text to the corrective tradition of the *Canonicianus* group. In this light, his repeated references to *codices graeci* – notably in the plural – are to be taken in a strictly literal sense: his edition emerged from a broad and carefully controlled engagement with multiple strands of the Greek textual tradition.

Against this background, the significance of Commandino's 1566 edition becomes fully apparent. It represents a distinctive moment in the Renaissance recovery of Greek mathematics: a fusion of textual scholarship and mathematical insight that established a lasting point of reference for later generations of mathematicians and editors.³³

5. *Typographical design and textual interventions*

The structure of the 1566 edition offers a concrete reflection of its philological ambition. Rather than conflating its sources, Commandino presents the text of Apollonius, Eutocius's commentary, his own annotations, and the lemmas drawn from Pappus's *Collection* as distinct textual layers, each marked by specific typographical features and coordinated through

Decorps-Foulquier, "La tradition manuscrite du texte grec des Coniques d'Apollonios de Pergé (livres I–IV)," 98.

³¹ This corrector carefully rectified the faults inherited from the Marcianus and inserted many variants belonging to the tradition made known to us by *Canonicianus* Gr. 106.

³² For examples of Commandino's corrections in *Vindobonensis Suppl. Gr. 9* and their incorporation into the Latin text, see the additions filling the lacuna in the preface (f. 1r, lines 13–15) and the marginal note on f. 1v; see Apollonius, *Apollonii Pergaei quae graece exstant*, II, 15–17, and Apollonius, *Apollonii Pergaei Conicorum libri quattuor*, 4r–4v.

³³ From a philological point of view, Commandino amends many corrupted passages he found in his *graeco exemplari*. Apollonius *Apollonii Pergaei Conicorum libri quattuor*. (I.12 (p. 15r), I.13 (p. 16r), I.41 (p. 30v), I.54 (p. 38v), I.55 (p. 38v), pp. 45v, 47v, 62r, II.51 (p. 65r), II.52 (p. 66r), II.53 (p. 67r et 67v), III.20 (p. 82r), III.21 (p. 82v), III.25 (p. 85v), III.36 (p. 91r).

alphabetical cross-references. This design – documented in manuscript Vitt. Em. 1510 of the Biblioteca Nazionale Centrale di Roma³⁴ – allows readers to navigate seamlessly among the Greek sources, the Latin translation, and Commandino’s explanatory apparatus.

From a textual standpoint, the edition is notable for the precision of its emendations. Drawing on multiple Greek witnesses, Commandino corrected numerous corrupted passages in Books I–III and restored the internal coherence of many demonstrations.³⁵ A particularly revealing case is that of Proposition II.4. The construction of a hyperbola through a given point with assigned asymptotes is present in the transmitted text, yet Eutocius – in his commentary on Archimedes, *De Sphaera et Cylindro* II.4 – explicitly states that this proposition is *not* found in the *Conic Elements*. He then provides a demonstration that coincides *verbatim* with the one printed in II.4.³⁶ Commandino thus concludes that the proposition must be a later interpolation, inserted either by Eutocius himself or by someone drawing directly on his commentary. A further argument comes from Pappus, who includes the same construction among the lemmas to *Conics* Book V – a task that would have been superfluous if the proposition had belonged to the authentic Book II.³⁷

The case of II.4 exemplifies Commandino’s critical method: meticulous attention to the manuscript tradition, systematic comparison with ancient sources, and a mathematically informed evaluation of the internal structure of the text. Through this synthesis of philological rigor and technical insight, the 1566 edition becomes not merely a Latin translation, but a genuinely critical reconstruction of Apollonius’s treatise.

6. Two different styles of mathematical language

Having outlined the general features of Maurolico’s and Commandino’s approaches to Apollonius in the previous section, we shall now turn to a closer comparison of their mathematical and linguistic styles.

At the beginning of the *Conics*, Apollonius introduces the basic geometrical notions—cone, axis, vertex, base, and the key fundamental elements of a conic, the diameter and

³⁴ This manuscript preserves traces of Commandino’s collaboration with the printer Alessandro Benacci, particularly regarding the typographical layout and the structure adopted for the 1566 edition. See A. Ciocci, “Federico Commandino and the Latin Edition of Apollonius’s *Conics* (1566),” 393–421.

³⁵ For selected corrected loci, see *Apollonii Pergaei Conicorum libri quattuor*, I.12, I.13, I.41, I.54–55, etc.

³⁶ For this concordance, see Wilbur Knorr, “The Hyperbola-Construction in the *Conics*, Book II: Ancient Variations on a Theorem of Apollonius,” *Centaurus* 25, 4 (1982): 253–291, 256.

³⁷ See Pappus, *Pappi Alexandrini Collectionis quæ supersunt e libris manu scriptis edidit latina interpretatione et commentariis instruxit Fridericus Hultsch* (Berolini, apud Weidmannos, 1877), vol. II, VII.204; Knorr, “The Hyperbola-Construction in the *Conics*, Book II,” 261–282.

the ordinates. The first ten propositions deal mainly with the properties of the cone and its sections, before the three curves are formally defined. It is only in propositions 11–14 that Apollonius introduces the parabola, ellipse, hyperbola, and the so-called “opposite sections,” together with another essential parameter, the *latus rectum*. As Fried and Unguru have observed, what Apollonius *shows* in propositions 1–10 is the way the cone is to be cut, whereas in propositions 11–14 what he *deduces* is the *symptōma*, the defining property of each section.³⁸ This brief reminder will suffice to situate the passages compared below, which concern the definition of the hyperbola in *Conics* I.12.

6.1. Mathematical and Philological Ways: Maurolico vs. Commandino on *Conics* I.12

Comparing the manuscript of Maurolico with Commandino’s printed edition, we can observe significant differences in expository style and in the Latin lexicon used to translate key Greek terms typical of conic geometry.

Let us consider, for instance, the two Latin versions of the enunciation (πρότασις) of *Conics* I.12 – the proposition that defines the hyperbola.³⁹

Maurolico, *Apollonii Conica Elementa*
(ms. Escor. J. III. 31, 7r-v)

Si conus plano secetur per **axim**, secetur autem et altero plano secante basim coni per rectam ad rectos existentem basi trianguli per axim: et diameter sectionis producta coincidat uni laterum trianguli per axim extra coni verticem; quae a sectione ducta est aequidistans communi sectioni secantis plani et basis coni usque ad diametrum sectionis, poterit id, quod superficies adiacens ad

Commandino, *Apollonius* (1566)

Si conus plano per **axem** secetur, secetur autem et altero plano secante basim coni secundum rectam lineam, quae ad basim trianguli per axem sit perpendicularis: et sectionis diameter producta cum uno latere trianguli per axem, extra verticem coni conveniat; recta linea, quae a sectione ducitur aequidistans communi sectioni plani secantis, et basis coni usque ad sectionis diametrum, poterit spacium adiacens linea, et ad quam ea, quae

³⁸ Michael Fried and Sabetai Unguru, *Apollonius of Perga’s Conica* (Leiden–Boston–Köln: Brill, 2001), 79–92.

³⁹ The comparison below concerns Maurolico’s and Commandino’s Latin renderings of *Con.* I.12. For the Greek text, see *Con.* I.12 in *Apollonius de Perge, Coniques, Texte grec et arabe, établi, traduit et commenté*, sous la direction de Roshdi Rashed, tome 1.2: *Livre I. Édition et traduction du texte grec par Micheline Decorps-Foulquier et Michel Federspiel* (Berlin, New York: Walter de Gruyter, «Scientia Graeco-Arabica», 2008). For an English paraphrase, see Thomas L. Heath, *Apollonius of Perga: Treatise on Conic Sections, Apollonius of Perga: Treatise on Conic Sections* (Cambridge: Cambridge University Press, 1896) 9–11. Our focus here is not on the Greek original, but on the lexical and stylistic choices made by the two Renaissance editors. See Fried and Unguru, *Apollonius of Perga’s Conica*, 79–90.

quamdam lineam, ad quam **rationem** habet linea in rectum manens diametro sectionis occurrensque exterius lateri trianguli, quam quadratum, quod sit a ducta a summitate coni penes diametrum sectionis in basim trianguli ad contentum sub basis segmentis a ducta factis, et latitudinem habens receptam sub ipsa a diametro ad verticem sectionis, excedens **specie** simili et similiter iacente, contento sub coincidente lateri trianguli, et sub illa, ad quam possunt ductae. Vocetur autem haec sectio hyperbole.

in directum constituitur diametro sectionis, subtenditurque angulo extra triangulum, eandem **proportionem** habet quam quadratum lineae, quae diametro aequidistans à vertice sectionis usque ad basim trianguli ducitur, ad rectangulum basis partibus, quae ab ea fiunt, contentum: latitudinem habens lineam, quae ex diametro abscinditur, inter ipsam et verticem sectionis interiectam; excedensque **figura** simili, et similiter posita ei, quae continetur linea angulo extra triangulum subtensa, et ea, iuxta quam possunt quae ad diametrum applicantur. Vocetur autem huiusmodi sectio hyperbole.

The comparison reveals both shared vocabulary and telling divergences. Maurolico translates λόγος as *ratio* and Commandino as *proportio* – a difference that is consistent throughout their works. More significant, however, is the rendering of εἶδος. In Apollonius, the term designates one of the key elements in describing the hyperbola and the ellipse, namely the rectangle defined by the *latus rectum* and the *latus transversum* – that is, by the diameter and its corresponding parameter in modern terminology. In Proposition I.12, however, the term refers specifically to the rectangle constructed from the abscissa and the *latus rectum*. Following Memmo, Maurolico uses *species*, the traditional Latin rendering of εἶδος. Commandino, by contrast, introduces the term *figura*, which refers more directly to the visible form as it appears in the diagram. His choice thus places emphasis on geometric visualization, whereas Maurolico's wording remains closer to the customary Latin vocabulary of the ancient tradition.

Now, let us get a look at the setting-out (ἐκθεσις) and determination (διορισμός) of the object.⁴⁰

⁴⁰ I provide in this note an English translation of Commandino's Latin text (1566), dealing with the setting-out (ἐκθεσις) and the determination (διορισμός): Let there be a cone whose vertex is the point *a* and whose base is the circle *bc*, and let it be cut by a plane through its axis, and let it make as a section the triangle *abc*. And let the cone also be cut by another plane, cutting the base of the cone in *de* perpendicular to *bc*, the base of the triangle *abc*, and let this second cutting plane make as a section on the surface of the cone the line *dfe*, and let the diameter of the section *fg* when continued meet *ac*, one [lateral] side of the triangle *abc* beyond the vertex of the cone at *h*. Let *ak* be drawn through *a* parallel to the diameter of the section *fg* and let it cut *bc* [at *k*]. And let *fl* be drawn from *f*, perpendicular to *fg*, and let it be contrived that as sq. (*ka*) is to rec. (*bkc*), so *fh* is to *fl*. And let some point *m* be taken, at random, on the section and through

Maurolico, Escor. J.III.31, 1547, 7r-v
 Conus, cuius vertex *a* basisque circulus *bg* secetur plano per axim: sitque sectio per 3^{am} $\triangle abg$ secetur et altero plano secante basim coni per rectam *de* ad rectos ipsi *bg* et faciente in conica superficie sectionem *dze* cuius diameter *zh* coincidat uni laterum, quod sit *ga* apud *t*. Item *ac* penes *zh*. Et *zl* ad rectos ipsi *zh* ita ut *tz* — *zl* sit sicut $\square ac$ — $\square bcg$. Et a contingenti puncto sectionis, utpote *m* ducatur *mn* penes *de*. Et connexa *tl* compleatur $\square zlon$. Et collapsis in unum *tl no* apud *x* compleatur $\square lpxo$.
 Dico iam quod *mn* potest $\square zx$ adiacens scilicet ad ipsam *zl* sub latitudine *zn* et excedens specie *lx* simili $\square^o tzl$.

Commandino, Apollonius 1566
 Sit conus, cuius vertex *a* punctum, basis circulus *bc*: et secetur plano per axem, quod sectionem faciat triangulum *abc*: secetur autem et altero plano secante basim coni, secundum rectam lineam *de* ad *bc* basim triangulo *abc* perpendicularem faciatque sectionem in superficie coni lineam *dfe*: et sectionis diameter *fg* producta cum ipso ac latere trianguli *abc* extra coni verticem conveniat in puncto *h*: deinde per *a* ducatur linea *ak* diametro aequidistans, quae fecit *bc*: et ab *f* ducatur *fl* ad rectos anguls ipsi *fg*: fiatque ut quadratum *ka* ad rectangulum *bkc*, ita *hf* linea ad lineam *fl*. Sumatur autem in sectione quodlibet punctum *m*; et per *m* ducatur *mn* aequidistans *de*; et per *n* ipsi *fl* aequidistans ducatur *nox*. Postremo iuncta *hl*, et ad *x* producta, per *lx* ipsi *fn* aequidistantes ducantur *lo*, *xp*. Dico *mn* posse spatium *fx*, quod quidem adiacet lineae *fl*, latitudinem habens *fn*, exceditque figura *lx* simili ei, quae *hfl* continetur.

A comparison of the setting-out (ἔκθεσις) and the determination (διορισμός) in Proposition I.12 reveals a further and more substantial divergence between Maurolico and Commandino.⁴¹ Although both mathematicians present the same geometric construction – a cone whose axial section is a triangle, a second plane perpendicular to the base of the triangle, which generates the section *dfe* (rendered as *dze* in Maurolico's version), whose

m, let *mn* be drawn parallel to *de*, and through *n*, let *nox* be drawn parallel to *fl*. Let *hl* be joined and continued to *x*, and *lo* and *xp* be drawn through *l* and *x* parallel to *fn*. I say that *mn* is equal in square to the rectangular plane *fx*, applied to *fl* having *fn* as breadth, and increased by a figure *lx* similar to rec. (*hfl*).

⁴¹ In the *Conics*, Apollonius generally follows the formal pattern of Euclid's *Elements*: πρότασις (general enunciation); ἔκθεσις (setting-out with reference to a diagram); διορισμός (a brief statement of what is to be proved or asserted about the configuration); and κατασκευή (auxiliary construction).

diameter fg is extended to meet one side of the axial triangle, along with the auxiliary lines required for the proof – their mode of exposition differs markedly (Fig. 1).

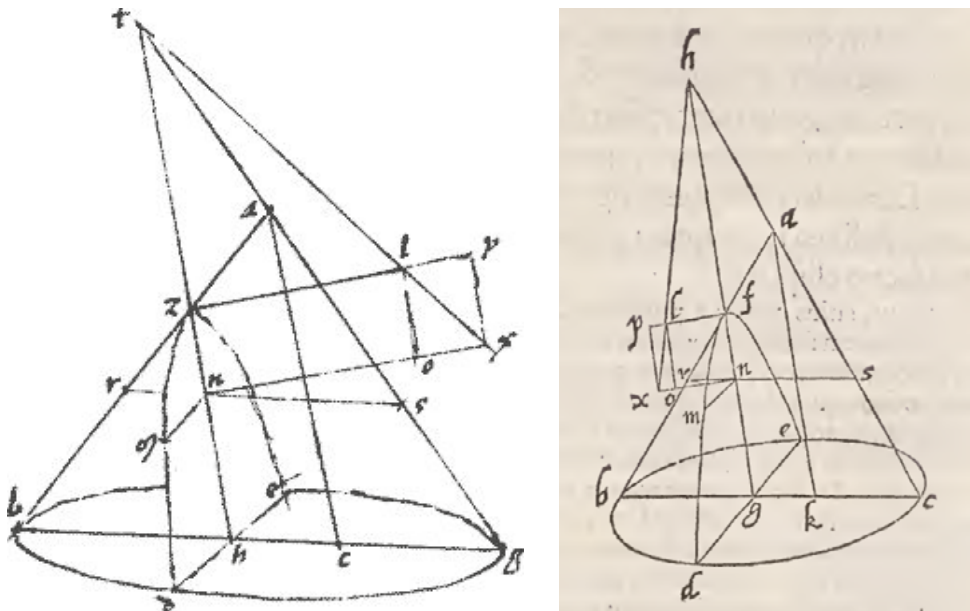


Fig. 1 – Diagrams of Proposition I.12. Maurolico (Escor. J.III.31) and Commandino (Apollonius 1566).

Commandino follows the Greek text with great fidelity. His Latin $\epsilon\kappa\theta\epsilon\sigma\iota\varsigma$ reproduces Apollonius' order clause by clause: the vertex a and base circle bc of the cone, the section abc cut by the axial plane, the second plane cutting dc perpendicular to bc , the curve dfe on the conical surface, and the construction of the diameter fg continued to h . Each step corresponds closely to the syntax and sequencing of the Greek, and Commandino regularly signals Euclidean dependencies in the margin. His aim is not to simplify the construction but to transmit it as part of a philologically reliable Latin version of Apollonius.

Maurolico's setting-out is of an entirely different nature. The Escorial manuscript compresses the same material into a remarkably compact paragraph, often into a single long sentence. Maurolico does not hesitate to depart from Memmo's earlier Latin version, nor does he follow the Greek clause structure. Instead, he reorganizes the construction according to his own mathematical idiom: symbols replace phrases, constructions are abbreviated, and references to Euclidean propositions are integrated directly into the flow of the text. The diagram plays a central role in this style, and Maurolico adapts the wording to

match closely what appears in the figure rather than what stands in Apollonius' wording.

A further technical difference concerns the transcription of the Greek letters in the diagrams. Commandino adopts a systematic Latin transliteration of the Greek alphabet, whereas Maurolico simply follows the conventions already used by Memmo in the 1537 Latin edition of the *Conics*. This confirms — in a minor but telling way — Maurolico's dependence on Memmo's text, which was his only source for Apollonius. For Maurolico, however, such conventions have no philological purpose: they form part of a more general tendency to streamline notation and to prioritise the clarity of mathematical reasoning.

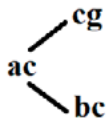
Gr. alphabet	A	B	Γ	Δ	E	Z	H	Θ	I	K	Λ	M	N	O	Π	X	P	Σ	T	Ξ	Υ	Φ	Ψ	Ω
Commandino	A	B	C	d	E	f	g	H	I	K	L	M	n	O	P	q	r	s	T	x	y	v	z	Ω
Maurolico	A	B	G	d	E	z	h	T	I	C	L	M	n	O	P	x'	r	s	Θ	x	y	f	Ψ	Ω

Such a difference emerges from a comparison of the text dealing with the construction (κατασκευή) and proof (ἀπόδειξις) in the two editions that Maurolico and Commandino elaborate. Let us look at the two approaches.

Maurolico shortens the construction (κατασκευή) to one line.⁴² Moreover, his proof (ἀπόδειξις), compared to the one by Apollonius, is abbreviated. Maurolico aims to take over Apollonius' results and demonstrates the proposition in his way, breaking away from

⁴² I provide in this note an English translation of Commandino's Latin text (1566), dealing with the construction (κατασκευή) and proof (ἀπόδειξις): For let *rns* be drawn through *n* parallel to *bc*. And *nm* is also parallel to *de*. Therefore [according to Proposition XI.15 of Euclid], the plane through *mn* and *rs* is parallel to the plane through *bc* and *de*, which is to the base of the cone. Therefore, if the plane is drawn through *mn* and *rs*, the section [according to Proposition I.4] will be a circle whose diameter is *rns*. And *mn* is perpendicular to it. Therefore rec. (*rns*) is equal to sq. (*mn*). And since as sq. (*ak*) is to rec. (*bkc*), so *fh* is to *fl*, and [according to Proposition VI.23 of Euclid] the ratio sq. (*ak*) to rec. (*bkc*) is compounded of [the ratios] *ak* to *kc* and *ak* to *kb*, therefore also the ratio *fh* to *fl* is compounded of [the ratios] *ak* to *kc* and *ak* to *kb*. But as *ak* is to *kc*, so *hg* is to *gc*, and *hn* is to *ns*; and as *ak* is to *kb*, so *fg* is to *gb* and *fn* is to *nr*. Therefore, the ratio *hf* to *fl* is compounded of [the ratios] *hn* to *ns* and *fn* to *nr*. And [according to Proposition VI.23 of Euclid], the ratio rec. (*hnf*) to rec. (*snr*) is compounded of [the ratios] *hn* to *ns* and *fn* to *np*. Therefore, as rec. (*hnf*) is to rec. (*snr*), *hf* is to *fl* and *hn* is to *nx*. But, with *fn* taken as common height [according to Proposition VI.1 of Euclid] as *hn* is to *nx*, rec. (*hnf*) is to rec. (*fnx*). Therefore also [according to Proposition V.11 of Euclid] as rec. (*hnf*) is to rec. (*snr*), so rec. (*hnf*) is to rec. (*xnf*), and [according to Proposition V.9 of Euclid] rec. (*snr*) is equal to rec. (*xnf*). But it was shown that sq. (*mn*) is equal to rec. (*snr*); therefore, also sq. (*mn*) is equal to rec. (*xnf*). But rec. (*xnf*) is the parallelogram *xf*. Therefore, *mn* is equal in square to *xf*, which is applied to *fl*, and having *fn* as breadth increased by the parallelogram *lx* similar to rec. (*hfl*). I will call such a section a hyperbola, and *lf* be called the straight line of application [of rectangular planes] to which the ordinates drawn to *fg* are equal in the square. I will call this straight line the *latus rectum* and the straight line *fh* the *latus transversum*.

Nam, ducta primum *rns* penes *bg* iam *per* 24^{am} 6ⁱ Euclidis ratio $\square ca - \square bcg$ componitur ex rationibus



Et propter similitudinem triangulorum [*acg*, *thg* atque *acb*, *zhb*] et proportionem laterum eadem ratio componetur ex rationibus *th*, *hg* atque *zh*, *hb* et similiter eadem componetur ex rationibus *tn* — *ns* atque *zn* — *nr*.

Verum, *per* 24^{am} predictam ratio

$\square tnz - \square snr$ componitur ex rationibus *tn* — *ns*, *zn* — *nr*.

Igitur $\square tnz - \square snr$ erit sicut

$\square ca - \square bcg$ et ideo sicut *tz* — *zl* ac sicut *tn* — *nx* propter $\triangle \triangle$ similitudinem.

Verum *per* primam sexti *tn* — *nx* sicut

$\square tnz - \square znx$.

Igitur $\square tnz - \square znx$ erit sicut

$\square tnz - \square snr$.

Quare, *per* 9^{am} 5ⁱ $\square snr -$ aequale — $\square znx$.

Cumque *per* 15^{am} 11ⁱ planum, in quo *mn*, *rs* aequedistet plano circuli *bgd* quae basis est coni.

Ideo *per* 4^{am} huius puncta *rms* erunt in periferia circuli, cuius diameter *rs*.

Ergo *per* 8^{am} 6ⁱ Euclidis $\square snr -$ aequum erit — $\square mn$ quare et $\square mn$ aequum $\square^o znx$.

Quod erat demonstrandum.

Vocetur autem talis sectio hyperbole: ipsa autem *lz* ad quam possunt ductae ad *zh* ordinatae: voceturque eadem et recta: transversa autem *zt*: ipsum autem $\square tzl$ species sectionis.

om.:

Et manifestum est, quod sicut *tz* — *zl* sic est $\square tnz$ ad $\square znx$ hoc est ad $\square mn$ namque eadem ratio fuit, quae *tn* — *nx*.

Ducatur enim *per* *n* linea *rns* aequidistans *bc*: est autem et *mn* ipsi *de* aequidistans. Ergo planum, quod transit *per* *mnrs* aequidistat plano *per* *bc*, *de*, hoc est basi coni [15. Undecimi]. Si igitur planum *per* *mnrs* producat, sectio circuli erit, cuius diameter *rns* [4 huius]: atque est ad ipsam perpendicularis *mn*. Ergo rectangulum *rns* aequale est *mn* quadrato. Itaque quoniam ut *ak* quadratum ad rectangulum *bkc*, ita est *hf* ad *fl*: proportio autem quadrati *ak* ad rectangulum *bkc* componitur ex proportione, quam habet *ak* ad *kc*, et ex ea, quam *ak* habet ad *kb*: et proportio *hf* ad *fl* composita erit ex proportione, *ak* ad *kc*, et proportione *ak* ad *kb*. Sed ut *ak* ad *kc*, ita *hg* ad *gc*, hoc est *hn* ad *ns*: et ut *ak* ad *kb*, ita *fg* ad *gb*, hoc est *fn* ad *nr*. Proportio igitur *hf* ad *fl* componitur ex proportione *hn* ad *nf*, et *fn*, ad *nr*. Ac proportio composita ex proportione *hn* ad *ns*, et *fn* ad *nr*; est ea, quam *hnf* rectangulum habet ad rectangulum *snr* [23 sexti]. Ergo ut rectangulum *hnf* ad *snr*, ita *hf* ad *fl*, hoc est *hn* ad *nx*. Ut autem *hn* ad *nx*, sumpta *fn* communi altitudine, ita *hnf* rectangulum ad rectangulum *fnx* [1 sexti]. Quare ut rectangulum *hnf* ad rectangulum *snr*, ita rectangulum *hnf* ad ipsum *fnx*: rectangulum igitur *snr* aequale est rectangulo *xnf* [9 quinti]. Sed quadratum *mn* ostensum est aequale rectangulo *snr*. Ergo quadratum *mn* rectangulo *xnf* aequale erit. Rectangulum autem *xnf* est parallelogrammum *xf*. Linea igitur *mn* potest spatium *xf*, quod lineae *fl* adiacet; latitudinem habens *fn*, excedensque figura *lx* simili ei, quae *hfl* continetur. Dicatur autem huiusmodi sectio hyperbole; et linea *lf*, iuxta quam possunt, quae ad *fg* ordinatim applicantur, quae quidem etiam recta appellabitur, transversa vero *hf*.

In sum, Proposition I.12 offers a clear illustration of the divergent editorial programs of the two mathematicians. Maurolico seeks to assimilate Apollonius' results into his own compact and operational mathematical idiom, rewriting the demonstrations in a style geared toward efficiency and internal coherence. Commandino, by contrast, seeks to transmit a faithful and carefully annotated Latin version of the Greek text, contextualized through Eutocius, Pappus, and the broader ancient tradition. The one rewrites in pursuit of mathematical clarity, the other edits in the name of philological accuracy.

7. *Drawing the diagrams: Maurolico and Commandino. From schematic medieval diagrams to Renaissance reconstructions*

The diagrammatic tradition of Greek mathematical texts follows stable and highly schematic conventions. Figures are drawn with ruler and compass, and even complex curves – such as conic sections or spirals – are represented through circular arcs or intersections of arcs, often producing the familiar lens-shaped forms. These diagrams are not intended to convey metrical accuracy but only to indicate the relative placement of points and lines within the argument.⁴³

This pattern is also evident in the humanistic transmission of Archimedes: in Iacopo da San Cassiano's Latin translation (ca. 1450), the diagrams of the conics still follow the schematic conventions of the medieval Greek tradition. Mathematicians such as Piero della Francesca – or even one of Regiomontanus' stature – reproduced them uncritically. Only with the Basel edition of 1544 were the curves relevant to Archimedes' arguments drawn in a geometrically meaningful way.⁴⁴

These limitations appear even more starkly in the manuscript tradition of Apollonius' *Conics*, where essential geometric relations – such as the inclination of the cutting plane or the mutual disposition of diameter, ordinates, and parameter – are rendered in schematic

⁴³ For general discussions of diagrammatic conventions in Greek and late-antique mathematical texts, see *inter alia*: Reviel Netz, *The Shaping of Deduction in Greek Mathematics: A Study in Cognitive History* (Cambridge: Cambridge University Press, 1999); Ken Saito, "A Preliminary Study in the Critical Assessment of Diagrams in Greek Mathematical Works," *SCIAMVS* 7 (2006): 81–144; Fabio Acerbi, "The mathematical *Scholia Vetera* to *Almagest* I.10–15," *SCIAMVS* 18 (2017): 133–259; Saito and Sidoli, "Diagrams and arguments in ancient Greek mathematics"; Sidoli, "The manuscript diagrams of Theodosios' *Spherics*," in *The History of Mathematical Proof in Ancient Traditions*, ed. by Karine Chemla (Cambridge: Cambridge University Press, 2012), 135–162; Reviel Netz, "Why Were Greek Mathematical Diagrams Schematic?" *Nuncius* 35 (2020): 506–535.

⁴⁴ On the figures in the manuscript tradition descending from Iacopo da San Cassiano's translation, see Paolo D'Alessandro and Pier Daniele Napolitani, *Archimede Latino / Archimedes Latinus. Iacopo da San Cassiano e il corpus archimedeo alla metà del Quattrocento con edizione della Circuli dimensio e della Quadratura parabolae* (Paris: Les Belles Lettres, 2012), 283–327.

ways that can contradict the very hypotheses of the propositions. The diagrams known to Maurolico (through Memmo's 1537 Latin translation) and to Commandino (through the *Vindobonensis Suppl. Gr.* 9) typically fail to reflect the structure of Apollonius' constructions.

Against this background, the work of the two Renaissance editors marks a decisive break with the medieval tradition. Maurolico systematically corrected the figures found in Memmo's edition,⁴⁵ while Commandino redrew the entire corpus of diagrams for Apollonius, Eutocius, Pappus, and Serenus. Their aims were convergent: to restore diagrams that were metrically consistent with the text and that rendered visually the logical structure of the demonstrations. Maurolico worked largely freehand, whereas Commandino relied on the constructive procedures described in his *Liber de Horologiorum descriptione* (1562); yet both inaugurated a new attention to the explanatory and epistemic role of the diagram, anticipating the more sophisticated iconography of seventeenth-century mathematics.

7.1 Maurolico and Commandino on the parabola (*Conics*, I.11)

The contrast between Maurolico and Commandino emerges most clearly when they confront the same defective material: the diagram of Proposition I.11, where Apollonius establishes the *symptoma* of parabola. Both mathematicians inherit schematic figures that crystallise the conventional features of the Byzantine tradition. In Memmo's 1537 edition, the parabola is drawn as a lens-shaped intersection of circular arcs; the diameter is not parallel to the side of the axial triangle; the line drawn parallel to the common section of the cutting plane is misplaced; and the axial triangle itself is geometrically inconsistent with the proposition's hypotheses. In short, the resulting figure cannot be regarded as a parabola in Apollonius' sense.

Maurolico reacts by reconstructing the pair of figures freehand, restoring the conditions that Apollonius' demonstration presupposes. He redraws the diameter so that it is parallel to the relevant side of the axial triangle; corrects the direction of the line drawn parallel to the common section; and replaces the medieval "lens" with a curve that is metrically coherent with the properties stated in the proposition. The result is not a revival of ancient drawing conventions, but an explicit analytic tool – a diagram constructed to express the logical structure of the proof (See Fig. 3).

Commandino, by contrast, approaches the same proposition through the constructive procedures outlined in his *Liber de Horologiorum descriptione* (1562). His redesign of the figure is regular, symmetrical, and metrically controlled. Yet in doing so, he tends to normalise the configuration: the diameter is presented in an axially oriented setting, and the

⁴⁵ On Maurolico's conception of the diagram as an epistemic component of geometrical reasoning, see Michela Malpangotto, "Graphical Choices and Geometrical Thought in the Transmission of Theodosius' Spherics from Antiquity to the Renaissance," *Archive for History of Exact Sciences* 64 (2010): 75–112.

figure is drawn in a manner that privileges orthogonal relations and Euclidean regularity.

The result is elegant, but less tightly aligned with the specific geometric relations of Apollonius' construction (See Fig. 4).

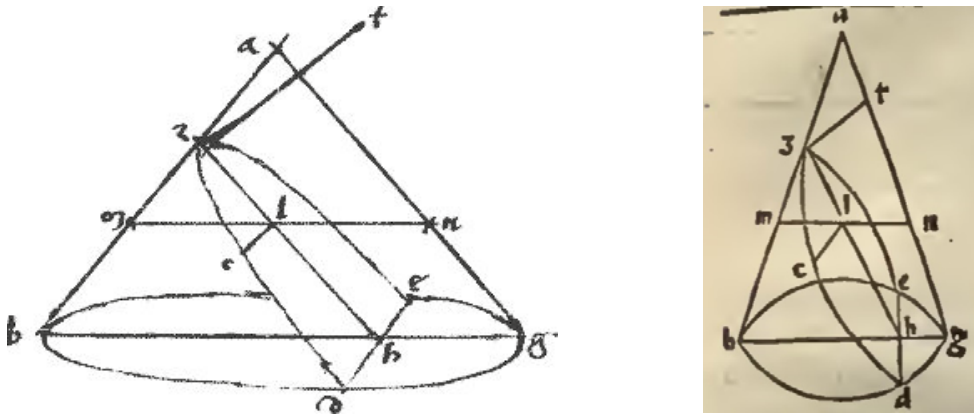


Fig. 3 – Diagrams of Proposition I.11: Maurolico (Escor. J.III.31) and Memmo (1537).

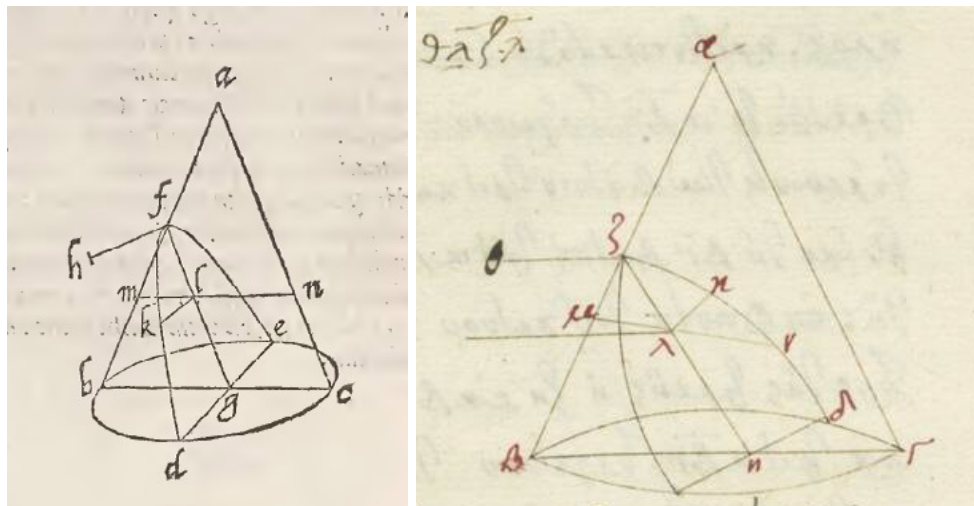


Fig. 4 – Diagram of the Proposition. I.11 parabola. Latin edition by Commandino (1566, 14r). In the diagram drawn by Commandino, hf is the *latus rectum*. On the right: diagram from the codex Vindobon. Suppl. Gr. 9, f. 10r.

The two solutions thus reflect two different editorial attitudes. Maurolico rewrites the inherited diagram to restore the geometry required by the proof; Commandino normalises the construction within a broader graphical method. Both reject the medieval schematic tradition – but they do so in opposite ways: Maurolico by recovering Apollonius' configuration, Commandino by embedding it in a more uniform constructive framework.

8. Maurolico and Commandino on the hyperbola (Conics, I.16)

Whereas in Propositions I.11–14 the difficulty lay in rendering spatial constructions involving axial inclinations, the subsequent propositions are confined to planar configurations. Yet these remain far from straightforward: the use of generic diameters means that the lines drawn ordinatewise are not conceived as perpendicular to the diameter, and the diagrams must capture a combination of orthogonal and oblique directions within the plane.

A representative case is Proposition I.16, where Apollonius determines the pair of conjugate diameters of the opposite sections (i.e. the two-branched hyperbola).⁴⁶ Here the diagram must capture a subtle configuration: a *generic* diameter cl of the hyperbola, two lines drawn ordinatewise (lt and ch) that are not orthogonal to cl , and a *latus rectum* ae drawn perpendicularly to the transverse side ab . This mixture of non-orthogonal and orthogonal relations, characteristic of Apollonius' construction, is essential to the logic of the proposition.⁴⁷

Maurolico represents this configuration with a striking degree of fidelity. In his diagram, the diameter is genuinely generic, not aligned with the axis of the hyperbola; the two ordinatewise lines meet the diameter obliquely, as Apollonius requires; and the *latus rectum* is drawn orthogonally to the transverse side. The figure thus expresses the precise interplay of relations on which the proof depends. Maurolico's drawing is freehand, yet it adheres closely to the Apollonian construction (See Fig. 5).

Commandino's solution reflects a different graphical philosophy. Applying his constructive method, he tends to align the diameter with the axis of the hyperbola and to regularise the configuration into an orthogonal framework. The resulting diagram is more symmetrical and metrically controlled, but it no longer reproduces the layout of lines required for Apollonius' determination of a conjugate diameter in the general case. In effect,

⁴⁶ For Apollonius, two diameters are *conjugate* when each bisects the lines drawn parallel to the other. His definition is explicit: "The two straight lines, each of which, being a diameter, bisects the straight lines parallel to the other, I call conjugate diameters of a curved line and of two curved lines." *Apollonius of Perga, Conics*, Books I–III, translated by R.C. Taliaferro (Santa Fe: Green Lion Press, 2000), 4.

⁴⁷ "If through the midpoint of the transverse side of the opposite sections a straight line be drawn parallel to a straight line drawn ordinatewise, it will be a diameter of the opposite section, conjugate to the diameter just mentioned." *Ibid.*, 33.

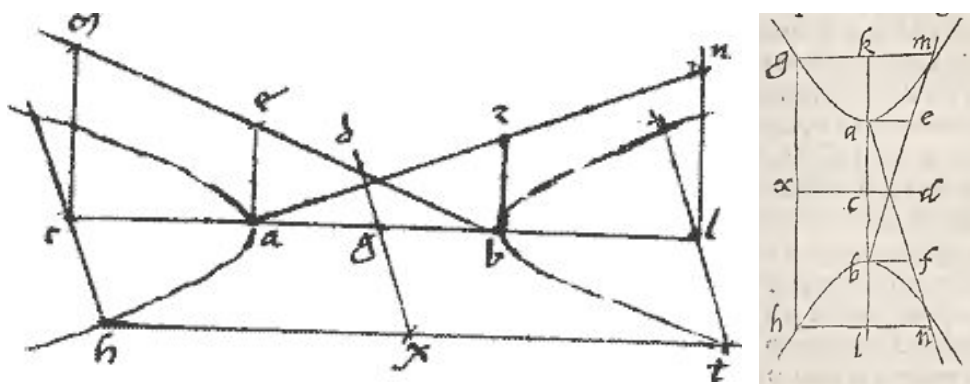


Fig. 5 – Diagram of Proposition I.16: Maurolico and Commandino.

Commandino assimilates the generic diameter of the hyperbola to the special case of the axis, thereby smoothing out a geometric distinction.

This was not due to a misunderstanding. In the diagrams he draws for Eutocius – in particular those accompanying Proposition I.16 in that commentary – Commandino clearly distinguishes between the axis and a generic diameter and accurately represents the diameter as non-axial. He was thus fully aware of the difference between the general and special cases, but in his edition of the *Conics* chose to favour a regularised and visually balanced construction over a faithful rendering of Apollonius' diagrammatic logic.

The difference, therefore, lies not in difficulty in representing two and three-dimensional visualisation, but in the interpretation of the configuration itself. Maurolico preserves Apollonius' use of a generic diameter and the associated non-orthogonal relations; Commandino normalises the figure by recasting it within his broader constructive method. The two diagrams reveal, with exceptional clarity, the divergent editorial strategies that guide their work: Maurolico reconstructs the configuration required by the proof, while Commandino regularises it according to a normative graphic procedure.

9. Two editorial philosophies of the diagram

The comparison of Propositions I.11 and I.16 reveals that the differences between Maurolico and Commandino are not merely graphical but reflect two distinct conceptions of what a mathematical diagram is and how it should function within a demonstration.

For Maurolico, the diagram is an essential component of the argument. It must express the configuration on which the proof depends and may be rewritten whenever the inherited tradition obscures or contradicts that structure. His drawings correct misalignments, restore missing relations, and adapt the visual layout to the internal logic of the demon-

stration. This approach reflects the broader character of his mathematical practice: text and figure are coextensive, and clarity in one demands clarity in the other. The diagram becomes an instrument of analysis and, in many cases, a means of reconstructing the reasoning that the damaged text no longer conveys.

Commandino's attitude is different. For him, the primary task of the editor is to transmit the text of Apollonius and the late-antique commentary in a philologically controlled form. The diagram serves as a regulated visual counterpart to the restored text. His figures are constructed according to a systematic Euclidean method, privileging axial symmetry, orthogonal relations, and metrical regularity. Even when Apollonius' construction implies a more generic configuration – as in the case of the conjugate diameters – Commandino tends to select the realisation that best fits within his normative graphical framework. The result is visual clarity anchored in textual fidelity, rather than in a reconstruction of the more complex geometric structure presupposed by the argument.

These two approaches should not be seen as opposing tendencies. Each reflects an attempt to recover Apollonius' geometry under different editorial priorities: Maurolico seeks to restore the operative structure of the arguments, while Commandino aims to preserve the textual and constitutive order of the ancient tradition. Their work thus offers two complementary ways of "rewriting" Apollonius: one mathematical, the other philological, both essential to the Renaissance reappropriation of the *Conics*.

10. Rewriting Apollonius

The comparison developed in this article shows that Maurolico and Commandino reopened access to Apollonius' *Conics* through two complementary strategies. Their work does not simply repair a damaged tradition: it reflects two coherent ways of making an ancient mathematical text intelligible, usable, and meaningful for sixteenth-century readership.

For Maurolico, recovering Apollonius meant reconstructing the operative structure of the arguments. His rewriting of proofs, his reorganisation of the material, and his deliberate redrawing of the diagrams all aim at restoring a mathematical text that could once again be *worked with*. Although his edition of the *Conics* remained unpublished until 1654 and therefore had no direct impact on the first phase of early modern geometry, the analytical attitude that shapes it circulated through other works – most notably the *De lineis horariis* – which influenced authors such as Clavius and, later, Borelli.

Commandino offered a different service to Renaissance mathematicians. His 1566 Latin edition stabilised the Greek text, integrated the late-antique commentary, and provided a regulated corpus of diagrams designed as a visual counterpart to the restored wording. For scholars, teachers, and students, it quickly became the standard gateway into Apollonius and shaped the reception of the *Conics* throughout the late sixteenth and seventeenth centuries.

Taken together, Maurolico and Commandino represent two distinct yet mutually reinforcing modes of Renaissance engagement with ancient mathematics: one oriented toward mathematical reconstruction, the other toward philological restitution. Their joint contribution made Apollonius available to two kinds of readers – those who wished to *work* with the *Conics* and those who wished to *study* them – and prepared the ground for the divergent appropriations of conic geometry in the seventeenth century.

In this sense, the two paths sketched in the introduction – an *imitatio* of the ancients that restores their words, and an *æmulatio* that rewrites their arguments as working mathematics – appear not as alternatives but as complementary forms of Renaissance engagement with Apollonius.

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The influence of practical arithmetic on the emergence of proto-symbolic algebra: A long-run survey

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Abstract

The paper provides a long-run overview of how the development of what we may term “proto-symbolic algebra” was profoundly shaped by the tradition of practical arithmetic, particularly as it evolved through abacus mathematics. Within this milieu, algebraic problem-solving techniques became increasingly sophisticated, and symbolic representations began to emerge. While the recovery of classical mathematical works provided an ideal of mathematical generality and abstraction, the mathematics practiced in abacus schools supplied computational techniques that were essential for the operationalization of algebra. Key figures such as Tartaglia, Cardano and Bombelli displayed influences of abacus mathematics in their algebraic works. The persistence of practical arithmetic techniques within these works suggests that the evolution of algebra was not a linear progression from classical to modern mathematics but rather a complex synthesis of diverse traditions.

Keywords

proto-symbolic algebra, abacus mathematics, hybridisation

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A possible starting point for the historiography on the emergence of new algebraic techniques in the European late Middle Ages can be identified in Pietro Cossali's *Origine, trasporto in Italia, primi progressi in essa dell'Algebra*, published in 1797–99.¹ Cossali was the first to identify Leonardo Pisano's *Liber abaci* as an important early source. Cossali correctly recognised the *Liber abaci* as an early thirteenth-century text, and was among the first to read its algebraic contents from extant manuscripts. Before Cossali's work, the history of algebra had been reconstructed solely from printed sources. The earliest and most famous of these was Luca Pacioli's *Summa de Arithmetica Geometria Proportioni et Proportionalità*, published in Venice in 1494, which Cossali used as a key comparison between Leonardo Pisano's and late fifteenth-century algebra.

Even though he was driven by an understanding of the evolution of mathematical thought that not all historians of mathematics would today subscribe to, Cossali correctly attributed the origins of these new algebraic techniques to Arabic mathematics, particularly al-Khwārizmī's *Al-jabr*, even though he did not know the existing Latin translations of that work. Nonetheless, his hypothesis that intermediary algebraic treatises must have existed between Fibonacci and Pacioli shaped historians' understanding for more than two centuries, despite the limitations of his evidence.

An important contribution was made by Guglielmo Libri's *Histoire des Sciences Mathématiques en Italie* (1838–1841).² Libri confirmed that a new form of algebra entered Europe through Latin translations of al-Khwārizmī, and published the text by Gerardo da Cremona. He noted that he found no evidence of algebraic texts in the thirteenth century beyond Leonardo Pisano's works, and reproduced in print the fifteenth chapter of his *Liber abaci*. Moreover, Libri identified a number of abacists from the fourteenth century, including Paolo dell'Abaco, Paolo Gerardi, and Giovanni Danti, as early transmitters of algebraic ideas, and mentioned having read a number of anonymous manuscripts. He noted that some of these abacus manuals discussed equations of third and higher degrees, but he described some of their algebraic formulas as “des règles bizarres fondées sur de faux principes.”³

Further progress was made by Baldassarre Boncompagni, who published the surviving works of Fibonacci and described numerous fourteenth- and fifteenth-century Italian manuscripts containing algebraic material.⁴ The editions sponsored by Boncompagni pro-

¹ Cossali, *Origine, trasporto in Italia, primi progressi in essa dell'algebra* (Parma: 1797). See also Franci, “Pietro Cossali storico dell'algebra,” in *Pietro Riccardi (1828-1898) e la storiografia delle matematiche in Italia*, ed. by Barbieri, Cattelani (Modena: Università degli Studi di Modena, 1989), 199–217.

² Libri, *Histoire des sciences mathématiques en Italie*, 4 vols. (Paris: Jules Renouard, 1838–1841).

³ *Ibid.*, vol. 2, 212–213.

⁴ Boncompagni, *Intorno ad alcune opere di Leonardo Pisano matematico del secolo decimoterzo* (Roma: Tipografia delle Belle Arti, 1854); *Id.* (ed.), *Scritti di Leonardo Pisano. Vol. I, Il Liber*

vided an important groundwork for later scholarship, and, for some of Fibonacci's works, they are still today the only modern editions available.⁵ It was not until the mid-twentieth century that Gino Arrighi resumed this line of research, transcribing and studying abacus treatises more broadly. In 1980, Warren Van Egmond published his seminal *Catalogue of Italian Abacus Manuscripts*, which provided for the first time a nearly comprehensive listing of known abacus manuals, several of which include algebraic sections.⁶

Building on this evidence, Laura Toti Rigatelli and Raffaella Franci initiated a series of studies on abacus treatises from the 1980s. Using Van Egmond's catalogue, they and their students transcribed and analysed several fourteenth- and fifteenth-century abacus manuals, many of which were published in the series *Quaderni del Centro Studi di Matematica Medioevale* of the University of Siena.⁷ By surveying and providing access to these primary sources, their research contributed to transforming the historiography of medieval Italian algebraic techniques from a largely speculative narrative into a documentary-based literature.⁸

Abbaci, Vol. II. *La Practica Geometriae*, Vol. III *Opuscoli*, III vol. (Roma: Tipografia delle scienze matematiche e fisiche, 1857); Id., "Intorno ad un trattato d'aritmetica stampato nel 1478," *Atti dell'Accademia Pontificia de' Nuovi Lincei* 16 (1862). See also Woepcke, *Sur l'introduction de l'arithmétique indienne en Occident et sur deux documents importants publiés par le prince don Balthasar Boncompagni et relatifs à ce point de l'histoire des sciences*. (Rome: Imprimerie des sciences mathématiques et physiques, 1859).

⁵ For a recent critical edition of the *Liber abaci*, see Giusti and D'Alessandro (eds.), *Leonardi Bigolli Pisani vulgo Fibonacci Liber Abbaci* (Firenze: Olschki, 2020).

⁶ Van Egmond, *Practical Mathematics in the Italian Renaissance: a Catalog of Italian Abacus Manuscripts and Printed Books to 1600* (Firenze: Giunti Barbera, 1980).

⁷ See, among others, Salomone, ed., *La reghola de algebra amuchabale: dal codice L.4.21 della Biblioteca comunale di Siena* (Siena: Università degli studi di Siena, 1982); Franci, ed., *Questioni d'algebra: dal Codice L.IX.28 della Biblioteca Comunale di Siena* (Siena: Università degli studi di Siena, 1983); Pieraccini, ed., *Chasi exenplari alla regola dell'algebra: nella trascelta a cura di m. Benedetto: dal codice L.VI.21 della Biblioteca comunale di Siena* (Siena: Università degli studi di Siena, 1983); Procissi, ed., *Ragionamenti d'algebra: i problemi. Dal codice Pal. 567 della Biblioteca nazionale di Firenze* (Siena: Università degli studi di Siena, 1983); Gori and Franci, eds., *Libro di ragioni e misure in sunto e a mente: dal Codice L.IX.30 della Biblioteca comunale di Siena* (Siena: Università degli studi di Siena, 1984); Franci and Pancanti, eds., *Il trattato d'algebra: dal manoscritto Fond. prin. 2. 5. 152 della Biblioteca nazionale di Firenze* (Siena: Università degli studi di Siena, 1988); Simi, ed., *Regole di geometria e della cosa: Codice Palatino 575 (sec. 15.) della Biblioteca Nazionale di Firenze* (Siena: Università degli studi di Siena, 1992); Franci, ed., *Maestro Dardi (s. xiv) Aliabrar Argibra (ms. LVII.17 Biblioteca Comunale di Siena)* (Siena: Università degli studi di Siena, 2001), vol. xxvi.

⁸ See, among others, Franci and Rigatelli, "Maestro Benedetto da Firenze e la storia dell'algebra," *Historia Mathematica* 10/3 (1983): 297–317; Id., "Towards a history of algebra from Leonardo of Pisa to Luca Pacioli," *Janus* 72 (1985): 17–82; Id., "Fourteenth century Italian algebra," in *Mathematics from manuscript to print, 1300-1600*, ed. by Hay (Oxford: Clarendon Press, 1988),

More recent studies, such as those by Albrecht Heeffer, Jens Høyrup, and Van Egmond himself, have contributed to highlight both the characteristics of this mathematical tradition and its internal evolution, bringing to the fore the ways in which it both laid the groundwork for what would eventually become European symbolic algebra, and how it was characterised by its own distinctive features.⁹ Heeffer, for example, has suggested that we could understand the algebraic techniques that emerged in abacus treatises as a form of “proto-symbolic” algebra, representing an important transitional stage between purely rhetorical methods and the emergence of symbolic notations.¹⁰

Scholars such as Cynthia Hay, Barbara Gärtner, Sabine Rommevaux, Maryvonne Spiesser, and Maria Rosa Massa Esteve, meanwhile, have studied mathematical works published beyond the Alps that also contributed to the emergence of new forms of algebra. Rather than a coherent body of mathematical knowledge, sixteenth-century algebra appears as a “plurality” of mathematical techniques characterised by a high degree of experimentalism.¹¹ In the meanwhile, new historiographical approaches proposed by Karine Chemla and others have stressed the historicity of mathematical knowledge, and the need to understand its evolution as embedded in its historical environment.¹² Rather than a

11–29; Franci, “Leonardo Pisano e la trattatistica dell’abaco in Italia nei secoli XIV e XV,” *Bollettino di storia delle scienze matematiche* 23/2 (2003): 33–54; Id., “The History of algebra in Italy in the 14th and 15th centuries: some remarks on recent historiography,” *Actes d’història de la ciència i de la tècnica* 3/2 (2010): 175–194; Id., “Il trattato d’arismetricha (Ms. Ricc. 2252 della Biblioteca Riccardiana di Firenze),” *Bollettino di Storia delle Scienze Matematiche* 38/1 (2018): 93–126.

⁹ Høyrup, “Jacopo da Firenze and the beginning of Italian vernacular algebra,” *Historia Mathematica* 33/1 (2006): 4–42; Van Egmond, “The Study of Higher-Order Equations in Italy before Pacioli,” *Acta historica Leopoldina* 54 (2008): 303–320; Heeffer, “On the Nature and Origin of Algebraic Symbolism,” in *New Perspectives on Mathematical Practices: Essays in Philosophy and History of Mathematics*, ed. by van Kerkhove (Singapore; Hackensack, NJ: World Scientific, 2009), 1–27; Id., “The Genesis of the Algebra Textbook: From Pacioli to Euler,” *Almagest* 3/1 (2012): 26–61; Høyrup, *Explorations and False Trails: The Innovative Techniques That Brought About Modern Algebra* (Cham: Springer, 2024); Id., *The World of the Abacus: Abacus Mathematics Analyzed and Situated Historically between Fibonacci and Stifel* (Cham: Springer, 2024).

¹⁰ Heeffer, “On the Nature and Origin of Algebraic Symbolism.”

¹¹ Hay, ed., *Mathematics from manuscript to print, 1300-1600* (Oxford: Clarendon Press, 1988); Gärtner, *Johannes Widmanns ‘Behende vnd hupsche Rechenung’: die Textsorte ‘Rechenbuch’ in der Frühen Neuzeit* (Tübingen: Niemeyer, 2000); Rommevaux, Spiesser, and Massa Esteve, eds., *Pluralité de l’algèbre à la Renaissance* (Paris: Honore Champion, 2012).

¹² Rommevaux, Spiesser, and Massa Esteve, eds., *Pluralité de l’algèbre à la Renaissance*; Cuomo, “Mathematical traditions in Ancient Greece and Rome,” *HAU: Journal of Ethnographic Theory* 9/1 (2019): 75–85; Chemla, “Cultures of Computation and Quantification in the Ancient World: An Introduction,” in *Cultures of Computation and Quantification in the Ancient World: Numbers, Measurements, and Operations in Documents from Mesopotamia, China and South Asia*, ed. by Chemla, Keller, Proust, 1–140 (Cham: Springer, 2022); Boucard and Morel, “New Ob-

history of origins and necessary progress, the history of mathematics appears in these accounts as a more complex – and in several cases more surprising – history of evolution of calculative practices.

Building on these approaches, and following a line initiated by Van Egmond,¹³ to understand the roots of the distinctive characteristics of the form of algebra that emerged in late medieval Italy, we first need to focus on the socio-economic setting in which abacus mathematics appeared. In other words, we need to discuss the deep connections between the emergence of abacus mathematics and the so-called “commercial revolution of the thirteenth century”: a period marked by rapid expansion in trade, finance, and urban enterprise.¹⁴ This phase of economic transformation was characterised by the rise of new corporate structures (such as the *compagnia* and the *accomandita*), the introduction of new accounting practices (such as double-entry bookkeeping), and the development of financial instruments such as bills of exchange and marine insurance.¹⁵ These innovations created an unprecedented demand for practical mathematical skills, which late medieval city-states satisfied with the foundation of vocational schools – the so-called “abacus schools” – which had the specific aim of securing the intergenerational transmission of a mathematics that had become central to commercial activity.¹⁶

Abacus manuals differ markedly from what we would today recognize as mathematical texts. Instead of providing theoretical expositions, these works consist largely of extensive collections of solved problems. These worked *ragioni* are typically grouped according to their domain of application – such as problems related to exchange, divisions of profits and losses, interest rates, barter, and related mercantile operations. A

jects, Questions, and Methods in the History of Mathematics,” *Histories* 2/3 (2022): 341–351; Rowe and Dauben, eds., *A Cultural History of Mathematics*, 6 vols. (London: Bloomsbury Publishing, 2024).

¹³ Van Egmond, “The Commercial Revolution and the Beginnings of Western Mathematics in Renaissance Florence, 1300–1500 (PhD dissertation),” University Microfilms International 1976.

¹⁴ De Roover, “The Commercial Revolution of the Thirteenth Century,” in *Enterprise and secular change: readings in economic history*, ed. by Lane and Riemersma, 80–85 (London: Allen and Unwin, 1953); Lopez, *The commercial revolution of the Middle Ages, 950-1350* (Cambridge; New York: Cambridge University Press, 1976).

¹⁵ Goldthwaite, *The economy of Renaissance Florence* (Baltimore, Md: Johns Hopkins University Press, 2009), 63–114.

¹⁶ Goldthwaite, “Schools and Teachers of Commercial Arithmetic in Renaissance Florence,” *Journal of European Economic History* 1/2 (1972): 418–433; Grendler, *Schooling in Renaissance Italy: literacy and learning, 1300-1600* (Baltimore: Johns Hopkins University Press, 1989), 306–332; Ulivi, “Scuole e maestri d’abaco in Italia tra Medioevo e Rinascimento,” in *Un ponte sul Mediterraneo: Leonardo Pisano, la scienza araba e la rinascita della matematica in Occidente*, ed. by Giusti and Petti, 121–159 (Firenze: Polistampa, 2002); Black, *Education and society in Florentine Tuscany* (Leiden; Boston: Brill, 2007); Masiero, *Literacy and learning in Latin and abacus schools in Verona (1405–1509)* (Leiden: Brill, 2026).

number of particularly sophisticated manuals also include sections devoted to algebra, demonstrating an early integration of advanced mathematical reasoning into a fundamentally practical framework. This organization renders abacus manuals more akin to recipe books than to systematic treatises. This mode of exposition aligns them with a broader genre of practical literature that appeared in increasing numbers from the late middle ages, which includes *pratiche di mercatura*, *books of arts*, *books of secrets*, *ricordanze*, and other instructional or technical compendia.¹⁷ Such texts share a common orientation toward utility and application, privileging procedural knowledge over theoretical explanations.

The broader commercial environment and the specific context of abacus schools in which these texts appeared helps illuminate the emergence of a form of mathematical knowledge characterized by distinct criteria for mathematical validity and standards of practice. For instance, Høyrup has recently noted how the competitive environment in which abacus masters operated encouraged them to conceptualise mathematical knowledge primarily in terms of a body of techniques for *solving problems*:

Abacus masters were members of a liberal profession in a market economy ... They competed with each other, either for students or for positions at municipally financed abacus schools. There, ability to solve problems proposed by the adversary at a competition or the aptitude to propose problems the adversary could not solve were useful ... Even school students had to be trained in solving problems – simple problems in their case. Competition as well as abacus-school teaching pushed toward seeing problem solution as the general format in which mathematical knowledge should be formulated.¹⁸

Such contextual considerations are crucial for understanding why the seeds of what would eventually become European symbolic algebra arose not within the tradition of learned mathematics pursued in late medieval universities, but within the practical, problem-oriented environment of commercial arithmetic. A useful point of departure for clarifying the nature of this transformation is offered by Mahoney's (1980) influential characterisation of the "algebraic mode of thought," that emerged in modern European algebra:

What should be understood as the "algebraic mode of thought"? It has three main charac-

¹⁷ Leong, *Recipes and everyday knowledge: medicine, science, and the household in early modern England* (Chicago; London: University of Chicago Press, 2018); Creager, Grote, and Leong, "Learning by the book: manuals and handbooks in the history of science," *BJHS Themes* 5 (2020): 1–13; Smith, *From lived experience to the written word: reconstructing practical knowledge in the early modern world* (Chicago: University of Chicago Press, 2022).

¹⁸ Høyrup, *The World of the Abbaco*, 113–114.

teristics: first, this mode of thought is characterized by the use of an operative symbolism, that is, a symbolism that not only abbreviates words but represents the workings of the combinatory operations, or, in other words, a symbolism with which one operates. Second, precisely because of the central role of combinatory operations, the algebraic mode of thought deals with mathematical relations rather than objects. Third, the algebraic mode of thought is free of ontological commitment ... In particular, this mode of thought is free of the intuitive ontology of the physical world. Concepts like “space,” “dimension,” and even “number” are understood in a purely mathematical sense, without reference to their physical interpretation.¹⁹

While Mahoney used this definition to describe modern European algebra, the seeds of this “mode of thought” can already be observed in the late medieval tradition of practical arithmetic. The adoption of Indo-Arabic numerals satisfied the fundamental precondition for the first characteristic listed by Mahoney – that is, the use of an operative symbolism. This is because Indo-Arabic numerals provide a numerical notation that is inherently operative, as the algorithms to carry out calculations with this notation require the practitioner to operate with the signs themselves.

Moreover, as Mahoney observed, the development of such a mode of thought presupposes a shift away from the classical, Euclidean ontology that long underpinned mathematical reasoning in the learned tradition. In the mercantile environment of the thirteenth-century commercial revolution, would-be merchants – i.e., the main constituency of students who attended abacus schools – required a set of mathematical tools to solve practical problems rather than a theoretical mathematical system. This environment favoured the emergence of a body of mathematical knowledge that broke decisively with the intuitive, physically grounded conception of mathematics characteristic of the Euclidean paradigm, and the development of an operative mathematical language.

Nevertheless, the absence of theoretical demonstrations in abacus manuals does not mean that there is no proof for their mathematics; rather, it means that their proof is to be found elsewhere. Merchants, bankers, and other practitioners prioritised the *effectiveness* of a rule over its thorough, geometrical demonstration. The mathematical knowledge included in abacus manuals reflects this prioritization of the application over the demonstration. Instead of being demonstrated deductively, the validity of the rules presented in abacus manuals was *corroborated* through successful application in long lists of worked examples. Rather than theorems, we should think of the rules included in abacus manuals

¹⁹ Mahoney, “The Beginnings of Algebraic Thought in the Seventeenth Century,” in *Descartes: Philosophy, Mathematics and Physics*, ed. by Gaukroger, 141–155, 142 (Brighton, Sussex: The Harvester Press, 1980). The paper was originally published as Mahoney, “Die Anfänge der algebraischen Denkweise im 17. Jahrhundert,” *RETE: Strukturgeschichte der Naturwissenschaften* 1 (1971): 15–31.

in terms of mathematical tools. The mathematics of these texts is thus best understood as a procedural and operational mathematical craft – rather than as a deductive system – whose validity is justified through use, in what Field has characterised as a form of “quasi experimental proof.”²⁰ This explains the distinctive recipe-like structure of these texts, composed predominantly of solved examples framed in the conditional form “if we are given this problem, *this* is what we need to do.”

Because the attention of these mathematical practitioners lay on solution rather than demonstration, abacus mathematics enjoyed a degree of exploratory freedom rarely available to mathematicians working in the wake of the Euclidean model. This freedom is also reflected in the flexible and selective manner in which abacus texts engaged with earlier sources, because they reveal a highly selective reception of prior mathematical works. A clear example of this is provided by the oldest abacus manual extant – the so-called *Livro de l'abbecho* (Florence, Biblioteca Riccardiana, Ricc. 2404) – whose text is the result of several stratifications. The text includes sections whose sources are not attested, and sections that are a vulgarization of the Latin text of Leonardo Pisano's *Liber abaci*. These latter sections, however, are not a simple vulgarization of Fibonacci's text. Rather, as Andrea Bocchi has shown, these sections are the outcome of a series of previous manipulations and reorganisations of their source.²¹

Similarly, the sections that do not derive from Fibonacci's work – hence, texts for which the *Livro* provides the first known source – reappeared with various modifications in several subsequent abacus manuals. For example, while Fibonacci introduced the rule of three with a thorough and systematic discussion,²² the formula used in the *Livro* prior-

²⁰ Field, “The Unhelpful Notion of ‘Renaissance man,’” *Interdisciplinary Science Reviews* 41/2–3 (2016): 188–201, 197–198.

²¹ Bocchi, ed., *Lo libro de l'abbecho* (Pisa: Edizioni ETS, 2017), 36–37.

²² Fibonacci discusses the rule of three in the eighth chapter of his *Liber abaci*. After two chapters outlining the possible combinations of wares, units, weights, and measures, Leonardo Pisano provides the following illustration of the rule, which is substantially more thorough than the concise formula of the *Livro*: “Sed primum ostendam unde hic modus procedit. Sunt enim, ut dixi, in negotiationibus quattuor numeri proportionales, scilicet ut sicut primus est ad secundum ita tertius ad quartum, hoc est sicut numerus alicuius quantitatis mercis est ad numerum quantitatis sui pretii, ita numerus cuiusvis quantitatis eiusdem mercis est ad numerum sui pretii; vel sicut aliqua quantitas cuiusvis mercis est ad quamvis quantitatem eiusdem mercis, ita est pretium unius ad pretium alterius. Et cum ita quattuor quantitates proportionales sunt, erit multiplicatio secunde in tertiam equa multiplicationi prime in quartam, ut in arismetris et in geometria probatum est. Quare si quarta quantitas est ignota tantum, ex multiplicatione quidem secunde quantitatis in tertia divisa per primam, nimirum ex divisione quarta quantitas proveniat; quia cum dividitur aliquis numerus per aliquem numerum et ex divisione aliquid proveniat, si proveniens in divisorem multiplicaveris, nimirum divisus numerus inde proveniet. Similiter si tertia quantitas ignoratur, dividenda est per secundam multiplicatio prime in quartam.” See Giusti and D'Alessandro, eds., *Leonardi Bigolli Pisani Liber Abbaci*, 141–142.

itises memorability over mathematical thoroughness, reflecting the didactic character of abacus manuals:

Lo p(r)imo chapitolo è-ne de le reg(o)le d(e) le tre chose.

Se ce fosse dicta alchuna ragione e · lla q(ua)-
le se proponesse tre chose, sì devemo m(ultip)licare
quilla chosa che noie volemo sap(er)e (contra) q(ui)lla
che non è de quilla medessme, a pa(r)ti(r)e nell'a-
ltra.²³

This formulation of the rule is attested in several subsequent abacus manuals with various modifications. These manuals include, among others, the so-called *Tractatus algorismi* by Jacopo da Firenze (Florence, Biblioteca Riccardiana, Ricc. 2236; Rome, Biblioteca Vaticana, Vat. Lat. 4826, Milan, Biblioteca Trivulziana, 90), the *Liber habaci* attributed to Paolo Gherardi (Florence, Biblioteca Nazionale Centrale, Magl. CI. XI. 88), the *Trattato di tutta l'arte dell'abacho* attributed to Paolo dell'Abaco (Florence, Biblioteca Nazionale Centrale, Fond. Naz. II. IX. 57, and several copies), and the *Trattato d'abaco* by Piero della Francesca (Florence, Biblioteca Mediceo-Laurenziana. Ash 359 (291)). Similar formulations of the rule of three also appeared in printed texts, including the so-called *Aritmetica di Treviso* (1478) and Luca Pacioli's *Summa* (1494). Across these texts, the formula first presented in the *Livero* was transmitted with various modifications that reflect, rather than the faithful reproduction of an authoritative model, what Bocchi has described as a form of "professional memory."²⁴

The above-mentioned *Trattato di tutta l'arte dell'abacho* attributed to Paolo dell'Abaco is another example of the weak authoriality of these sources. A complete copy is included in the oldest manuscript preserving the text: the manuscript Fond. Naz. II. IX. 57 of the Biblioteca Nazionale Centrale of Florence. More than half of the other manuscripts preserving the text (six out of ten) include either partial copies of the text or excerpts.²⁵ In the fifteenth century, the *Trattato d'abacho* by Maestro Benedetto was a particularly successful text, with over fifteen manuscript copies surviving to this day. Most of these copies only include parts from Benedetto's text, which were probably selected following the needs and interests of the users of these texts.²⁶

²³ Bocchi, ed., *Lo libro de l'abbecho*, 163.

²⁴ Ibid., 65.

²⁵ Ceccherini, "Come nasce un libro d'abaco. Struttura, tradizione e storia del ms. Firenze, Biblioteca nazionale centrale, II.IX.57," *Scrineum* 22/1 (2025): 1–79, 58–59.

²⁶ Van Egmond, *Practical mathematics in the Italian Renaissance*, 356; Danna, *The Craft of Indo-Arab Numerals: How Practical Arithmetic Shaped Commerce and Mathematics in Western Europe, 1200-1600* (Cambridge, MA: Harvard University Press, 2026), 148–149.

As these examples show, rather than transmitting earlier material verbatim, the writers of abacus manuals adapted, modified, and reorganized previous abacus works with a degree of freedom comparable to how artisans would adapt, depending on convenience, tools and practices inherited from their peers. This adaptive and experimental attitude was central to the development of the operative mathematical language in which the seeds of European algebraic symbolism could be developed.

This environment – characterised by a degree methodological freedom and a limited emphasis on the rigour of mathematical demonstrations – created the conditions for the emergence of a form of algebra that experimented with new algorithms and symbolic practices. The different ontology of this mathematics is already signalled by the terms used to describe algebraic quantities: reflecting the Arabic origins of their algebra, abacists called the unknown “thing” (*cosa*, derived from the Arabic *shay*) and its square “wealth” (*censo*, derived from the Arabic *māl*).²⁷ The most significant consequence of this shift in mathematical reasoning was the abandonment of geometrical methods in favour of operational and algorithmic ones. Whereas the algebraic sections of Fibonacci’s *Liber abaci* had offered a unique synthesis of Arabic algebra and Euclidean-style geometric demonstrations, the algebraic sections of abacus manuals adopted a decisively non-geometric orientation. Their authors privileged algorithmic procedures over spatial reasoning, and the criteria for mathematical validity shifted accordingly. In place of thorough geometrical demonstrations, these texts relied on the successful application of a rule to a particular problem as the primary warrant of validity. What counted as proof, therefore, differed fundamentally from the expectations governing learned mathematical texts: it was not a matter of deductive demonstration, but of effective performance.

As noted by Franci and Toti Rigatelli, abacus manuals with algebraic sections exhibit these characteristics already from the fourteenth century:

In Italy, little more than one century after the *Liber abaci*, algebra had already developed sufficiently to permit the authors a free and easy use of the calculation of rationalization processes, radicals, and algebraic fractions, while completely omitting, among other things, the geometric references so dear to Leonardo Pisano.²⁸

An early example of abacus algebra can be found in Jacopo da Firenze’s *Tractatus algorismi*, first written in Montpellier in 1307. The algebraic section of this text presents no

²⁷ These terms already appeared in al-Khwārizmī’s algebraic work. On the reception of these mathematical techniques, see Moyon, “La restauration et la comparaison, ou l’art de résoudre des équations quadratiques dans l’Europe latine, *Revue d’Histoire des Mathématiques* 23/2 (2017): 233–299; Sammarchi, “Which al-Khwārizmī? Divergent Scholarly Conceptions of a Canonical Figure,” *Arabic Sciences and Philosophy* (forthcoming).

²⁸ Franci and Rigatelli, “Fourteenth century Italian algebra,” 29.

geometric proofs, and lists solution formulas for twenty equation forms, including some cubic and quartic equations that can be solved by reducing them to a lower degree. Instead of being introduced by a theoretical premise that demonstrates their validity or explains their technical terms, these rules are simply stated, and are subsequently applied to the solution of a number of practical problems.²⁹

While the precise date of the algebraic section of Jacopo da Firenze's *Tractatus algorismi* is debated,³⁰ we find an unquestionably early fourteenth-century example of abacus algebra in the *Libro di ragioni* written by Paolo Gherardi in 1328, another Florentine active in Montpellier. This text includes fifteen solution rules to linear, quadratic, and cubic equations, six of which had already appeared in al-Khwārizmī's and Fibonacci's works. As in the previous example, these rules were not introduced by a theoretical premise, were not supported by geometric diagrams, and were directly applied to the solution of practical problems. Most interestingly, some of Paolo Gherardi's rules were not valid. The presence of false rules is particularly interesting, because it reflects the high degree of experimentalism that characterised the early tradition of abacus algebra.³¹ Other early abacus texts that include a treatment of algebra, such as the Manuscript Ricc. 2252 of the Biblioteca Riccardiana of Florence and the Manuscript 1754 of the Biblioteca Statale of Lucca, presented and rearranged the rules of previous algebraic texts.³²

The first mathematical text entirely dedicated to algebra was probably written in Venice in the 1340s. The so-called *Aliabraq argibra* is attributed to a maestro Dardi da Pisa, and includes 198 rules, four of which for the solution of cubic and quartic equations. The author did not know a general method for solving such equations, and could handle only those with particular combinations of coefficients. Yet this limitation itself underscores the experimental character of the tradition of abacus mathematics, and its ongoing exploration of algorithmic and algebraic procedures.³³ Starting from a survey of the algebraic

²⁹ Franci, "The History of algebra in Italy in the 14th and 15th centuries: some remarks on recent historiography," 179–180; Høyrup, *The World of the Abbaco*, 198–209; 432–433.

³⁰ The date of the algebraic section of the text is debated. See Høyrup, *Jacopo da Firenze's Tractatus Algorismi and Early Italian Abbacus Culture* (Basel: Birkhäuser, 2007); Van Egmond, "Jacopo da Firenze's Tractatus Algorismi edited by Jens Høyrup," *Aestimatio: Critical Reviews in the History of Science* 6 (2009): 37–47; Oaks, "Essay review: Medieval Italian practical mathematics," *CHSPM/SCHPM Bulletin* (2009); Høyrup, "A response to Van Egmond on Høyrup, Jacopo da Firenze's Tractatus Algorismi," *Aestimatio: Critical Reviews in the History of Science* 6 (2009): 116–126.

³¹ Franci, "The History of algebra in Italy in the 14th and 15th centuries: some remarks on recent historiography," 178–179; Høyrup, *The World of the Abbaco*, 209–215.

³² Franci, "The History of algebra in Italy in the 14th and 15th centuries: some remarks on recent historiography," 181–182.

³³ Van Egmond, "The algebra of master Dardi of Pisa," *Historia Mathematica* 10/4 (1983): 399–421; Franci, ed., *Maestro Dardi (s. xiv) Aliabraq Argibra (ms. LVII.17 Biblioteca Comunale*

contents of these early texts, Van Egmond suggested a possible classification of their rules into different “families,” and noted that they were generally characterised by accretional and incremental experimentation. He argued that the distinctive “pattern” of this tradition consisted of “direct copying mixed with reordering and the addition of new terms and cases, so that overall [one finds] a steady increase in the highest power, number of terms, and total number of equations.”³⁴

The fifteenth century is the period in which these sprawling experimentations started to be selected and systematised. This is clearly exemplified by the so-called “abacus encyclopaedias” written in this period, such as the ms L.IV. 21 of the Biblioteca Comunale of Siena, the ms. Pal. 573 of the Biblioteca Nazionale Centrale of Florence, and the ms. Ottobon. Lat. 3307 of the Biblioteca Apostolica Vaticana. These large and precious manuscripts comprise lengthy and encyclopaedic treatments of abacus mathematics, and reflect the changing use of abacus texts around the middle of the 15th century, as they were probably designed as collectable objects for prestigious libraries. These manuscripts also include algebraic sections, where one can find a critical selection of the algebraic rules attested in the previous centuries. For example, these texts do not feature Paolo Gherardi’s invalid rules, and present a selection of algebraic rules that are attributed to abacus masters of the previous generations.³⁵

The fifteenth century is also the period in which we find evidence of experimentation in mathematical notation. These were again carried out by practitioners who had either had their foundational mathematical training, or who wrote themselves works of practical arithmetic. The former case is exemplified by Giovanni Bianchini, a merchant and administrator with an interest in astronomy, whose astronomical tables display remarkably early experimentation with decimal notation.³⁶ The latter case is exemplified by the famous artist Piero della Francesca. In addition to being among the most important painters of the fifteenth century, Piero della Francesca was also an accomplished writer of mathematical texts. His *Trattato d’abaco* – which was probably among his earliest works – falls squarely into the tradition of practical arithmetic, and includes a thorough algebraic section.³⁷ This

di Siena); Wagner, “Mordekhai Finzi’s translation of Maestro Dardi’s Italian Algebra, a Partial Edition,” in *Latin-into-Hebrew: Texts and Studies. Volume 2*, ed. by Fontaine and Freudenthal, 195–212; 437–501 (Leiden; Boston: Brill, 2013).

³⁴ Van Egmond, “The Study of Higher-Order Equations in Italy before Pacioli,” 306.

³⁵ Franci, “The History of algebra in Italy in the 14th and 15th centuries: some remarks on recent historiography,” 187–188. For a discussion of the algebraic contents of these manuscripts, see Høyrup, *The World of the Abbaco*, 267–304.

³⁶ Van Brummelen, “Decimal fractional numeration and the decimal point in 15th-century Italy,” *Historia Mathematica* 66 (2024): 1–13.

³⁷ Piero della Francesca’s *Trattato d’abaco* is published in Arrighi, ed., *Piero della Francesca. Trattato d’abaco: dal Codice Ashburnhamiano 280, 359*-291*, della Biblioteca medicea laurenziana di Firenze* (Pisa: Domus Galilaeana, 1970); Dalai Emiliani, Besomi, Maccagni, Gamba, Montebel-

text includes sixty-one algebraic formulas for solving specific forms of quadratic, cubic, quartic, and quintic equations, most of which draw heavily on earlier algebraic manuals, and displays an experimental symbolic notation for the powers of the unknown.³⁸ Specifically, Piero used geometrical signs to represent the powers of the unknown: he represented the first degree of the unknown with a superscript “-”; the square of the unknown with a superscript square “□”; the cube of the unknown with a superscript “c” or “Δ”; and the fourth degree with a double square “□□.” Hence, Piero wrote what we would represent as $5x$ as “5-”; $5x^2$ as “5x□”; $5x^3$ as “5xc” or “5xΔ”; and $5x^4$ as “5x□□.” While experimenting with this symbolism, Piero also used the standard names given to the degrees of the unknown in other abacus texts, i.e. *cosa*, *censo*, *cubo*, and *censo di censo*.³⁹

Luca Pacioli’s *Summa de Arithmetica Geometria Proportioni et Proportionalità*, published in Venice in 1494, closed the fifteenth century by making an encyclopaedic mathematical work appear in print. Moreover, by noting that “thus far” no general rules for the third and fourth degree equations had been identified – and by considering the possibility that these might be found in the near future – Pacioli anticipated the important developments of the following decades. The process through which the general solutions to cubic and quartic equations were found was highly influenced by the methods and experimentations described above. The process that led to this discovery – the first major mathematical breakthrough since antiquity – involved in different degrees Scipione dal Ferro, Antonio Maria Fior, Niccolò Tartaglia, Girolamo Cardano, and Ludovico Ferrari.⁴⁰ While their social profiles were not identical, these were all mathematicians who hybridized elements of the practical and the learned tradition, and were active in different degrees between the world of practical arithmetic, the court, and the university.

Scipione dal Ferro was the first to identify a solution formula for the equation that we would now write as $x^3 + px = q$. As far as we know, he never published this result, and only shared it with pupils and close relatives. The likely reason for this is that monopoly over this formula gave Scipione dal Ferro a competitive advantage during mathematical challenges, allowing him to consolidate a strong reputation, and to be appointed in the Bolognese *studium* with increasing salaries – even though with a chair of lower status than that of astronomy and other established disciplines.⁴¹ A pupil of Dal Ferro’s, An-

li, Derenzini, Mattesini, Valerio, and Sorci, eds., *Piero della Francesca. Trattato d’abaco*, III vol. (Roma: Istituto poligrafico e zecca dello stato, 2012).

³⁸ Giusti, “Fonti medievali dell’Algebra di Piero della Francesca,” *Bollettino di Storia delle Scienze Matematiche* 13/2 (1993): 199–250; Field, *Piero Della Francesca: a mathematician’s art* (London, New Haven: Yale University Press, 2005).

³⁹ Arrighi, ed., *Trattato d’abaco*, 12–13.

⁴⁰ Toscano, *The Secret Formula: How a Mathematical Duel Inflamed Renaissance Italy and Uncovered the Cubic Equation* (Princeton, NJ: Princeton University Press, 2020).

⁴¹ Bianca, “Dal Ferro, Scipione.”

tonio Maria Fior was one of the few people with whom the Bolognese mathematician shared his solution method.

In 1535, Fior challenged Niccolò Tartaglia to solve a set of mathematical problems. While not much about his life is known with certainty, Tartaglia was probably active as an abacus master, and was the author of several innovative works of vernacular mathematics.⁴² Spurred on by the challenge posed by Fior, Tartaglia understood that the problems Fior had submitted to him could be reduced to the same equation form, and worked out by himself the solution formula. He thereby won the challenge, and made public the existence of a thus far unknown algebraic formula. News spread out and reached the attention of Girolamo Cardano around 1539. The illegitimate son of a Milanese doctor, Cardano graduated in medicine from the university of Padua, was active as a teacher of arithmetic in Milan, and managed to have important connections with the court.⁴³ Cardano's *Practica arithmeticae* had just been published, consolidating his standing as a distinguished mathematician and his connections with the Milanese court. Cardano started corresponding with Tartaglia, and – probably exploiting his ties with the court – managed to persuade Tartaglia to at least partially share his discovery with him. In exchange, Cardano promised never to reveal Tartaglia's discovery.

Cardano, however, suspected that Tartaglia knew more than he had shared, and hence continued to work on Tartaglia's formula with the assistance of his pupil Ludovico Ferrari. While also active as a practical mathematician and surveyor, Ferrari eventually managed to be appointed at the chair of arithmetic of the Bolognese *studium*, and died in unclear circumstances.⁴⁴ Together with finding out about the original discovery of the formula by Scipione dal Ferro, in their research Cardano and Ferrari were able to find the general solution to both cubic and quartic equations. In 1545, these results were made public for the first time in Cardano's *Ars Magna*. Cardano credited the discovery of the initial formula to both Dal Ferro and Tartaglia, but the latter saw this publication as a breach of the vow, and openly denounced Cardano for appropriating his work.⁴⁵

While this opened one of the most bitter mathematical disputes of the sixteenth century, it is important to note that Tartaglia and Cardano had much that united them, as they were both heirs of the practical tradition of abacus mathematics. While their social profiles differed, they embodied the increasingly diverse social standing of the practical

⁴² Nenci, "Tartaglia, Niccolò."

⁴³ Grafton, *Cardano's cosmos: the worlds and works of a Renaissance astrologer* (Cambridge, MA: Harvard University Press, 1999); Baldi and Canziani, eds., *Girolamo Cardano. Le opere, le fonti, la vita* (Milano: Franco Angeli, 1999).

⁴⁴ Fiocca, "Alcune opere inedite di Ludovico Ferrari," *Bollettino di Storia delle Scienze Matematiche* VIII/2 (1988): 239–305.

⁴⁵ Gabrieli, *Niccolò Tartaglia: invenzioni, disfide e sfortune* (Università degli studi di Siena, 1986); Toscano, *The Secret Formula*.

mathematician, as they both engaged with the traditional role of teachers of arithmetic as well as with diverse roles, including appointments in private families, at the university, and the court. This is also reflected in the kind of mathematics which they both practiced. Tartaglia actively contributed to expanding the field of application of abacus mathematics. His *Nova scientia*, published in 1537, is the first printed mathematical work dealing with artillery and, by providing some of the earliest experiments in formalising the motion of projectiles, reflects the increasing diffusion of firearms. Tartaglia never used geometrical methods in his algebraic texts, thereby fully adhering to the model of abacus algebra.

Cardano's works were also founded in the tradition of practical arithmetic, though in a more nuanced way. His *Practica arithmeticae* aimed at distinguishing between mathematical theory and its applications, but was clearly rooted in the tradition of abacus mathematics and its practical applications.⁴⁶ This is reflected in his Latin language, as Cardano was compelled to Latinize terms that had previously existed only in the vernacular, in some cases with rather amusing results. For example, Veronica Gavagna has noted the term *schisatio*, with which Cardano clearly latinised the Italian *schisare*, a technical term which abacus masters used to indicate the simplification of a fraction to its lowest terms – and which had no translation into Latin.⁴⁷

This discussion brings us to the role of the rediscovery of classic mathematical works which unfolded in the same period and interacted in significant ways with these developments. By writing in Latin, Cardano aimed not only to make his mathematics accessible to a European audience, but also to give it a higher cultural standing. His “great art” (*ars magna*) exemplifies the hybridization of vernacular and learned mathematics, as Cardano rendered in Latin the language and methods of practical arithmetic. This hybridization of classical models and practical traditions is even more evident in Cardano's *Ars magna*. In his processes, Cardano coupled the algorithmic methods characteristic of abacus mathematics with geometrical demonstrations, clearly following the Euclidean model.⁴⁸

However, when it came to demonstrations for which there is no straightforward geometrical representation (such as negative numbers, or quartic equations), Cardano had to resort to operative and algorithmic methods. For example, Cardano's “mercantile interpretation” of negative numbers (i.e. treating them as debts) reveals a clear influence

⁴⁶ Gavagna, “Alcune osservazioni sulla *Practica Arithmetice* di Cardano e la tradizione abachistica quattrocentesca,” in *Girolamo Cardano. Le opere, le fonti, la vita*, ed. by Baldi, Canziani, 273–312 (Milano: Franco Angeli, 1999).

⁴⁷ Gavagna, “Medieval Heritage and New Perspectives in Cardano's *Practica arithmetice*,” *Bollettino di storia delle scienze matematiche* XXX/1 (2010): 61–80, 67–68.

⁴⁸ Girolamo Cardano, *Artis magnae, sive De regulis algebraicis, liber unus* (Nuremberg: Johannes Petreus, 1545), fols. 29v–49r. A modern edition of the text is published in *Girolamo Cardano. Artis magnae, sive De regulis algebraicis liber unus*, ed. by Tamborini (Milano: Franco Angeli, 2011), vol. xxv.

from the practices of abacus mathematics.⁴⁹ Likewise, Cardano managed to treat the so-called “irreducible case” of cubic equations, which implied working with the square root of negative numbers. Instead of resisting the possibility of square roots of negative numbers on the ground of the impossibility to represent them, Cardano called these numbers *quantitates sophisticae*, and manipulated them as any other number. In so doing, he followed the standards and practices of the generations of abacus masters before him, who had been tinkering with algorithmic methods without concerning themselves with their geometrical interpretation. At the same time, however, his use of geometrical demonstrations reflects Cardano’s attempt to present algebra as a body of knowledge characterised by the same epistemological standing as classical mathematics. While he only wrote in the Italian vernacular, Tartaglia edited and commented the first Italian edition of Euclid’s *Elements* in 1543, as well as a collection of writings by Archimedes. Both authors, hence, were in different ways aware of classical models, and their mathematics can be understood as an early example of the hybridizations of the two mathematical cultures that emerged in the sixteenth century.

The figure who best exemplifies this convergence, however, is Raphael Bombelli, whose *L'algebra parte maggiore dell'aritmetica* is probably the most advanced work of sixteenth-century Italian algebra. Printed in 1572 in Bologna, Bombelli’s work aimed to define algebra as a distinct branch of mathematical theory. Bombelli was the son of a textile merchant, and was himself active as a practitioner, working as an engineer and architect. As shown by R. A. Jayawardene, however, while Bombelli’s work had a marked theoretical character, its roots were clearly grounded in the practical tradition of abacus mathematics. A preparatory manuscript for the third book of Bombelli’s *L’Algebra* preserved in the Biblioteca dell’Archiginnasio of Bologna, in fact, reveals that Bombelli’s initial draft for the work used extensively the practical problems characteristic of abacus mathematics.⁵⁰ Bombelli decided to overturn the character of his text after he had the opportunity to study a manuscript preserved at the Biblioteca Vaticana which recorded a copy of Diophantus’s “algebra” – a work which had been identified by Regiomontanus in the fifteenth century. The example of Diophantus’s mathematics brought Bombelli to expunge all practical aspects from his algebra, and to present it as a worthy heir to what he perceived to be a classical model of mathematical purity.⁵¹

These examples show how the tools developed across a tradition that had first

⁴⁹ Gavagna, “Radices Sophisticae, Racines Imaginaires: The Origins of Complex Numbers in the Late Renaissance,” in *The Art of Science*, ed. by Angelini, Lupacchini, 165–190, 166 (Cham: Springer, 2014).

⁵⁰ The text of the manuscript is edited in Fiocca and Leone, eds., *L’inedito terzo libro de L’Algebra di Rafael Bombelli* (Pisa: Edizioni della Normale, 2017).

⁵¹ Jayawardene, “The Influence of Practical Arithmetics on the Algebra of Rafael Bombelli,” *Isis* 64/4 (1973): 510–523.

emerged to solve problems that primarily concerned merchants and other practitioners were hybridised and repurposed in the sixteenth century. This transformation occurred together with the increasing social standing of the practical mathematician, who began to work beyond the abacus school and to appear in university rooms and courtly halls.⁵² This transformation also occurred together with the increasing influence of a perceived model of classical mathematical rigour. This model, however, was grafted onto a substratum of a mathematical practices that had emerged in the previous two centuries to address practical problems. The classics exemplified an ideal of mathematical coherence and thoroughness, and practical mathematics provided an experimentalist and algorithmic model.

Original results in European mathematics developed at the convergence of these two paradigms: a “native” European model transmitted through scholarly practices, and a “foreign” model that originated in southern Asia, was introduced into western Europe through Mediterranean exchanges, and was first developed by practical mathematicians, until it eventually also influenced learned mathematical practices. The persistence of practical arithmetic techniques within foundational works of “modern” European algebra shows how the evolution of this mathematical knowledge was not a linear progression from “classical” to “modern” mathematics, but rather a complex synthesis of various traditions.

⁵² Biagioli, “The Social Status of Italian Mathematicians, 1450–1600,” *History of Science* 27/1 (1989): 41–95.

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The several faces and the interlaced roots of the “New Algebra”

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Abstract

Who compares Euler’s algebra with that of al-Khwārizmī will see more differences than kinship. Al-Khwārizmī uses natural language, while Euler calculates within the syntax of algebraic symbolism. Al-Khwārizmī has a single unknown (a word), Euler as many as he needs, represented by non-linguistic signs. Al-Khwārizmī deals with the unknown and its second power, Euler knows no limits. Al-Khwārizmī’s coefficients are numerically fixed, those of Euler may have undetermined values. When al-Khwārizmī operates on a composite expression, he needs roundabout ways; Euler has the parenthesis. The parenthesis mostly goes unmentioned when the characteristics of the “new algebra” are discussed. The rest is familiar. However, the unfolding of the various characteristics is largely left in the dark. The whole seems to have emerged fully grown from the minds of Viète and Descartes. The aim of the paper is to trace the emergence of the various characteristic features of the New Algebra from the 14th-century beginnings of abacus algebra. The process is far from linear – before the arrival of German *coß* we cannot even speak of “stops and goes” on the road toward some aim. After Christoph Rudolff we probably can; in this final phase, the mostly neglected roles of Michael Stifel and Valentin Mennher are taken up.

Keywords

Descartes, Viète, abacus algebra, Rechenmeister algebra, Stifel, Mennher, humanist mathematics

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Algebras

“Algebras” constitute what Ludwig Wittgenstein would call a *family* (by commentators often expanded into “natural family”) – held together not by a definition but by “a complicated network of similarities overlapping and criss-crossing: sometimes overall similarities, sometimes similarities of detail.”¹ It encompasses B. L. van der Waerden’s *Moderne Algebra*, built on the work of Emmy Noether and Emil Artin; that of René Descartes and François Viète; furthermore, German *coß* of the 16th century, and Italian abacus algebra with the culmination brought about by Girolamo Cardano and Nicolò Tartaglia – and of course Arabic *al-jabr*, which gave the family its name; not to speak of Diophantos and algebra-like techniques in Sanskrit and classical Chinese mathematics. Disagreements exist whether the techniques of a single and a double false position or the Old Babylonian area technique are to be counted as algebra – Wittgensteinian families have no clear boundary.

The “new algebra” of my title (henceforth capitalized) is that which was created between *ca* 1600 and *ca* 1700 – by Viète, Descartes, John Wallis and others. At a time when few earlier sources were studied at least by those (few) who were interested in the matter – say, al-Khwārizmī’s algebra, Leonardo Fibonacci’s *Liber abbaci* (supposed to be more original and more influential than it actually was), Cardano’s *Ars magna* and Tartaglia’s *General trattato* – this really seemed to be a *new* discipline, and its creation a fabulous leap. A late reflection of this attitude is Michael Mahoney’s essay review² of the reprint of Neugebauer’s *Vorgriechische Mathematik I*, which relegated anything preceding Viète to the realm of “algebraic mode of thought.”

In mythology, Athena can be presumed to have sprung fully armed from Zeus’s head. In the evolution of mathematics, such miracles are dubious, and the following pages try to dig below the surface, drawing on the much larger amount of sources that are now available. But in order to see what should be looked for, it may be useful to compare an algebra which doubtlessly precedes the supposed leap (al-Khwārizmī’s early ninth-century algebra, the earliest extant *treatise* dealing with the topic) to a text which just as obviously postdates it (Leonhard Euler’s *Analysis infinitorum*).

First al-Khwārizmī, as Latin Europe encountered him, that is, as translated by Gerard of Cremona; a characteristic passage shows the style:³

Divide ten in two sections and divide one of the two parts by the other, and four result.

¹ Ludwig Wittgenstein, *Philosophische Untersuchungen – Philosophical Investigations* (New York: Macmillan, 1953), §§ 66–67.

² Michael S. Mahoney, “Die Anfänge der algebraischen Denkweise im 17. Jahrhundert,” *Rete* 1 (1971): 15–31.

³ Barnabas Hughes, ed., “Gerard of Cremona’s Translation of al-Khwārizmī’s *Al-Jabr*,” *Mediaeval Studies* 48 (1986): 248; my translation, as all English translations below.

Whose rule is that you posit one of the two sections to be a *thing* and the other ten less a *thing*. Then divide ten less a *thing* by a *thing* so that four result. However, now you have known that when you multiply what comes out of a division by the same by which you divided, it will give back your amount which you divided. But what results from the division in this question was four, and that by which was divided was the *thing*. Multiply therefore four in a *thing*, and they will be four *things*. Thus four *things* are made equal to the amount which you divided, which is ten less a *thing*. Restore thus ten by a *thing*, and add the same to the four [*things*]. It will hence be that ten are made equal to five *things*. Now you have thus reduced this question to one of the six chapters, which is that roots are made equal to number. [...]

Then two passages from Euler's *analysis infinitorum*, also showing his style of argument when taken together. First vol. I §139:⁴

Let now in the formulas of § 133 n be an *infinitely small* number, or $n = \frac{1}{i}$, i being an *infinitely large* number. Then

$$\cos nz = \cos \frac{z}{i} = 1 \quad \text{and} \quad \sin nz = \sin \frac{z}{i} = \frac{z}{i}$$

The sine of a vanishing arc $\frac{z}{i}$ is indeed equal to the arc itself, whereas the cosine = 1. Once that is posited one has

$$1 = \frac{(\cos z + \sqrt{-1} \sin z)^{\frac{1}{i}} + (\cos z - \sqrt{-1} \sin z)^{\frac{1}{i}}}{2}$$

and

$$\frac{z}{i} = \frac{(\cos z + \sqrt{-1} \sin z)^{\frac{1}{i}} - (\cos z - \sqrt{-1} \sin z)^{\frac{1}{i}}}{2}$$

[...]

Next the “general equation” for lines of the second order from vol. II §101.⁵

$$yy + \frac{\varepsilon + \gamma}{\zeta} y + \frac{\delta xx + \beta x + \alpha}{\zeta} = 0$$

There is no need to go into the mathematical substance of these excerpts. Even a superficial glance will reveal that there are decisive differences and little conspicuous kinship.

1. Al-Khwārizmī's arguments are rhetorical, that is, framed within the syntax of natural language; Euler calculates within the syntax of algebraic symbolism.

⁴ Leonhard Euler, *Introductio in analysin infinitorum*, vol. 1 (Lausanne: Bousquet, 1748), 104–105.

⁵ *Ibid.*, vol. 2, 48.

2. Al-Khwārizmī operates with a single unknown named by a word, Euler’s range of unknowns has no pre-determined limits, and his unknowns and operators (his “algebraic lexicon”) are represented by non-linguistic signs.
3. Al-Khwārizmī’s rules and problems deal with the unknown and its second power, Euler again knows no limits.
4. Al-Khwārizmī’s coefficients are numerically fixed, those of Euler may have undetermined values.
5. When al-Khwārizmī wants to operate on a composite expression, he needs to take roundabout ways; Euler has the parenthesis.

The parenthesis⁶ mostly goes unmentioned when the characteristics of the New Algebra are discussed, and the distinction between symbolic syntax and symbolic lexicon rarely is taken care of. For the rest, the list itself is familiar.

Actually, there is one more fundamental difference: Euler deals with the infinite – here the infinitely great and the infinitely small, elsewhere also with infinite sums. The New Algebra, however, did not do so in general, this only characterized its offspring, “infinitesimal calculus.” Moreover, the general parenthesis emerged within infinitesimal calculus, and was only broadly adopted as an algebraic tool after Euler’s time.

Euler is thus an approximate representative of the full-fledged New Algebra but does not provide an exact limit *ante quem*. As to *post quem*, we should also be aware that the algebra of the European Middle Ages (and thus ultimately the New Algebra) did not descend from the translations of al-Khwārizmī, nor from the *Liber abbaci*.⁷ With minimal lateral input from these, it had sprung from the algebra that was borrowed by abbasus masters in the early 14th century. Ultimately it was based on Arabic algebra, in a tradition which I have characterized as “diluted al-Karajī,” but apparently via an earlier Romance-speaking algebra tradition based in the Ibero-Provençal area, which has left no further surviving traces. Since this background is of no importance for the present enquiry, I shall only refer to my earlier writings on the topic.⁸

The earliest abbasus algebra – well represented by that of Jacopo da Firenze, in all probability written in 1307⁹ – corresponds well to the characteristics ascribed above to

⁶ In ordinary text as well as mathematics, a *parenthesis* is that which is enclosed (for instance) in a pair of brackets, not these brackets themselves.

⁷ This is thoroughly argued in Jens Høyrup, *The World of the Abbaco* (Basel: Birkhäuser, 2024).

⁸ See Jens Høyrup, “A Diluted al-Karajī in Abbasus Mathematics,” in *Actes du 10ième Colloque Maghrébin sur l’Histoire des Mathématiques Arabes (Tunis, 29-31 mai 2010)* (Tunis: Publications de l’Association Tunisienne des Sciences Mathématiques, 2011), 187–197; Jens Høyrup, *Jacopo da Firenze’s Tractatus Algorismi and Early Italian Abbasus Culture* (Basel: Birkhäuser, 2007), 153–169; Høyrup, *The World of the Abbaco*, 198–230.

⁹ Even if the algebra chapter in the Vatican manuscript of Jacopo’s *Tractatus algorismi* should not come from Jacopo’s hand, it can be seen to antedate 1328; and even if it may not be the very first

al-Khwārizmī. It is rhetorical, and makes use of no standard abbreviations that might (or might not) serve as algebraic symbols; it gives rules and illustrates these by means of solved problems; it operates with a single unknown and its powers; its coefficients are numerically fixed; and it has no parenthesis function. The only difference is that its range of powers goes on until the fourth – the *cosa* ('thing'), the *censo* (from Latin *census*, translating Arabic *māl*), the *cubo* and the *censo di censo*. The latter term is meant as a product, $censo \times censo$, and not as what we might understand as a function, $censo(censo)$. It gives rules for solving three-member equation types ('cases') involving these powers that can be solved by root extraction or be reduced to the second degree. In Paolo Gherardi's *Libro di ragioni* from 1328 we also find rules provided with examples for solving irreducible cases – obviously wrong, but since they involve radicals and since approximation was not considered that was not immediately conspicuous.

Initial transformation within abacus algebra

The abacus school can be traced back to *ca* 1260 – *not* to Fibonacci, as regularly claimed. It taught artisans' and merchants' sons for 1½–2 years, mostly 11 to 12 years old. At that age they had already learned to read and write, the abacus school taught them how to calculate with Hindu-Arabic numerals on paper (*not* using any kind of abacus) in commercial arithmetic: the rule of three, metrological conversions and shortcuts; simple and composite interest; alloying; perhaps the simple false position; and some rudiments of practical geometry, in particular simple area calculations. Neither the double false position nor algebra were taught. All this follows from two surviving expositions of the curriculum, on the whole agreeing except for minor differences in the order of topics.¹⁰

However, from the early 14th century onward, such matters entered many of the so-called "abacus books."¹¹ Why, if they did not serve in school?

Abacus master, the teachers of the school, were members of a craft, and as such they were in competition – in small towns, where abacus schools were organized by the municipality, for positions at these, and in large cities like Venice and Florence, where independent schools could thrive, competition for paying students. For both purposes, formal or informally arranged contests served to display the abilities of the competitors, and these contests had to concern what not every bungler in the trade or any average student

abacus algebra, it represents the type well.

¹⁰ One, from the first half of the 15th century, is a proper description of how teaching was in the "Pisa way"; the other describes the teaching duties of an assistant in Venice in 1519. Details in Høyrup, *The World of the Abbaco*, 6–7.

¹¹ "Abacus books" is a vague term, referring to all kinds of writings somehow connected to the abacus school: teachers' books or books for self-study; more or less orderly problem collections; writings by mathematical dilettanti once trained in an abacus school.

would master. Here algebra came to serve, and several writings show that it was indeed a prestige topic.¹²

This explains that false solutions could flourish – neither municipal councils nor the fathers of hoped-for students would look through them, and only with bad luck would one encounter a competitor who could. It also provided the drive for genuine development, however (more on this below). But let us first look at our chosen basic parameters.

Within a single generation, some writers began using letter abbreviations or non-letter signs for operations (plus, minus, root extraction) and for the unknown and their powers – first for instance ρ or c for the *cosa*, later for instance ce or $\square$ for the *censo*. In some cases these functioned as genuine *symbols*, in others not.

Since this distinction is not standard, an explanation may be needed. At first we may look at a passage from Biagio “il vecchio”¹³

Somebody makes two travels. In the first travel he earns 8, in the second he loses at the same rate as he has earned in the first. And then he finds to have 12 *forini*. It is asked with how much he set out. Posit that he set out with one *cosa*, at the first travel, and he earned 8, he will thus have a *cosa* and 8. And with this he left the first travel and goes to the second, at which it is said that he loses at the same rate as he has earned. You will say thus: at the first travel he made, from one *cosa*, a *cosa* and 8. And if at the second travel he loses at the same rate, he must do the opposite, that is that you will say, if from one *cosa* he earned 8, he earned $\frac{8}{1\rho}$ of a *cosa*, that is, $\frac{8}{1\rho}$ of his capital, where you will take $\frac{8}{1\rho}$ of a *cosa* and 8, which are 8 *cose* and 84 divided in a *thing*, that is, this fraction $\frac{8\rho\ 64}{1\rho}$.¹⁴ And this is that which he find to have done in the second travel, and we said that he finds 12.¹⁵ $\frac{8\rho\ 64}{1\rho}$ are thus equal to 12. And in

¹² Thus Vatican, Ottobon. lat. 3307 (henceforth Ottoboniano *Praticha*) from ca 1458, which on fol. 303^r introduces algebra with the words “all that which has been said until this point would be in vain without the present.” Luca Pacioli, *Summa de arithmetica, geometria, proportioni et proportionalita* (Venezia: Paganino de’ Paganini, 1494), fol. 144^r, “having with God’s assistance come to the much desired place: that is, to the mother of all the cases by common people called ‘the rule of the thing’ or ‘the major art’, that is, theoretical practice, also called *algebra et almucabala* in Arabic language.”

¹³ Licia Pieraccini, ed., *M° Biagio, Chasi exenplari alla regola dell’algibra nella trascelta a cura di M° Benedetto dal Codice L. VII. 2Q della Biblioteca Comunale di Siena* (Siena: Servizio Editoriale dell’Università di Siena, 1983), 110f. This “old” Biagio died before ca 1340. We know an extract from his *Praticha* from Benedetto da Firenze’s *Trattato di praticha d’arismetricha*, written in 1463 (this extract fol. 403^{r-v}). Since Benedetto’s own notation differs somewhat from what he uses when copying Biagio (and others), we can probably consider him faithful to the original.

¹⁴ Addition is indicated by juxtaposition, in agreement with the understanding of addition as aggregation of the constituents.

¹⁵ This is actually wrong, it is the loss at the second travel and not what remains; since the rate of loss is 50%, the two coincide, and a numerical check would not reveal the mistake.

order not to have fractions, multiply both sides by 1 *cosa*, you will have 12 *cose* to be equal to 8 *cose* and 64, where you shall confront the sides, removing from both sides 8 *cose*, and we shall have that 4 *cose* are equal to 6, where the *cosa* is worth 16. And we made the position that he set out with a *cosa*, he thus set out with 16.

As we see, Biagio uses the sign ρ within “formal fractions” and operates on them according to the same rules as for ordinary fractions. Others (and sometimes also Biagio) do the same with the *cosa* written in full; that is, they use the *symbolic syntax* without any “symbolic lexicon.”¹⁶

In contrast we may look at Dardi of Pisa, writing in 1344 in a Venetian linguistic environment (where *censo* becomes *çenso*).¹⁷ He uses *c* for *cosa* and *ç* for *çenso* (for ease of distinction I shall write *Ç*), but does so in a way that *excludes* symbolic operation. When providing them with coefficients he uses a fraction-like notation – instead of 30*c* Dardi mostly writes $\frac{30}{ç}$. The underlying idea is that the denominator of a fraction is understood not as a divisor but as a denomination, that is, as an ordinal. Similarly, indeed, in other abacus texts, for instance the third of three men can be referred to as “the $\frac{1}{3}$,” while $\frac{1}{grana 2}$ may stand for “1 *grana* and $\frac{1}{2}$ [of a *grana*].” This notation, not being *meant* to render a calculation, cannot serve for that purpose; that is, it is *not* symbolic, merely an abbreviation – as made even more obvious by the mixed writing of “1 *censo of censo*” as “ $\frac{1}{ç}$ de *Ç*.” Apart from use in manuscripts of Dardi’s work I have not noticed this notation elsewhere in Italian manuscripts, but it must have survived – it turns up repeatedly in the wildly eclectic “German algebra” from 1481 (called thus

¹⁶ Formal fractions written with full words are used in the *Alcibra amuchabile*, a composite Florentine manuscript which on the faith of indirectly connected watermarks is conventionally dated to ca 1365 (also by myself on several occasions) but may be as early as 1335. It explains how to add

$$\frac{100}{\text{per una cosa}} \quad \text{and} \quad \frac{100}{\text{per una cosa e più 5}}$$

by means of the parallel $2^4/4 + 2^4/6$, using the rules (syntax) according to which position in fractions determines the roles of and relations between the numbers, just as ordinary syntax determines the relations to ascribe the roles of subject, object and verb (etc.) of words in a sentence, and prescribes rules for performing the operation (here addition), just as ordinary syntax tells us (e.g.) how to change a sentence from the active to the passive voice. The manuscript is transcribed in Annalisa Simi, ed., *Anonimo (sec. XIV), Trattato dell’alcibra amuchabile dal Codice Ricc. 2263 della Biblioteca Riccardiana di Firenze* (Siena: Servizio Editoriale dell’Università di Siena, 1994), 41.

¹⁷ Raffaella Franci and Marisa Pancanti, eds., *Anonimo (sec. XIV), Il trattato d’algibra dal manoscritto Fond. Prin. II. V. 152 della Biblioteca Nazionale di Firenze* (Siena: Servizio Editoriale dell’Università di Siena, 1988), contains an edition of a Tuscan manuscript. Since the abbreviations and the full words do not agree in this Tuscan text, I use the Vatican manuscript Chigi M.VIII.170, where they do. See Høyrup, *The World of the Abaco*, 230.

because it is in German).¹⁸

Even though signs that *might* serve as symbols were much more common, there was probably little understanding of this distinction at the time. In any case, in the *Divina proportione* Pacioli explains that sundry professionals, among whom the *mathematici per algebra*, use specific *caratteri e abbreviature* “in order to avoid prolixity in writing and also of reading” – no symbolic use is hinted at.¹⁹ And indeed, even signs that *might* have served as symbols were mostly used as abbreviations only.

Even though formal fractions are regularly used within the running text, they appear more often as marginal annotations. More complex symbolic calculations are always kept there. We may look at an example from the Ottoboniano *Praticha*, fol. 331^v. It deals with a problem which is first expressed in words (as all problems are), but which we may render

$$\frac{100}{1\rho} + \frac{100}{1\rho+7} = 40$$

This is a problem type which since Biagio’s time had called for the use of formal fractions. In the margin we actually find the two formal fractions

$$\frac{100}{1\rho} \quad \text{and} \quad \frac{100}{1\rho+7}$$

and then this scheme

$$\begin{array}{r} 100\rho \\ \hline 100\rho \quad 700 \\ 200\rho \quad 700 \\ \hline 1\sigma \quad 7\rho \quad \quad \quad 40 \end{array}$$

$$200\rho \quad 700 \quad \text{—} \quad 40\sigma \quad \langle 280\rho \rangle$$

corresponding to the calculation

$$\frac{100}{1\rho} + \frac{100}{1\rho+7} = \frac{100\rho+100\cdot(\rho+7)}{(1\rho)\cdot(1\rho+7)} = \frac{100\rho+(100\rho+700)}{(1\rho)\cdot(1\rho+7)} = 40$$

Already Dardi had used schemes to indicate the cross-multiplication of binomials (in his case only arithmetical binomials, but others soon applied them to algebraic binomials, see presently) – for instance,²⁰ for $(3-\sqrt{5})\cdot(3-\sqrt{5})$

¹⁸ Kurt Vogel, ed., *Die erste deutsche Algebra aus dem Jahre 1481* (München: Verlag der Bayerischen Akademie der Wissenschaften, 1981), 10.

¹⁹ Luca Pacioli, *Divina proportione*, I (Venezia: Paganino de’ Paganini, 1509), fol. 3^v. The manuscript (Milan, Biblioteca Ambrosiana, MS 170 Sup., fols. XI^v–XII^r) specifies, listing abbreviations and signs for *radice*, *più*, *meno*, *quadrato* (*cosa* and *censo* are absent), together (*inter alia*) with abbreviations for *linea*, *geometria* and *arithmetic*. Obviously, not all of these could serve in symbolic calculations.

²⁰ Vatican, Chigi M.VIII.170, fol. 6^r.

$$\begin{array}{ccccccc} 3 & \tilde{m} & R & d e & 5 & & \\ 3 & \tilde{m} & R & d e & 5 & \rightarrow & 14 \tilde{m} R d e 180 \end{array}$$

(*R* abbreviates *radice*, ‘root’, *m̃* *meno*, ‘less’). For tri- or longer polynomials this would not do, and no later than the outgoing 14th century additions and multiplications of these were made (still in marginal calculations) in schemes that emulated those used for the corresponding operations with Hindu-Arabic numerals.²¹

Until Pacioli’s time, there was no agreement between those who used signs representing powers of the unknown or algebraic operations on *which* signs to use – the only exception being *R*; in Pacioli’s words, “as many heads, so many opinions.”²² Few used them *systematically* as abbreviations, and almost none as symbols.

Several unknowns were used by a few, but so few that we cannot establish the connections between them. All those who operate with two unknowns have the *cosa* as their first unknown. In first-degree problems, an entity occurring in the problem – for instance, a price – may be taken as a second unknown. Around 1380, Antonio de’ Mazzinghi solved a number of difficult second-degree problems by means of a second unknown called *quantità*. This happens in his *Fioretti*. It is copied conscientiously by Benedetto da Firenze in his *Trattato di Pratica*, which allows us to see that these “small flowers” were a work in progress; here we can follow how the idea of using a second unknown (thus certainly original) develops stepwise. Others must have used the treatise, but before Pacioli we do not know them.

Benedetto himself created a system allowing him to solve first-degree problems by means of symbolic algebra with many unknowns in 1463.²³ Nobody seems to have noticed, and his technique was forgotten.

Pacioli shows us another development. He does so clearly in the *Summa* (just to show how it is done, he explains), but already though confusedly in a large manuscript from 1478–1480 known after the dedication as *Suis carissimis discipulis* (Vatican, Vat.Lat. 3129). It is thus clear that the technique was not invented by Pacioli (it was also used by Nicholas Chuquet in 1484). This technique still uses *quantità* as a second unknown – but then eliminates it, allowing recycling of the name. So, Pacioli and those who inspired him knew to solve first-degree problems with at least three unknowns.

Already Jacopo had made use of four powers of the unknown. That number was soon to grow, and in parallel to the way higher powers were expressed in Arabic algebra (for

²¹ Thus, frequently, in a *Tratato sopra l’arte della arismeticha* (Florence, BNC, Fondo princ. II.V.152) from the outgoing 14th century, which for binomials uses the same scheme as Dardi. The extensive algebra part of this treatise is transcribed in Franci and Pancanti, *Il trattato d’algibra*.

²² Pacioli, *Summa*, fol. 67^v.

²³ See Høyrup, *The World of the Abbaco*, 315–325.

instance by al-Karajī), higher powers were initially expressed as products – beyond the *censo di censo* known to Jacopo the *cubo di censo* (or *censo di cubo*), the *cubo di cubo* (or *censo di censo di censo*), and so forth. In the outgoing 14th century, certain writers began transforming this system, and to use sometimes composition based on embedding instead; the above-mentioned Florentine *Tratato* thus uses *censo di cubo* for *censo(cubo)*, that is, the sixth power, but *cubo di censo* for *cubo×censo*, that is, the fifth power.²⁴ In the late 15th century, embedding had taken over completely, with the consequence that prime powers had to get independent names – the fifth power becoming the *primo relato*, the seventh the *secondo relato*, etc.

Abstract coefficients were only invented by Viète, so there is no reason to discuss them at this point. One “special-purpose parenthesis” was invented, however, first apparently by Dardi. When he wants to indicate that a root is to be taken of a binomial he uses the phrase *R de zonto*, “root of, joined, ...” – for instance,²⁵ “*R de zonto cò ¼ R de 12*” standing for

$$\sqrt[1/4]{12}$$

Later writers until Cardano would mostly speak of a “universal” or “bound” root with the same meaning. In principle the notation is ambiguous – how long is the expression meant to serve as radicand? However, only in extremely rare cases is more than a binomial meant.

Antonio goes further in his *Fioretti*. He uses $\textcircled{R}$ or an encircled fully written *radice* for the universal root. When dealing with an expression corresponding to our

$$\sqrt{\textcircled{\text{cosa} + \sqrt{350-1 \text{ censo}}}}$$

he refers to the “outer brackets” as “general root,” “general root of one *cosa* plus root of 350 less one *censo*” (underlining represents encircling). This terminology was taken over (but misunderstood) by his student Giovanni di Bartolo, after which we have no more traces of it (nor of calculations where it would be needed). Cardano, when eventually encountering the need in the *Ars magna* (fol. 14^r), had to reinvent.

The numerator as well as the denominator of formal fractions are also, if composite, special-purpose parentheses, as is the fraction as a whole.

At a pinch, we may also consider the notation for higher powers by embedding (e.g., *censo(cubo)*) as a parenthesis; however, the only “argument” (to use the later language of functions) that can be embedded is a power – we never find the *censo* taken of a composite expression, for instance *censo(5+R5)*; this would have asked for a transfer of the notion of “universal” from the domain of roots, but nobody seems ever to have been tempted to

²⁴ The catalyst for this inconsistent system was probably the quest for a way to express roots beyond the square root of the square root.

²⁵ Vatican, Chigi M.VIII.170, fol. 9^v.

make that step.

Already al-Khwārizmī sometimes needed to operate on a composite algebraic expression. Then he would interrupt the flow and make a separate calculation; we may see this as a substitute for a parenthesis. In abacus algebra, such separate calculations often made use of schemes for adding or multiplying polynomials emulating those used in the arithmetic of Hindu-Arabic numerals.

Counting generously, abacus algebra thus made use of four kinds of special-purpose parentheses; but nobody at the time would see them as instantiations of a single category. There was no general need for that, just as there was no need for a general notion of “function” before the number of these grew beyond powers, sine, tangent, logarithm and exponential.²⁶

So, on most of our chosen parameters, abacus algebra presents us with small steps on the road leading (in retrospect) toward the New Algebra; but they were small, unsystematic, not elements of a search for anything (whatever it might be). To these characteristics we may add an instance of mathematical acuteness. It is a spin-off from the search for solutions to irreducible higher-degree equations, and it turns up twice – first in Dardi.

Dardi offers rules for solving a huge number of seemingly abstruse equation types involving radicals; all are correct, apart from two where he has no name for a fifth and a seventh root. Beyond these he offers rules for solving four irreducible equation types, pointing out explicitly (Vatican, Chigi M.VIII.170, fol. 100^r) that they only work in special cases; they differ in style from the rest, and are almost certainly not his own inventions. He does not explain where they come from, but analysis of the coefficients (made by means of modern letter algebra) reveals the trick: namely a change of variable.²⁷ Whoever invented these rules must have had a fair grasp of polynomial algebra – a *good* grasp, given that he did not have our symbolism.

The other instance is found in the Florentine *Tratato*. As confirmed by other sources, in the late 14th century a “solution” to equations of the type $K = \alpha t + N$ circulated (N stands for “number,” t for *cosa*, K for t^3). As explained in the *Tratato*, the “cube root of 44 with 5 added” is 4 because $4^3 = 44 + 5 \cdot 4$.²⁸ Similarly, the cube root of 65 with 12 added is 5 because $5^3 = 65 + 12 \cdot 5$. This is not very interesting. It may be said to offer just a name to the solution; moreover, as the author of the *Tratato* says, it is sometimes but not always possible for given t and N to find a fitting (integer) value of α . Mathematically interesting,

²⁶ The general parenthesis only emerged in the company of the explicit function concept within infinitesimal analysis in the later 17th century; see Jens Høyrup, “How Did the All-Purpose Parenthesis Come About in European Algebra?” *Gaṇita Bhāratī* 45 (2023): 45–75.

²⁷ The principle, expressed in modern notation: The solution to the equation $x^3 = a$ is obviously $x = \sqrt[3]{a}$. Therefore, if $x = y + e$, $y^3 + 3y^2e + 3ye^2 + e^3 = a$, and $y = \sqrt[3]{a - e}$. A proper analysis is offered in Høyrup, *The World of the Abaco*, 262.

²⁸ Franci and Pancanti, *Il trattato d'algebra*, 98.

however, is his explanation (p. 99) of how other equation types – for instance $K + \beta C = N$ – can be transformed into the shape $K = \alpha t + N$. He does not explain in detail, but analysis of the coefficients shows once again that polynomial algebra has been made use of.²⁹

Naively we may wonder that Dardi and the anonymous Florentine did not display their ability (or, even more naively, help the collective advance of mathematical knowledge) by explaining their method. Taking into account the social setting of abacus mathematics we should rather ask, why should they? The abacus masters, as said, were competing members of a craft, and complicated algebra served in their competitions. Advanced insights within this domain were therefore business secrets. We may remember the Cardano-Tartaglia clash. Cardano, in agreement with the norm system of university learning, published the solution to the cubics; Tartaglia, still living on the conditions of an abacus master however much he strived to be recognized as a scholar, had needed the solutions to compete.

The lack of uniformity of notations and of systematic use *may* also have something to do with this setting, but it is probably in the main to be explained by the low density of interactions. The use of ρ a symbol/abbreviation for *cosa* was handed down in a tradition reaching from Biagio to Benedetto, but did not diffuse much laterally – other masters belonged to other school traditions and stuck to their own ways. After all, abacus communication beyond the single school was mainly carried by manuscripts,³⁰ and these had limited circulation – only rarely, more than a few copies were made. That was to change when Pacioli's *Summa* was printed in 1494, perhaps in 2000 copies, all of which were sold.³¹ After that, the notation for powers found there was adopted by Tartaglia as well as Cardano.

Transmission to German land, and the creation of Coss

Tartaglia and Cardano, however, are outside the main lines of this story. That line, instead, goes through southern German areas.

At least since the early 14th century it had been the habit of long-distance merchants from Nürnberg and other south-German cities to send sons and apprentices (we do not

²⁹ Analysis in Høyrup, *The World of the Abbaco*, 264.

³⁰ Mainly, not exclusively. Jacopo da Firenze must have learned his basics at home and then went to Montpellier, apparently to learn more; but his *Tractatus* was then spread in manuscript copies, and his preface in many more. Dardi, from Pisa, seems to have gone to Venice, but again his marvellous treatise went around as manuscripts (four, and a Hebrew translation, have survived).

³¹ Alan Sangster, “The Printing of Pacioli's *Summa* in 1494: How Many Copies Were Printed?” *Accounting Historians Journal* 34 (2007): 125–145. The complete print run must have been sold over the years since the printer made a new edition at his own costs in 1523.

know how many) to Venice in order that they might learn the language and the abbasus.³²

Only rich merchant houses could afford that, and that provided the occasion for a new profession to arise – the *Rechenmeister*. In 1457, three of these are known to have held school in Nürnberg.³³

Rechenmeister translates *maestro d'abbaco*, but the style of the teaching as well as the social setting were different. True enough, even the *Rechenmeister* were in competition – the three Nürnberg masters were litigating in court. But one of them published a book, and that symbolizes that competition had moved to a different arena.

Over the next centuries, hundreds of *Rechenbücher* were published, almost invariably claiming on their title page to contain material which is totally new (which is rarely the case). Yet while abbasus algebra thrived within the framework of the abbasus school, German 16th-century algebra (the *coß*) was largely independent. Extremely few *Rechenbücher* present the technique.

The *coß* was an offspring of abbasus algebra, but through particular channels. Around 1460, interest was kindled among (a few) university scholars in a new mathematical technique – new with respect to the mathematics cultivated in their environment. The first phase was eclectic; those who were involved had to draw on whatever they could get hold of, and that can be seen to have been neither coherent nor of high quality. Two participants are known by name: Johannes Regiomontanus, the Vienna astronomer, and Friedrich Amann, who may have gone through Leipzig University but learned (maybe later, when a monk in the St Emmeran monastery in Regensburg) from the Viennese Gmunden tradition.³⁴ The others are anonymous but are likely also to have come from a university setting.

Among other writings, Regiomontanus studied a manuscript containing Gerard's translation of al-Khwārizmī. The only surviving manuscripts of Robert of Chester's translation of the same work were also written in south German areas.³⁵ However, influence from either of these in the later *coß* tradition is negligible, restricted to the use of *dragma* as the unit for pure numbers, and widespread awareness that the proper name of the technique is *algebra et almuchabala*.

The names *coß* and *zensus* for the unknown and its square show that northern Italy

³² Bettina Pfotenhauer, *Nürnberg und Venedig im Austausch* (Regensburg: Schnell & Steiner, 2016), 72–76.

³³ Eberhard Schröder, ed., *Ulrich Wagner, Das Bamberger Rechenbuch. Facsimile-Druck der Ausgabe von 1483* (Weinheim: VCH Verlagsgesellschaft, 1988), 301.

³⁴ See Armin Gerl, "Fridericus Amann," in *Rechenbücher und mathematische Texte der frühen Neuzeit*, ed. by Rainer Gebhardt (Annaberg-Buchholz: Adam-Ries-Bund, 1999), 1–12; Dana Bennett Durand, *The Vienna-Klosterneuburg Map Corpus of the Fifteenth Century* (Leiden: Brill, 1952), 73–76.

³⁵ See Barnabas B. Hughes, ed., *Robert of Chester's Latin Translation of al-Khwārizmī's Al-jabr* (Wiesbaden: Franz Steiner, 1989), 11–13.

(Venice etc.) and not Tuscany provided the main inspiration – in the north they were written *cossa* and *zenso* in the 15th century.

The details of this eclectic phase do not concern us here.³⁶ The first extant orderly presentation – and the earliest extant representative of the *coß* – is a Latin algebra³⁷ contained (together with numerous other mathematical treatises) in the manuscript Dresden, Sächsische Landesbibliothek, C 80. This manuscript was once in the possession of Johannes Widmann, and the Latin algebra is likely to have been the basis for the algebra lectures he offered in Leipzig in 1486. We have no evidence that these lectures had any influence, but the contents of the Latin algebra had.

Actually, the treatise in itself is a conglomerate, but the three components supplement each other without contradicting, showing that this contents already existed as a whole.

The treatise makes systematic use of abbreviations for the first powers of the unknown:

φ	<i>numerus</i>
$\mathcal{R}$	<i>res</i>
$\mathfrak{Z}$	<i>zensum</i>
$\mathcal{C}$	<i>cubus</i>
$\mathfrak{Z}\mathfrak{Z}$	<i>zensum zensorum</i>

Less systematic is the use of a dot «·» for *radix*, “root,”³⁸ and «··» for “root of root.” Cube root, instead, when not written in full, is “ $\mathcal{R}$ cubica”; the square root also occasionally appears as $\mathcal{R}$.

The Italians, when abbreviating *più* (‘more’, whence ‘plus’) and *meno* (‘less’, whence ‘minus’), had used letter abbreviations. The present manuscript instead uses + and –. Like the transformed dot, even these signs, mostly serving as symbols, were also taken over by others and ultimately by modern mathematics.

Where they can serve, formal fractions are used. No other instances of symbolic calculations are present.

The next, and decisive, step toward the formation of the *coß* was undertaken by Andreas Alexander, one of the first specialist lecturers on mathematical topics (meaning

³⁶ They are described in Høyrup, *The World of the Abbaco*, 389.

³⁷ Edited in Hermann Emil Wappler, “Zur Geschichte der deutschen Algebra im 15. Jahrhundert,” in *Gymnasium zu Zwickau. Jahresbericht über das Schuljahr von Ostern 1886 bis Ostern 1887* (Zwickau: R. Zückler, 1887), 1–32.

³⁸ This dot, when written on uneven paper, was often written with a down- and an upstroke, as $\sqrt{}$. This gave rise to the modern root sign $\sqrt{}$. Once this transformation was made, for instance in Christoph Rudolff, *Behend und hübsch Rechnung durch die kunstreichen Regeln Algebra, so gemeinlich die Coss genennt werden* (Straßburg: Johann Knoblauch, 1525), and Michael Stifel, *Arithmetica integra* (Nürnberg: Johan Petreius, 1544), a dot after the root sign could be used to indicate a universal or bound root.

he was not allowed to go on with other, more lucrative topics; certainly no promotion). Before 1504 he wrote a Latin algebra manuscript. One version (Leipzig, Hs. 1696) was discovered by Menso Folkerts. Beyond that, we have a German commentary (*Initium Algebrae*, “Algebra’s Introduction”) surviving in four copies, which explains a somewhat different, later version of the Latin treatise.³⁹ The existence of at least two different manuscripts, and of a number of copies of the German commentary, explains that Alexander’s work could become influential.

Alexander extends the sequence of names of powers and appurtenant signs

φ	1. dragma or numerus
$\mathcal{C}$	2. res
$\mathcal{Z}$	3. zensus
$\mathcal{C}$	4. cubus
$\mathcal{Z}\mathcal{Z}$	5. census de censo
$\mathcal{B}$	6. sursolidum
$\mathcal{Z}\mathcal{C}$	7. cencicubus
$\mathcal{B}\mathcal{B}$	8. bissursolidum
$\mathcal{Z}\mathcal{Z}\mathcal{Z}$	9. census censu de censu
$\mathcal{C}\mathcal{C}$	10. cubus de cubo

As we observe, he also numbers them – unfortunately counting *number* as the first, which makes the formulation of rules for their product cumbersome; Pacioli had made the same unlucky choice a few years earlier, but whether there is any influence is undecidable. In any case, in the later version which is commented upon in *Initium Algebrae* he drops this infelicitous numbering. Formal fractions are not only used but also explained; operations on polynomials are made within schemes emulating those used for calculation with Hindu-Arabic numerals. In *Initium Algebrae* it is even shown how to multiply two formal fractions whose numerators themselves are formal fractions.⁴⁰ Calculations with quadratic and cubic surds are explained with reference to geometric diagrams (the latter in perspective drawing).

The Latin algebra from C 80 had offered 24 rules – the 22 rules for reducible cases that had been current in abbas algebra (20 of them since Jacopo), and two (equally reduc-

³⁹ This follows from ongoing work by Martin Hellmann, who also corrected the interpretation of the title (Martin Hellmann, “Von Pythagoras zum Liber de cosa: Bemerkungen von Nicolaus Matz über die Geschichte der Rechenkunst,” in *Bewahren und Erforschen: Beiträge aus der Nicolaus-Matz-Bibliothek (Kirchenbibliothek) Michelstadt*, edited by Wolfgang Schmitz (Michelstadt: Stadt Michelstadt, 2003), 88). Maximilian Curtze, ed., *Urkunden zur Geschichte der Mathematik im Mittelalter und der Renaissance* (Leipzig: Teubner, 1902), 435–600, is an edition of what he named *Initius Algebras*. In its prologue (fol. 41^v) and explicit (fol. 127^v), even the Latin text refers to the work as that of *Algebras arabus*.

⁴⁰ Curtze, *Urkunden*, 517.

ible) involving roots; at the same time it had explained how to reduce higher-order cases. Alexander cuts down their number to eight, leaving out those that can be easily reduced.

The first to bring German algebra into print (now German also in language) was Heinrich Schreyber alias Grammateus, in *Ayn new kunstlich Buech, welches gar gewiß und behend lernet nach der gemainen regel Detre, welschen Practic, regel falsi unn etlichen regeln* Cosse from 1521. Quite unusually he did so as part of a *Rechenbuch* (obviously, norms for what belonged within the genre had not yet crystallized). He identified powers not by means of signs but by numbers (coinciding with our exponents). Since no repercussions of his work can be traced there is no reason to discuss it further.

The book that instead came to define *coß* was Christoph Rudolff's *Behend unnd hübsch Rechnung durch die kunstreichen Regeln Algebra, so gemeincklich die Coss genennt werden* from 1525 (henceforth Rudolff's *Coss*). Beyond its being printed and its lucid style, one reason was probably that Rudolff offered a large number of problems illustrating the use of the eight rules.

Rudolff owes much to Alexander. He presents the same eight cases (making fun of those who make empty generalizations to 24, offering a hundred if wanted); he uses the same names and signs for the powers of the unknown; and his arithmetic for monomials and binomials is similar. He eliminates the geometric demonstrations used by Alexander to support the explanation of the arithmetic of mono- and binomials (as well as those serving in the *Initium Algebrae* for the fundamental algebraic cases).

In a way we might claim that there is nothing new in what was done by Alexander and Rudolff. Nothing they do had not been done already by *some* abacus writer. We may even observe that we find in the cossic writings nothing similar to the secret explorations of polynomial algebra and its application to irreducible higher-degree equations. There is, however, a fundamental difference: Since the Latin algebra of C 80, cossic algebra had used the same notation (leaving aside Schreyber's vain attempt to introduce a theoretically based alternative), and it was using it systematically – so systematically that it is often difficult to find out which verbal names for the powers were intended.

Most of what Rudolff offers (though not his abundance of examples) goes back to Alexander. Not everything, however, does, Rudolff also adds. In particular – of importance for the future of algebra – he adds the *regula quantitatis* and uses it extensively.

The *regula quantitatis* was not new. It is simply the technique to eliminate and then recycle a second unknown – the *quantitet*, thereby allowing the operation with more than two unknowns. The technique was mentioned above in connection with Pacioli, together with its use by Chuquet in 1484 – thus after the writing of the *Suis carissimis*, which already suffices to show that Chuquet was not Pacioli's source. Inspiration the other way can also be excluded, Pacioli's presentation in *Suis carissimis* being confused.

The “rule” – and now also the name, unknown to Chuquet and Pacioli – are also used

by Étienne de la Roche in 1520.⁴¹ Albrecht Heeffer takes this to be an indication that de la Roche was Rudolff's source,⁴² but at the same time points out that a marginal note written into the Chuquet manuscript,⁴³ almost certainly by de la Roche, states that *ceste regle est appelee la Regle de la quantite*, "this rule is called the rule of the quantity," showing that de la Roche has discovered use of a rule he already knows under this name. None the less, striking unlikely similarities between de la Roche's text and that of Rudolff make it next to certain that Rudolff knew de la Roche's book.⁴⁴

De la Roche uses the method much less than Rudolff, however, and his book was not nearly as well diffused. Already for this reason there is little doubt that the after-world came to know the method mainly through Rudolff.

This "completion of the *coss*, indeed in truth a completion without which it would not be worth much more than a trifle"⁴⁵ became really important through the transformation it underwent in Michael Stifel's *Arithmetica integra* from 1544 and in particular in his re-edition of Rudolff's *Coss* from 1553.

Stifel's "Whole of arithmetic" consists of three books. Book I may be characterized as a regular though broadly considered arithmetic. Book II is, so to speak, an arithmetical transformation of *Elements* X. Book III (fols. 227^r–319^r), finally, is an orderly algebra in the *coß*/Rudolff tradition.

Already therefore it was an important book. Few outside German lands at the time were able to read German, so the improvements of *coß* algebra over the *abbacus* tradition – namely the insight that a symbolic notation is next to worthless if it is not used systematically, and in the same way by everybody – only reached them through Stifel.

Even more important, however, was certainly his transformation of the *regula quantitat*is (as I shall argue). Stifel was the first to propose a system for the use of many unknowns (Benedetto had not explained a system, he just developed the technique, soon to be forgotten). On fol. 251^v we find the heading *De secundis radicibus*, "on second(ary) roots." Stifel still uses *res* (written $\mathcal{R}$) as his primary unknown, but instead of a *quantitet* that can be recycled he proposes an open-ended sequence of secondary unknowns, using the letters of the alphabet: "1A (that is, $1A\mathcal{R}$), 1B (that is, $1B\mathcal{R}$), 1C (that is, $1C\mathcal{R}$), 1D etc."; for their second powers he uses $1A\mathcal{Z}$ etc. For the product of $\mathcal{R}$ and A he suggests $\mathcal{R}A$,

⁴¹ Étienne de la Roche, *Larismethique nouvellement composee* (Lyon: Constantin Fradin, 1520), fols. 42^r, 61^r.

⁴² Albrecht Heeffer, "The Rule of Quantity by Chuquet and de la Roche and Its Influence on German Cossic Algebra," in *Pluralité de l'algèbre à la Renaissance*, edited by Sabine Rommevaux, Maryvonne Spiesser, and Maria Rosa Massa Esteve (Paris: Honoré Champion, 2012), 127–147.

⁴³ Paris, BNF, français 1346, fol. 169^v.

⁴⁴ Jens Høyrup, *Explorations and False Trails* (Cham: Springer, 2024), 106.

⁴⁵ Rudolff, *Behend unnd hübsch Rechnung*, fol. I.vi^v.

while that of A and B will be written AB .⁴⁶

The first illustrating example asks for the splitting of a given number in parts with given ratio. It is taken from Rudolff, who solves it on fol. 64^r by means of a single unknown, as everybody had done since al-Khwārizmī. The second asks for seven numbers (here supposed to be debts owed to Stifel), given the sums of all except the first, except the second, except the third, etc. That type (we may call it “all except each”) was traditionally solved without any algebra.⁴⁷ Stifel uses his new technique, in a way which would easily have yielded to the traditional *regula quantitatis*.

The third example is a reducible quartic, thus more advanced. Actually, apart from Antonio (and Pacioli borrowing one of his problems), nobody known to us in the abacus

1 r	39-1B	1 z
1 A	78+1B-1z	39-1B

or derived traditions had used several unknowns in non-linear problems. The problem (fol. 254^r) asks for two numbers (say, P and Q) fulfilling the condition

$$P^2 + Q^2 - (P + Q) = 78, PQ + (P + Q) = 39.$$

Stifel chooses the first number to be 2ℓ , the second to be A and their sum to be B . He proceeds in a way that owes more to *Elements* II or square-grid geometry than to algebra, using the adjacent diagram.

The second condition gives him $2\ell A = 39 - 1B$. Thereby he can complete the square, etc., without using algebra. As we see, the geometric interpretation allows

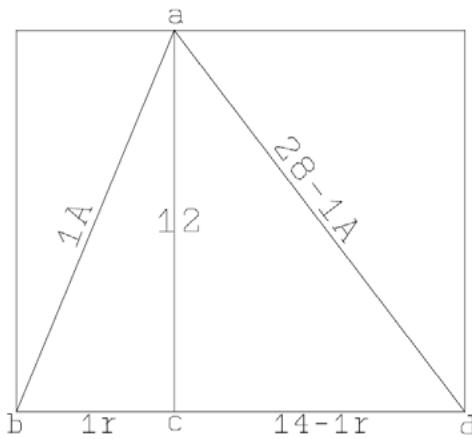
Stifel to take over from geometry the habit of naming more than a minimal set of unknowns by letters (a point to which we shall return).

The end of the *Arithmetica integra* is a collection of supplementary problems. Most of those on fols. 292^r–301^r make use of the new system, and with two exceptions they are all of the first degree (and thus uninformative).

One non-linear problem (fol. 292^r) asks for three line segments for which the areas of the rectangles they contain pairwise are given. They are named 2ℓ , A and B . The solution follows from another square-grid diagram, without any use of algebra or formulation of an equation.

⁴⁶ The explanation of $1A$ as standing for $1A2\ell$ shows that A , B , etc. are thought of as markings. So, all the first powers are 2ℓ , but they are distinguished as $^A2\ell$ ‘the A -kind of 2ℓ ’), $^B2\ell$, etc. When standing to the left, on the other hand, 2ℓ is meant as a factor. This system might easily produce mistakes, and in 1553 Stifel would abandon it.

⁴⁷ The sum of all given numbers is seven times the sum of the debts minus a single time this sum, that is, six times the sum of the debts.



The final problem (fol. 300^v) is more intricate than the others, also according to Stifel, and meant to illustrate the “immense usefulness of the second roots.” It is properly geometric but solved by means of algebra. It deals with a rectangle with sides 12 and 14, subdivided into two rectangles, the sum of whose diagonals is 28. From the Pythagorean theorem follows that ab^2 is $12^2 + (1^2\mathcal{C})^2 = 1\mathcal{Z} + 144$, identified with $1A\mathcal{Z}$ (i.e., $(1A)^2$). ad^2 is found in a similar way to be $340 - 28^2\mathcal{C} + 24\mathcal{Z}$; but it is also

$(28 - A)^2 = 784 - 56A + 1A\mathcal{Z}$. Therefore, $1A\mathcal{Z} = 56A + 1\mathcal{Z} - 28^2\mathcal{C} - 444$. But $1A\mathcal{Z} = 1\mathcal{Z} + 144$, so A can be eliminated. The problem is thus of the second degree, but only in $^2\mathcal{C}$.

In 1553, Rudolff’s *Coss* had become so rare that it was no longer to be bought “even if one wanted to pay three or four times” the price, for which reason Stifel was asked to prepare a new edition (as Stifel tells in his preface). This new edition appeared as *Die Coss Christoffs Rudolffs. Die schönen Exemplen der Coss gebessert und gemehrt*. Here, Stifel replaces Rudolff’s use of the *Regula quantitatis* with his new system, also using it in a number of problems where Rudolff had used a single unknown.

That, of course, does not tell us much. More interesting is a collection of 24 new sophisticated problems added in the end – appropriate to the different context of the book, where problem solution and not the presentation of theory is central.

Half of the new problems (all of higher degree) make use of several unknowns. One of them appears to be borrowed from Pacioli (*Summa*, fol. 148^v), who had already borrowed it directly or indirectly from Antonio: a question for two numbers fulfilling the conditions $mn = 96$, $m^2 + n^2 = 292$. Stifel makes the positions $m := ^2\mathcal{C} + A$, $n := ^2\mathcal{C} - A$. Pacioli’s parameters are different but his positions the same. The remaining 11 problems appear to be new. Two are solved by means of geometric diagrams similar to those used in the *Arithmetica integra*, the others by means of algebra. All would be very difficult without use of the new technique.

Stifel has now simplified his notation, leaving behind the idea that all unknowns are different kinds of $^2\mathcal{C}$. Instead of $A\mathcal{Z}$ Stifel now writes AA (etc.); powers until k , $\mathcal{Z}A$, $^2\mathcal{C}AA$ and AAA appear (fol. 469^v).

The echo was faint, few German writers on algebra believed they had any use for Stifel’s system (we shall return to French writers). Caspar Peucer’s Latin *Logistice Regulae Arithmeticae, quam Cossam & Algebram quadratam vocant*, the second part of his *Logistice Astronomica Hexacontadōn et Scrupulorum Sexagesimorum* [...], contains a section *De radicibus secundis*,

“on second roots” (fol. T.vi^r). It refers to Rudolff, Cardano⁴⁸ and Stifel. The notation is that of the *Arithmetica integra* (“1 A, id est 1 A²℄,” etc.). The book was meant for the higher Lutheran educational system and had scant influence; its examples are utterly simple.

Mattheus Nefe’s *Zwey neue Rechenbuecher* from 1565 does not treat algebra in general (that, as said, falls outside the scope of standard *Rechenbücher*); once, however, Nefe gives a single example of *Regula quantitatis* of type “all except each.”⁴⁹ In so far it is uninteresting; Nefe, however, has seen that there is no reason to distinguish between primary and secondary unknowns, so his unknowns are simply A, B, C and D.

Finally, in his *Algebra*, Christopher Clavius (German at least by birth, though working in Rome and writing in Latin) presents the system from the *Arithmetica integra* faithfully.⁵⁰ On pp. 56–70, 15 problems illustrate its use. The first three coincide with Stifel’s first three (the first and second changing the parameters but nothing else); the fourth is a very simple second-degree problem.⁵¹ 11 are of traditional recreational type (“give-and-take,” “finding of a purse,” “purchase of a horse”). No wonder perhaps that nobody seems to have taken notice among the many students of the Jesuit schools who were taught by Clavius’s book (including Descartes, as we shall see).

Rounding off, we observe once more that *Rechenmeister* algebra was not closely associated with general *Rechenmeister* mathematics. *Rechenbücher* might present the double false position, even though even that was not of real practical use; but only Schreyber dealt with algebra.

Some *Rechenmeister* may have followed the arts course of a university, but as a rule they had not. Writers on algebra, to the contrary, had fair Latin learning (whether obtained during a regular inscription or not).

Algebra was known as an art serving to solve problems, and *Rechenmeister* algebra did so too. In so far it followed the norm system of *Rechenbücher* – after all, this orientation agrees well with the inherited Italian model.

University mathematics was different.⁵² At the lower level it taught Boethius’s *Introduction to Arithmetic*, and also algorism, the calculation with Hindu-Arabic numerals includ-

⁴⁸ Girolamo Cardano, *Practica arithmetice, et mensurandi singularis* (Milano: Bernardini Calusco, 1539), fol. L.viii^r, follows Pacioli, referring to the second unknown as *quantitas surda*. In Girolamo Cardano, *Artis magnae sive de regulis algebraicis, liber unus* (Nürnberg: Johan Petreius, 1545), fol. 21^v, it has become *secunda quantitas incognita*.

⁴⁹ Mattheus Nefe, *Arithmetica: Zwey neue Rechenbuecher* (Breslau: Crispin Scharffenberg, 1565), fol. P.iii^r.

⁵⁰ Christopher Clavius, *Algebra* (Roma: Bartolomeo Zanetti, 1608), 72–76.

⁵¹ The structure is $x^2 + a^2 = 340$, $xa = \frac{6}{7}x^2$. Clavius may have borrowed it from Jacques Peletier, *L’algebre* (Lyon: Ian de Tournes, 1554), 102.

⁵² An overview can be found in Jens Høyrup, “Mathematics Education in the European Middle Ages,” in *Handbook on the History of Mathematics Education*, ed. by Alexander Karp and Gert Schubring (New York: Springer, 2014), 109–124.

ing sexagesimal fractions. At the higher level, at least the first part of the *Elements* or Wite-lo's *Perspective communis*, and (probably for the few, but well known to the best Vienna mathematicians) *theory* – that of Jordanus of Nemore, and of the Oxford *calculatores* and continental followers like Albert of Saxony and Nicole Oresme. Lectures on theory were certainly followed by disputations, but from their reflection in collections of *quaestiones* we see that they would concentrate on foundational problems (Oresme's questions on Euclid being in part an exception, since Oresme used the opportunity to produce *more theory*). There was no opportunity to develop a culture of problems.

This aspect of university mathematics influenced even that of the writers on *Rechenmeister* algebra. Firstly, many of them, from Alexander onward, presented the arithmetic of irrationals much more thoroughly than had been done in *abbacus* algebra.⁵³ Secondly, the problems presented in *Rechenmeister* algebra are really meant as illustrations, there is little agonistic ostentation; we may even say that Rudolff, when poking fun at the 24 rules, censures them as empty display.

Thirdly, as far as it goes the use of symbols (signs for the unknown and for operations, the use of formal fractions and of schemes for operations on polynomials) is systematic, and very homogeneous. As pointed out by Franz Woepcke in connection with his description of the Maghreb algebraic notation,⁵⁴

the indispensable condition for characterizing a set of conventional signs as a notation is that they are always used when it is fitting, and always in the same way.

The role of a single book – that of Rudolff – as the one everybody had read and emulates (a “paradigm” in Thomas Kuhn's original sense) of course helped; but only because Rudolff himself had been systematic within that book.

Repercussions of Stifel's system outside the German area

As we have seen, there were almost no repercussions of Stifel's new system within the German area. Elsewhere?

I have only been able to trace four pertinent authors. Two seem to have been uninfluential, a third *may* have provided Viète with some inspiration, the fourth though almost unnoticed was probably quite important.

⁵³ In a way, Benedetto is an exception – but what we find in his *Trattato di prattica* is a copy of an arithmetical interpretation of *Elements* X which he has found ready-made (Høyrup, *The World of the Abbaco*, 330).

⁵⁴ Franz Woepcke, “Recherches sur l'histoire des sciences mathématiques chez les Orientaux, d'après des traités inédits arabes et persans. Premier article. Notice sur des notations algébriques employées par les Arabes,” *Journal Asiatique*, Se série, 4 (1854): 355.

The first was Jacques Peletier, who follows the *Arithmetica integra* without adding anything conspicuous. We shall say more about him below.

The second is Jean Borrel (Latinized Buteo), who used several unknowns in his *Logistica, quae et arithmetica vulgò dicitur* from 1559. Borrel is one of those who rarely cite predecessors except in order to castigate them. Moreover, he often changes the terminology he has inherited in order to make the exposition look more Greek (the *Logistica* of his title is an example). We must therefore rely on indirect arguments in order to determine from where he got his ideas.

On p. 189 he introduces “another way of calculation which *vulgò* is called the Rule of quantity.” He states that he will *not* follow the shape given to it by Luca [Pacioli] or Stephanus [de la Roche] or others because that shape is extremely troublesome [*moles-tissima*]. Since Borrel is not afraid of mentioning Cardano (he does so on p. 188), the unidentified “others” can hardly be anybody but Stifel (or Peucer or Valentin Mennher, on whom imminently, both of whom follow Stifel). Since his unknowns are 1 A, 1 B, etc., Borrel has simply done as Nefe was to do again a few years later: he gives up the idea that one unknown in primary and the others secondary.

On pp. 189–196, four examples are offered, and on pp. 357f a fifth. They are all linear and variations of such classics as “give-and-take” and “finding of a purse.” Hardly likely to provoke much interest.

The third is Guillaume Gosselin, to whom we shall also return. Even he censures predecessors (Pacioli, de la Roche, Cardano, Borrel “and others” without specifying in what they err).⁵⁵ In any case, he appears to imitate Borrel with a small admixture from Pacioli or Cardano. The topic fills book IV of the work. At first, under the heading “De quantitate absoluta” (fol. 80^r), five problems are dealt with in Borrel’s notation; next, with heading “De quantitate surda” (fol. 84^e), four problems are dealt with as Pacioli would have done, the second unknown being *q* (without recycling, however). All problems are of the first degree, of classical recreational types though formulated in terms of pure numbers and not as dealing with monetary possessions or prices. It is noteworthy that A and B are used to represent numbers in *arithmetical* argument on fols. 27–33 and 58–62.

Mennher’s *Arithmétique seconde* from 1556 did provoke interest – exceptionally long-lasting interest indeed. In 1620, Didier Henrion was to publish *Deux cens questions ingénieuses et récréatives extraites et tirées des oeuvres mathématiques de Valentin Menher*, in part drawn from the *Arithmétique seconde*, in part from Mennher’s spherical trigonometry;⁵⁶ at his death in 1637, Descartes’ friend and mentor Isaac Beeckman possessed a

⁵⁵ Guillaume Gosselin, *De arte magna, seu de occulta parte numerorum quae & algebra, & Almu-cabala vulgo dicitur* (Paris: Egide Beys, 1577), fol. 80^r.

⁵⁶ Half drawn indirectly, however, from the similar collection of 100 problems published by Michiel Coignet (who had probably studied with Mennher) in 1573 in Antwerp. See Ad Meskens, *Practical Mathematics in a Commercial Metropolis* (Dordrecht: Springer, 2013), 59.

copy;⁵⁷ and as late as 1666, John Collins points to Mennher together with Viète and Viète's translator Jean-Louis Vaulezard (etc.) as good introductions to algebra.⁵⁸

Mennher was born in Schwaben in 1520.⁵⁹ He entered the Fugger firm as an accountant, which brought him to Antwerpen. He settled there and opened a school, and also started mathematical publishing in French (thereby opening the world of *Rechenmeister* mathematics to French readers). According to the preface to his "second arithmetic" he had published a first arithmetic in 1550 – probably part of his *Praticque brifue*, of which I have only been able to see a re-edition of the book-keeping part.

A first part of the *Arithmetique seconde* is a good *Rechenbuch* in German style. The third is a geometry going well beyond traditional abacus geometries – it contains a Euclidean *proof* of the Pythagorean theorem (fol. S.i^r) as well as an Archimedean determination of the ratio between the perimeter and the diameter of a circle (our π). This is not our only evidence that Mennher was a competent geometer: in 1564 he published "practice of spherical triangles, the distances on the globe, clocks, shadows, and other ingenious and new mathematical questions"⁶⁰ – adding, as said in the preamble, to what had been done by the very learned Regiomontanus (*viz*, in *De triangulis*) the labour of calculation.

Between the first and the third part we find an algebra. As in the case of the spherical triangles, Mennher is not afraid of identifying sources. In the present case the "style and manner" of the very renowned Christoph Rudolff has been of great help, and Mennher has found him very competent. In consequence, he says, he has not deviated much from him, knowing well that the very renowned Michael Stifel has renewed and augmented him much in the same High German language with several beautiful examples – from which, however, "I have extracted a fair part of the best only, adding other matters needed by merchants."

At first comes a presentation of traditional *coß* with a single unknown, which fits the reference to Rudolff. It is followed by an exposition of the *regle de la quantité ou seconde radix*. It is inspired by Stifel, in agreement with what was announced, but Mennher does not copy. His notation is Stifel's improved version from 1553. Since no existing books apart from that of Stifel treated the topic in depth, Mennher must have been working on it directly.

The first illustrative example (fol. O.i^r) is an "all less each" problem with four participants. It is similar to one in Stifel's Rudolff-edition (fol. 312^r), but the parameters are not

⁵⁷ See *Catalogus variorum et insignium librorum clarissimi doctissimique viri D. Isaaci [...]* (Dordrecht: Isaac Andreae, 1637), B.iv^r.

⁵⁸ Philip Beeley and Christoph J. Scriba, eds., *The Correspondence of John Wallis*, vol. II (1660–September 1668) (Oxford; New York: Oxford University Press, 2005), 315.

⁵⁹ See Meskens, *Practical Mathematics*, 14f and *passim*.

⁶⁰ Valentin Mennher, *Praticque des triangles sphériques, des distances sur les globes, et autres ingenieuses et nouvelles questions mathematique* (Antwerpen: Gilles Coppenius, 1564).

the same; even the choice of the unknowns is different, for which reason Mennher cannot follow Stifel's procedure. The third example (fol. O.ii^r) is similar to Stifel's two-number problem from the *Arithmetica integra* (fol. 254^v), but again the parameters are different; the argument is similar (including the use of a convenient though somewhat superfluous third unknown). Other two-number problems follow patterns we know from Stifel while being solved differently – sometimes making a more convenient choice of the algebraic unknowns, sometimes by using a diagram instead of algebra; still other examples have no analogue in Stifel. Mennher was to present the *regle de la quantité* once again in 1565,⁶¹ by then expanding the treatment though not much. However, the book possessed by Beeckman and referred to by Collins appears to be the *Arithmetique seconde*.

Mennher thus produced no striking innovations; but he, and apparently nobody else, took care that the mature and full version of Stifel's technique from 1553 could reach French readers. Actually he was widely plagiarized also for other matters, including in Flemish, thereby diffusing more basic *Rechenmeister* mathematics in the Netherlands and France, but that is irrelevant for the present purpose.

France before Viète

The New Algebra emerged in France. That is the reason that Mennher's transmission of *coß* and of Stifel's system should be taken note of. But there was also interest in algebra in 16th-century France proper.

The earliest manifestation of this interest was de la Roche, whose book, as we have seen, probably provided some information to Rudolff. He may also have had peripheral influence on Mennher – the latter speaks of *racine universelle* as well as *racine liee*. Both terms are used by de la Roche. But Mennher uses *racine liee* about the sum of two roots – that is, if borrowing the terms from de la Roche he reinterprets them so as to fit a distinction made in *coß* algebra.

De la Roche's influence in later properly French algebra seems to have been equally modest. The lack of system in his notations – in absolute contrast to what we find in the *coß* – will hardly have helped. The next evidence of algebraic interest is a re-edition in 1551 in Paris of Johann Scheubel's algebraic introduction to his edition of *Elements* I–VI.⁶² Since it was republished in 1552, there must have been an interested audience – but Scheubel's rather idiosyncratic notation left no visible traces, so even *his* influence must have been modest.

⁶¹ Valentin Mennher, *Praticque pour brievement apprendre à ciffer, et tenir livre de compte, avec la regle de coss, et geometrie* (Antwerpen: Aegidius Diest, 1565), fols. F.ii^r–G.gii^v.

⁶² Johann Scheubel, *Algebrae compendiosa facilisque descriptio, qua depromuntur magna arithmetices miracula* (Paris: Guillaume Cavellat, 1551).

In 1554, Peletier published *L'algebre* in Lyons. On the first pages he lists those writers on the topic whom he knows about – also those whose works he has not seen, including Rudolff; noteworthy is that he does not know about de la Roche.⁶³

His own work builds on Stifel's *Arithmetica integra*, of which he makes no secret (including the arithmetical treatment of the matters of *Elements* X); from Stifel he has taken over the insight that a notation has to be used systematically, only making some purely graphical changes of the signs; for the notation of “second roots” he follows Stifel precisely.⁶⁴

Peletier thus introduced the style and insights of German *coß* to French readers; but since nobody followed him he can hardly be said to have *brought them* to France.

Borrel's *Logistica* from 1559, also published in Lyons, was already mentioned. Borrel's principal aim is to assimilate arithmetic and algebra to some Greek-sounding model. Like Peletier (and Scheubel) he accepts that a notation has to be systematic. But he replaces the already current + and – with *P* and *M*; for the powers he uses the Florentine ρ (which he is likely to have known from de la Roche) for the first power, $\diamond$ for the second and $\square$ for the third. All in all the book is a protracted primer, and it left no obvious traces.

Petrus Ramus's utterly elementary *Algebra* was published anonymously in Paris in 1560. Ramus uses + and –, “seized upon from common usage.” Since he pretends implicitly in the *Scholarum mathematicarum libri unus et triginta* from 1569 not to know Stifel we may safely assume (given what else he pretends not to know there) that he has drawn on him (possibly through Peletier, but even then he would know about Stifel whom Peletier cites repeatedly). Ramus invents new signs (simple and composite letter symbols) for the powers, and employs the usual schemes for the arithmetic of polynomials. Even this book got no following.

Guillaume Gosselin's *De Arte magna, seu de occulta parte numerorum, quae & Algebra, & Almucabala vulgo dicitur* was published in Paris in 1577; it presents us with another abortive fresh start. After the appearance of Xylander's Latin translation of Diophantos⁶⁵ it takes over much of Xylander's terminology and notation but modifies it as actually required (Diophantos had produced higher powers as products, Gosselin as everybody since a century by embedding); whereas Xylander had used + and –, Gosselin writes *P* and *M*.

In 1578, Gosselin published an abbreviated French translation of the first and second part of Tartaglia's *General trattato di numeri, et misure* (the second part of which deals with

⁶³ The Latin version from 1560 does mention de la Roche in its preface.

⁶⁴ Omitting however Stifel's first example because it is easily solved by means of a single unknown, and replacing it with the problem later used by Clavius, see note 51. We may assume that it was created by Peletier, since two other problems are correctly said on p. 107 to be borrowed from Cardano (namely, from Cardano, *Practica arithmetice*, fols. L.vii^r and HH.vi^r).

⁶⁵ Wilhelm Xylander, ed. and trans., *Diophanti Alexandrini Rerum arithmeticarum libri sex* (Basel: Eusebius Piscopus, 1575).

algebra and its foundations)⁶⁶ with added commentaries from his own hand and using his own algebraic notation.⁶⁷

There was thus a continuous interest in algebra in the French area ; to the actual writers about the topic we may add Pierre Forcadel, who in 1556 announced on the title page that his *Arithmetique* would allow to “easily obtain knowledge of algebra” without actually dealing with that subject.⁶⁸ But there was very little continuity, and thus *no algebraic tradition* from which the New Algebra could emerge.

Schematically – where $\ncong$ indicates lack of connection:

Étienne de la Roche (1520) $\ncong$ [Scheubel (1551)] $\ncong$
 Jacques Peletier (1554, Latin 1560) $\ncong$ Jean Borrel/Buteo (1559) $\ncong$
 Pierre de la Ramée (1560) $\ncong$ Guillaume Gosselin (1577)

A new cultural setting

That emergence, instead, owed much to two aspects of a new cultural setting.

One aspect is the advent of “Humanist mathematics.” Original Humanism (that of Petrarca, Boccaccio, etc.) could not have cared less about mathematics. In spite of its professed admiration for Greek Antiquity, it read Latin, and that gave access to nothing beyond Boethius (Campanus’s Latin translation of the *Elements* belonged to the university tradition). After 1450, that began to change.

From its beginning, Humanist thought had been interested in “utility,” understood as civic utility – and that again understood as that which could embellish the state or serve its diplomatic or military strength. During the later 15th century it became clear that this service had to include (applied) mathematics – in great drainage projects and other engineering undertakings, in fortification, and (in particular in the Atlantic states) in navigation. That created the new social role of a “court mathematician”⁶⁹ – and that established the contact between Humanism and mathematics, and made mathematics a Humanistically legitimate interest.

Legitimacy takes its time to be distilled into results. The first Humanist translations of Greek mathematics were published only from around 1500 onward:

- Giorgio Valla, 1501: Euclidean and Archimedean/Eutocian fragments;
- Bartolomeo Zamberti, 1505: problematic translations of Euclid;

⁶⁶ Guillaume Gosselin, ed. and trans., *L'arithmétique de Nicolas Tartaglia* (Paris: Gilles Beys, 1578).

⁶⁷ With one exception. In 1577 he had used *L* (for *latus*) for the first power, in the Tartaglia translation he uses *R*.

⁶⁸ Pierre Forcadel, *L'arithmétique* (Paris: Guillaume Cavellat, 1556).

⁶⁹ See Mario Biagioli, “The Social Status of Italian Mathematicians, 1450–1600,” *History of Science* 27 (1989): 41–95.

- Giovanni Battista Memmo, 1537: translations of Apollonios's *Conics* I–IV;
- Federico Commandino, 1566: another translation of Apollonios;
- Wilhelm Xylander, 1575: Diophantos;
- Federico Commandino, 1588: Pappos.

Various printings of Archimedean works (in Moerbeke's translation) were published by Luca Gaurico in 1503 and Tartaglia in 1543.

Finally, there were the editions of Greek texts:

- The Grynaeus edition of Euclid with Proclus's commentary in 1533;
- Ptolemy's *Almagest* in 1538;
- Archimedes in 1544.

In the later 16th century, a fairly full Greek mathematical canon was thus available.

The other aspect of the new cultural setting of mathematics in France and neighbouring areas was the reappearance of an agonistic mathematical culture centred on the solution of problems. It is reflected in a familiar story:⁷⁰

Viète's mathematical reputation was already considerable when the ambassador from the Netherlands remarked to Henry IV that France did not possess any geometers capable of solving a problem propounded in 1593 by Adrian van Roomen to all mathematicians and that required the solution of a forty-fifth-degree equation. The king thereupon summoned Viète and informed him of the challenge. Viète saw that the equation was satisfied by the chord of a circle (of unit radius) that subtends an angle $2\pi/45$ at the center. In a few minutes he gave the king one solution of the problem written in pencil and, the next day, twenty-two more.

The story illustrates, firstly, that mathematical tournaments were not only a matter for mathematicians and mathematical dilettanti⁷¹ but were by now a state affair; that would not have been the case a century before, or after. Secondly, that Viète (and his peers) were known to the court, and linked to it.

The present problem came from advanced algebra (though ultimately from trigonometry); in that respect it was not quite typical of the challenges exchanged between French dilettanti. Viète's *Book 8 of various responses about mathematical matters* from 1593 indi-

⁷⁰ I quote Hubert L. L. Busard, "Viète, François," in *Dictionary of Scientific Biography*, vol. XIV (New York: Scribner, 1976), 22.

⁷¹ Viète, Fermat, and Descartes were all mathematical dilettanti; professional mathematicians of the time would be mathematical practitioners or teachers of these; at most professors like Oronce Finé. None of them were at the mathematical level of the best dilettanti, and these were the ones engaged in the challenges here concerned.

cates (since it contains “responses”) what was typical:

- two intermediate proportionals;
- squaring and rectification of the circle and of circular segments, using Archimedean spirals and the quadratrix;
- construction of a regular heptagon;
- lunules; etc.
- In the end Viète deals with spherical trigonometry, a topic that had his special interest, and which may have helped him to recognize what was behind van Roomen’s problem.

Why algebra?

So, the challenges that were *à la mode* were rooted in Greek geometry, which had become known thanks to the Humanist editions. Nothing seems to lead toward innovative interest in *algebra*.

But the challenges were *problems*, and algebra had always been known as a tool for solving problems.

Neither Viète nor Descartes say much about their inspiration except when they can castigate; in that respect they are similar to Borrel and Gosselin (and to many others). Descartes, however, left letters and other writings, and Beeckman wrote about him. So, we may start with him, and then see how Viète fits the pattern.

Peripherally we may observe at first that what Descartes makes in the *Geometrie* differs so completely from what Viète had done that there is no reason to expect even the slightest inspiration from him.

The *Geometrie* was purportedly meant as an illustrative appendix to Descartes’ *Discours de la methode* from 1637. Actually, as shown by Christian van Randenborgh,⁷² Frans van Schooten knew the finished manuscript already in 1632. But the *Discours* itself provides a first key. In part 1 (p. 18) he explains that when younger (namely at the Jesuit school) he had studied *un peu* logic and, “among the mathematics, the analysis of geometers and algebra,” sciences that seemed to promise something for his project. But logic mostly served to express what one already knows (or does not know); analysis only deals with geometric figures; and the rules and signs of algebra (which Descartes had been taught on the basis of Clavius’s book) have made it “a confused and obscure art that puts the mind in difficulty instead of a science that cultivates it.”

This expresses the opinion Descartes had reached when writing the *Discours*. In any

⁷² Christian van Randenborgh, “Frans van Schootens Beitrag zu Descartes’ *Discours de la methode*,” *Mathematische Semesterberichte* 59 (2012): 225.

case, in a letter to Beeckman from 26 March 1619⁷³ he speaks of his intention to create “a wholly new science by which it will be possible in general to solve all questions that can be proposed dealing with any kind of quantities, continuous as well as discrete.” In this letter, Descartes makes use of algebra in Clavius’s style, employing his symbolism⁷⁴ – by the way without betraying any kind of familiarity with several unknowns (he may be right that he had studied the algebra textbook just *un peu*).

After a long separation Descartes met Beeckman again on 8 October 1628. This time Beeckman’s *Journal*⁷⁵ informs us that Descartes has invented a “general algebra;” but his notation is still that of Clavius.

That, as we know, has changed in the *Geometrie*. Between 1628 and 1632, Descartes has thus learned about the use of multiple unknowns. He might of course have invented them on his own, but there is evidence that Descartes learned from Beeckman – shortly afterwards, thus Beeckman, Descartes proposed either to send him from Paris his *Algebra* for correction or to visit him for collaboration (the former must have happened; the manuscript in question has disappeared, apart from likely extracts copied by Beeckman⁷⁶).

Since Beeckman owned a copy of Mennher’s *Arithmetic*, this book is a likely source for Descartes new insight (still unless we believe in an independent discovery); inspiration from Peletier, Borrel or Gosselin seems unlikely.

Descartes himself has this to say in the *Geometrie*:⁷⁷

When wishing to solve some problem, one should first look at it as already solved, and give a name to all the lines that seem to be needed in order to construct it, those that are unknown as well as the others. Then, without making any difference between these known and unknown lines, one should run through the difficulty according to the order it shows, the most natural of all, in which way they depend mutually on each other, until the point where one has found a way to express one and the same quantity in two ways: which is called an equation.

Descartes (as already Stifel) thus does not look for a minimal set of unknowns – what appears to play a role gets a name. Once he applies algebra with several unknowns to geometric problems, quasi-abstract names (that is, not specified numbers but names linked

⁷³ Charles Adam and Paul Tannery, eds., *Oeuvres de Descartes*. Vol. X: *Physico-mathematica. Compendium musicae. Regulae ad directionem ingenii. Recherche de la vérité. Supplément à la correspondance* (Paris: Léopold Cerf, 1908), 154–160.

⁷⁴ Cf. p. 155, note d in the edition.

⁷⁵ Cornelis De Waard, ed., *Journal tenu par Isaac Beeckman de 1604 à 1634*, vol. III (La Haye: Nijhoff, 1939–1953), 94–97.

⁷⁶ *Ibid.*, vol. IV, 135–139. The fragment does not elucidate the questions that concern us here.

⁷⁷ René Descartes, *Discours de la methode* (Leiden: Ian Maire, 1637), 300.

to specific entities appearing in a diagram) would be used for everything pertinent – no distinction being made between what would turn up as coefficients and constants in the equations and what would turn up as unknowns.

Slightly later, without explanation Descartes starts following the principle to use letters from the end of the alphabet (first *z*, then if needed *y*, etc.) for the unknown magnitudes and letters from its beginning for those that are known.

The words “all the lines that seem to be needed” are significant. Traditional lettered diagrams mostly give letter-names to *points*, not to the segments. Algebra, however, applies to quantities, and these are the segments. Descartes therefore tacitly changes the way lettering is made; already Stifel had done so, and Viète does so too.

Viète, like Descartes, speaks badly about the algebra he has inherited, namely as “a new art, or rather so old and so defiled and polluted by barbarians that I have found it necessary to bring it into, and invent, a completely new form,” in the words of his *Zetetics*.⁷⁸ He may well have adhered to the widespread Humanist claim that algebra was invented by Diophantos and then distorted by the Arabs, but he says nothing more precise. He takes over Diophantos’s (that is, Xylander’s) names for the powers, and borrows the idea of *analysis* from Greek authors. He also mentions Cardano’s *Practica arithmeticae* twice, censuring (fols. 9^v, 10^r). That is all.

He is unlikely not to have known Cardano’s *Ars magna*. Equally implausible is that he should not have known Gosselin. Viète was engaged in spherical geometry, and he is thus likely to have known Mennher’s book about that topic; possibly therefore also his *Arithmetique seconde*.

Irrespective of which predecessors have inspired Viète’s use of several unknowns, his own first step as he describes in the *Isagoge* is similar to that of Descartes:⁷⁹

Magnitudes, those which are known as well as those which are asked for, should be combined and compared, adding, subtracting, multiplying and dividing, always observing the law of homogeneity.

For both, the decision to apply algebra with several unknowns to geometric problems entailed as an obvious consequence the abstract coefficients. *It came automatically as a gift not asked for*. Both were self-conscious to the point of being haughty, none of them afraid to claim to have done completely away with the filth, pollution and confusion of the algebra they had inherited. But precisely *this* point, which in later historiography is seen as the most important aspect of the New Algebra, they hide within subordinate clauses, “those which are known as well as those which are asked for” respectively “those that are

⁷⁸ François Viète, *Zeteticorum libri quinque* (Tours: Jamet Mettayer, 1591), fol. 2^v.

⁷⁹ François Viète, *In artem analyticem isagoge* (Tours: Jamet Mettayer, 1591), fol. 7^r.

unknown as well as the others.”⁸⁰

Most of the other characteristics that separate the New Algebra from that of al-Khwārizmī had accumulated gradually since the 14th century, as we have seen. One was not yet ripe: the general parenthesis. Descartes as well as Viète still made use of formal fractions. Descartes introduced the new root sign which made nesting much easier; he also used braces to define a new special-purpose parenthesis (containing a polynomial that serves as a factor), but used it only for that purpose. Oughtred, Newton and others started using a vinculum (a stroke above a composite expression) for the same purpose, but it remained a special-purpose tool. Only in infinitesimal operations would Wallis and Newton go cautiously beyond this boundary, soon to be followed more courageously by Johann Bernoulli and Leibniz; the abundance of new functions and the explicit function concept created a need to allow composite expressions as arguments. Then, after the mid-18th century, ordinary algebra learned from its infinitesimal offspring to see the parenthesis as so obvious that it needed no explanation. This is another complicated story, which I have told elsewhere.⁸¹

⁸⁰ Viète, *In artem analyticem isagoge* fol. 5^r, does give the new type a name, *speciosa*, “which shows things by the species or forms of things, possibly by the elements of the alphabet.” The name reappears on fol. 8^r and in the titles of *Ad Logisticem speciosam notae priores* and ... *posteriores*, announced on fol. 1^v as part of the total algebraic project. On fol. 8^r it is said to be simple whereas Diophantos’ *logistica numerosa* is so difficult that it has come to be much admired. The term does not serve as a fanfare of victory but as a sober description.

⁸¹ Høyrup, “How Did the All-Purpose Parenthesis Come About in European Algebra?”

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Descartes and the law of refraction

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Abstract

Historical research on refraction in the modern era has often focused on the question of attributing the law of refraction to Descartes, Snel and Harriott. The discovery of Ibn Sahl's work has put an end to the dispute over priority between these authors, leaving another problem largely unnoticed: most of the literature refers to this discovery without distinguishing the native form of the statements on refraction from the current conception of the law of refraction. This anachronism is problematic. Ancient texts do not contain a single statement of the law of refraction as we know it today. Descartes' contribution is in line with Maurolico's and Kepler's work on refraction. Like them, Descartes makes no mention of sines or refractive indices, nor does he formulate a physical law of refraction. Since these ideas developed gradually between the late 17th century and the 19th century, the recognition of a "law of sines" or a "law of refraction" is not an idea native to the 16th and 17th centuries. We suggest that the work of Descartes does not fit into the modern quest for the laws of nature, but rather into the framework of the theory of proportions.

Keywords

refraction, law, proportion, Descartes, Maurolico, Kepler

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Introduction

The “law of refraction” has long been regarded as the great optical discovery of the 17th century, and its invention is still one of the most hotly debated issues in the history of optics. While the many studies devoted to the law of refraction have clarified the relationships between certain contributions and authors, they have sometimes had a detrimental effect on the history of science, due to a presupposition inherent in any search for anteriority.

Historically, research into the law of refraction began with Descartes’ *Dioptrique* (1637). It then turned to Willebrord Snel’s *Libellus de refractione* (1621/6), then to Harriot’s *Optics and caustics* notebook (circa 1605). The discovery of an early expression of the law of refraction in Ibn Sahl’s *Book of burning <instruments>*, ca. 984,¹ struck a definitive blow to the search for priority among 17th-century scholars, but left the original presupposition intact. In order to demonstrate that one author preceded another, any claim to priority must assume that the scientific content of the competing authors is identical. Authors can only be aligned in the same narrative if they are pursuing the same intellectual quest and achieve comparable results.² This requires a minimum degree of alignment between the concepts and research goals.

As for the law of refraction, it is by no means certain that these conditions are met. Since this identity is assumed by those who wish to identify precursors or predecessors, it is possible that certain differences in concept or purpose – deemed minor – were downplayed to align the authors within the same series.

Ibn Sahl came close to discovering the law of refraction by studying the properties of burning lenses;³ Grosseteste approached the law of refraction while seeking to explain the rainbow;⁴ Kepler was motivated in his research by astronomical concerns, then by his discovery of the Galilean telescope;⁵ Snel, a specialist in geodesy, wanted to explain the strange lifting of objects when viewed from a certain angle through water;⁶ Descartes

¹ Roshdi Rashed, “A pioneer in anaclastics: Ibn Sahl on burning mirrors and lenses,” *Isis* 81 (1990): 464–491.

² Remember here Georges Canguilhem’s criticism of the precursors, *Études d’histoire et de philosophie des sciences* (Paris: J. Vrin, 1968), 21.

³ Rashed, “A pioneer in anaclastics.”

⁴ Carl B. Boyer, “Robert Grosseteste on the rainbow,” *Osiris* 11 (1954): 247–258.

⁵ Johannes Kepler, *Ad Vitellionem Paralipomena, quibus Astronomiae pars optica traditur...* (Frankfurt: C. Marnius, 1604); *Optics. Paralipomena to Witelo & Optical Part of Astronomy*, trans. W. H. Donahue (Santa Fe: Green Lion Press, 2000). On these aspects: Carlos Alberto Cardona, “Kepler: analogies in the search for the law of refraction,” *Studies in History and Philosophy of Science* 59 (2016): 22–35, 34.

⁶ Cornelis De Waard, “Le manuscrit perdu de Snellius sur la réfraction,” *Janus* 39 (1935): 51–73; Johan Adriaan Vollgraff, “Snellius’ notes on the reflection and refraction of rays,” *Osiris* 1 (1936): 718–725; See in particular the emphasis on motivations by Klaus Hentschel, “Das

studied refraction in connection with the anaclastic problem,⁷ etc. Why should we assume that authors who do not have the same mathematical skills and do not pursue the same scientific goals would have exactly the same understanding of the “law of refraction”?

All those involved in this search for priority – setting aside their disagreements over attribution – have jointly defended the same view: namely that Ibn Sahl, Harriot, Snel or Descartes were all seeking the same law of refraction. And yet, as soon as we delve into material from any historical source, we find that the “law of refraction” does not appear as we understand it today – that is, as a physical law involving sines and refractive indices. The aim of this article is to characterize, through a careful study of texts, the exact status of statements concerning refraction.

Since there are so many texts, a selection is needed. The question of the contribution of 15th- and 16th-century mathematics to modern science defines the general time frame. After careful consideration, I chose Descartes as focal point – even though he falls slightly outside this period – because he is one of the key figures of modernity and one of those credited with discovering (or at least understanding) the law of refraction. To provide a contrast with how refraction was previously understood, the study of Descartes’ work is preceded by a review of some earlier works on refraction: Maurolico, *Diaphana* (1554) and Kepler, *Paralipomena* (1604) and *Dioptrice* (1611).⁸

1. Maurolico, *Diaphana*, 1554

Francesco Maurolico wrote the *Diaphana* in January 1523: *Diaphaneon... libellus completus est... anno 1523... die 30 Ianuarii* (Paris, BnF, latin 7249, fol. 11v). The text was revised in Catania in 1554, before being passed on to the publisher on August 26, 1555. The text was finally edited in Naples the next century (1611). Bellè and Takahashi have recently published a critical edition of Maurolico’s *Optics*.⁹

The *Diaphana* are written *more geometrico*. They open with a set of definitions and postulates (*supposita*) whose status oscillates between rules and observational statements.

Brechungsgesetz in der Fassung von Snellius,” *Archive for History of Exact Sciences* 55 (2001): 297–344.

⁷ René Descartes, *Règles utiles et claires pour la direction de l’esprit en la recherche de la vérité*, trad. J. L. Marion and P. Costabel (La Haye: M. Nijhoff, 1977), 27.

⁸ I have omitted Willebrord Snel’s *Libellus de refractione* so as not to reopen the debate over priority between Snel and Descartes. This work is known only from an abridgment (Amsterdam, Universiteitsbibliotheek, Bc 1), which may not accurately reflect Snel’s ideas on refraction.

⁹ Francesco Maurolico, *Diaphana* (1554), *Photismi de lumine & umbra... Diaphanorum partes, seu libri tres...* (Naples: T. Longo, 1611), 31–80; *Francisci Maurolyci Optica*, ed. by Riccardo Bellè et Ken’ichi Takahashi (Pisa: F. Serra, 2017), text 95–133, commentary 83–93.

The most important of these postulates is P3: “If the angle of incidence¹⁰ is multiplied, the angle of deviation¹¹ must also be multiplied equally” *Multiplicato angulo inclinationis, angulum quoque fractionis aequaliter multiplicari*. This postulate echoes Euclid’s theory of equimultiples (*Elements* V, defs. 4–7).¹² The importance of postulate P3 lies in the fact that it leads to a false statement about refraction, which is set out in Theorem 10.

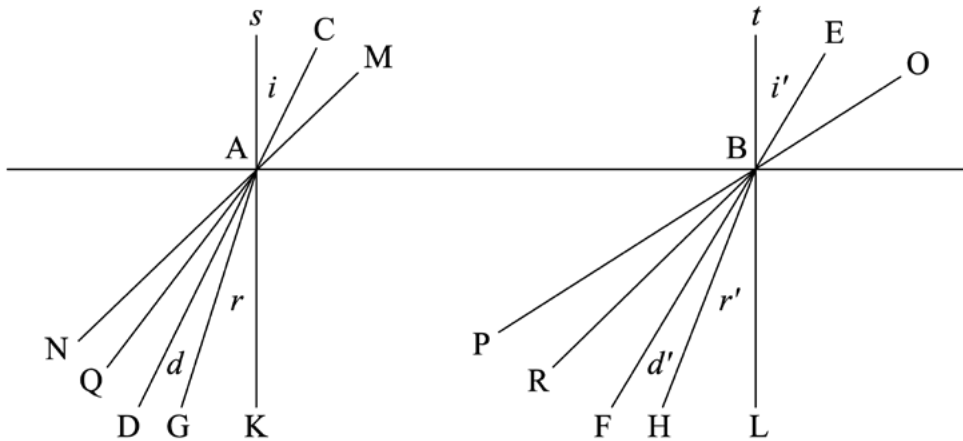


Fig. 1 – Maurolico’s diagram of refraction¹³.

Theorem 10: Angles of incidence¹⁴ are proportional to angles of deviation¹⁵
*Anguli inclinationum sunt fractionum angulis proportionales*¹⁶ (Figure 1).

Let two rays MAN and OBP fall obliquely on the plane interface AB. Theorem 10 is demonstrated using the theory of proportions (postulates P1 to P3):

[Assume that medium ABKL is denser than ABCE]

Thus incident rays are refracted towards AK and BL normals (postulate P1)

Let the ray CAD refracted along AG

¹⁰ inclinatio] lit. inclination, i.e. incidence

¹¹ fractio] deviation $d = i - r$

¹² Euclide, *Les Eléments*, trad. B. Vitrac (Paris, PUF, 1994), vol. 2: Livres V–VI, 35–47.

¹³ Inadvertently, the figure in the MS BnF, Latin 7249, fol. 2v, represents MA, OB rays with the same obliquity on the interface. I take angles such that $\angle DAK < \angle FBL$. I add the letters $s\ t$ and the angles of incidence i , deviation d and refraction r for discussion.

¹⁴ inclinatio] incidence

¹⁵ fractio] deviation $d = i - r$

¹⁶ Maurolico, *Diaphana*, 35; *Maurolyci Optica*, 99.

And EBF refracted along BH

[We want to show that the angles of incidence are like the deviations]

That is $\angle DAK : \angle FBL :: \angle DAG : \angle FBH$

Take two other rays: MAN refracted towards Q, OBP towards R

$\angle DAK = k \angle NAK$ and $\angle FBL = k \angle PBL$; with any k

Therefore $\angle DAG = k \angle NAQ$, $\angle FBH = k \angle PBR$ (postulate P3)*

If $\angle NAK > \angle PBL$

Then $\angle NAQ > \angle PBR$ (postulate P2)

If $\angle NAK = \angle PBL$

Then $\angle NAQ = \angle PBR$

If $\angle NAK < \angle PBL$

Then $\angle NAQ < \angle PBR$

Thus $\angle DAK : \angle FBL :: \angle DAG : \angle FBH$.

Corollary: The angle of incidence and the angle of deviation are always in the same ratio. As we proved by experiment in the transparent sphere, it is double and $\frac{2}{3}$ superpartiens [8 : 3]

*Ergo et angulus inclinationis ad angulum suae fractionis semper unam servat rationem. Sicut experimento in crystallina sphaera probavimus, estque dupla et $\frac{2}{3}$ superpartiens.*¹⁷

Since $\angle CAs = \angle KAG = i$ and $\angle EBt = \angle LBF = i'$ (angles opposite by the vertex), Maurolico's relationship can be written:

$$i : d :: i' : d' :: 8 : 3$$

$$d = i - r$$

$$r = i - d$$

$$i : r :: i' : r' :: 8 : 5.**$$

Commentary

* Theorem 10 is based on postulate P3, which is false. If we note the angle of incidence i and the deviation $d = i - r$, postulate P3 states that if $i' = ki$, then $d' = i' - r' = k(i - r)$. Relying on the theory of equimultiples, Maurolico establishes a proportion that incorrectly describes the relationship between the angle of incidence and the angle of deviation. Fermat was one of the first to notice the error. In a letter to Mersenne dated April-May 1637, he wrote:

Maurolic Abbé de Messine, en son traité posthume *De lumine & umbrâ*,¹⁸ a soutenu que les angles qu'il appelle d'incidence, sont proportionaux à ceux qu'il nomme de refraction. Si cette

¹⁷ Maurolico, *Diaphana*, 36; *Maurolyci Optica*, 100.

¹⁸ *Diaphana* are included with *Photismi de lumine et umbra* in the 1611 edition.

proposition estoit vraye, elle suffiroit pour nous marquer les vrayes figures que doivent avoir les corps diaphanes qui produisent tant de merveilles. Mais pource qu'elle n'a pas esté bien démontrée par Maurolic, & que l'expérience mesme semble la convaincre de faux, il en est resté assez à Monsieur Descartes pour exercer son esprit [a discussion of *Dioptrique* follows].¹⁹

** Maurolico studies a crystal sphere (*crystallina sphaera*). A modern interpretation would say that the ratio of the refractive indices of air and crystal is 8 : 5, and since $n_{\text{air}} = 1$, it follows that $n_{\text{crystal}} = 1.6$. However, Maurolico stops short of characterizing the double and $\frac{2}{3}$ super-partiens ratio without using the notion of refractive index, introduced much later, without distinguishing the index of crystal and air, and without involving other refractive media.²⁰

In 1554, the very year of the *Diaphana* revision, Maurolico also wrote *De proportionem sermo noster*, in which he sought to simplify the theory of proportions. His aim was to establish the notion of proportion independently of commensurability: "It was therefore necessary to find a condition common to these and those [commensurable and incommensurable magnitudes] which is proper to proportional quantities" *Cogitandum igitur fuit de aliqua conditione, quae his et illis communis esset, solisque proportionalibus propria quantitativibus* (1554, fol. 22r).²¹ At this time, Maurolico tried to transcend the difference between discrete and continuous magnitudes by means of the theory of equimultiples: the four terms of a proportion are not necessarily homogeneous with each other; it is sufficient that they are homogeneous two by two. Maurolico adhered to this theory before abandoning it in his 1567 *Compendium of the Elements* (Paris, BnF, Latin 7463).²² The same treatment of the two types of magnitude enables him to write: "This is why proportion is the equality or identity between two ratios" *unde proportio est rationum aequalitas sive identitas* (1554, fol. 21r). Maurolico thus reduces the gap between the concepts of identity and equality and, as a result, promotes the assimilation of proportion (as a comparison of qualitative ratios $a : b :: c : d$) and equality of ratios (as numerical equality of quantitative ratios $a / b = c / d$).

Maurolico's application of proportions to refraction was not an isolated gesture. The same theory is applied to mechanics. In *De momentis aequalibus*, Maurolico similarly shows that "gravities suspended at equal distances are proportional to their moments."²³

¹⁹ *Correspondance de Descartes*, ed. Charles Adam et Paul Tannery (Paris, Cerf, 1897), vol. 1, 356.

²⁰ Bellè and Takahashi, *Maurolyci Optica*, 178.

²¹ Maurolico, *De proportionem sermo noster* (1554), MS: Molfetta, Biblioteca del Seminario Minore Vescovile 5–7 H 15, fol. 22r; ed. Pier D. Napolitani, *Francisci Maurolyci Opera Mathematica*, <http://www.dm.unipi.it/pages/maurolic/intro.htm>

²² Maurolico, *Compendium* (1567), MS: Paris, BnF, latin 7463, ed. Jean-Pierre Sutto, "Le compendium du 5e livre des Éléments d'Euclide de Francesco Maurolico," *Revue d'histoire des mathématiques* 6 (2000): 59–94.

²³ Sutto, "Le compendium," 66.

The confidence with which proportions were applied to various subjects is probably an effect of the immense credit it enjoyed during the Renaissance. The explanatory power of this theory has, so to speak, suspended its critical verification. Maurolico, who occasionally resorts to experimental data, *Sicut experimento... probavimus*, did not test the relationship between the angle of incidence and the angle of deviation. P3's error was therefore transmitted to Theorem 10 and its corollary.

2. Kepler, *Ad Vitellionem Paralipomena*, 1604

Kepler's *Ad Vitellionem Paralipomena* is a massive 487-page work published by Marnius in Franckfort (1604). Its modern, logical plan evades the conventions of ancient and medieval optics. Kepler first examines the physics of light (chapter 1) and the formation of images (chapter 2), revisits the theory of reflection (chapter 3) and refraction (chapter 4) and then applies these principles to vision (chapter 5). Refraction is the subject of chapter 4. After introductory considerations, Kepler formulates Propositions IV, 1 and IV, 2, dealing with incidence and density, and then presents their synthesis in Proposition IV, 3, which can be seen as a precursor to the modern law of refraction.

Proposition IV, 3: The angles of deviation²⁴ increase by greater increments²⁵ than the angles²⁶ of incidence

*Refractionum anguli crescunt maioribus rationum incrementis, quam obliquitas incidentiae (Paralipomena IV, 3).*²⁷

Kepler then writes:

The angle of deviation²⁸ is therefore composed of a part proportional to the incidence and a part proportional to the BM lines. The BM lines increase very little at first, but a lot at low incidence,²⁹ as shown by the secant table, where equal degrees correspond to larger and larger secants. Thus, part of the deviation angle is proportional to incidence, and part increases with increasing increments. Thus, the whole angle increases with larger increments

²⁴ refractionum anguli] angles of deviation, *corr.* refraction! *Donahue*

²⁵ Competing translations: "Les angles de réfraction augmentent dans des proportions plus grandes que l'obliquité de l'incidence" (Simon) or "Le rapport sous lequel croissent les angles de déviation est supérieur à celui sous lequel croissent les angles d'incidence" (Simon) lose the idea of increment, i.e. the difference between two consecutive values.

²⁶ obliquitas] lit. angle of incidence, sg.

²⁷ Kepler, *Ad Vitellionem Paralipomena*, 111; *Paralipomena to Witelo*, 124.

²⁸ angulus refractionis] angle of deviation

²⁹ in humili incidentia] with low incidence

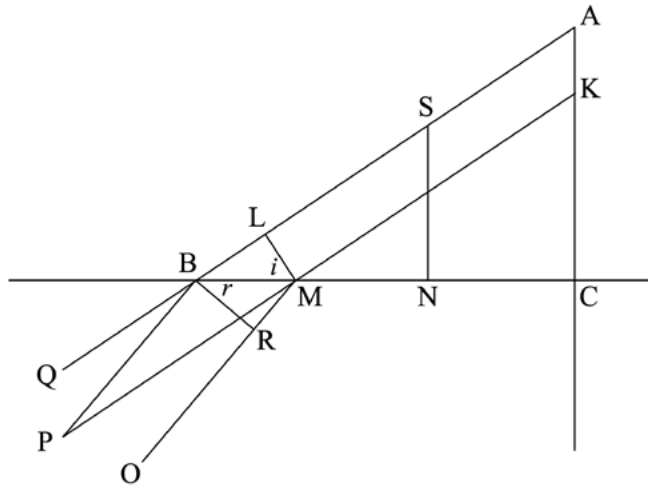


Fig. 2 – Kepler's diagram of refraction³⁰.

Componitur ergò angulus refractionis ex aliquo, quod est proportionale incidentiis, et aliquo, quod est proportionale lineis BM. At lineae BM crescunt initio parum, in humili incidentia crescunt multum, ut ostendit tabula secantium, ubi aequalibus gradibus semper maiores atque maiores secantes respondent. Ergò pars anguli refractionum proportionatur incidentiis, pars maioribus rationis incrementis crescit. Totus igitur angulus maioribus incrementis crescit. (Paralipomena IV, 3; Figure 2)³¹

This passage has been variously interpreted.

Gérard Simon's thesis that Kepler did not use the angle of incidence, as is our custom, but the complementary angle $90^\circ - i$, can be quickly dismissed.³² This interpretation is refuted by the very title of the column in the refraction table: "distance [angular] from source to vertex," *distantia radiantis à vertice*,³³ which refers to the angle of incidence. Kepler was therefore trying to express the deviation as a function of the angle of incidence.

Kepler's conjecture involves a combination of factors: the deviation $i - r$ is proportional to both the angle of incidence i and the secant of the angle of refraction $\sec r$. Several historians have attempted to mathematize this relationship:

³⁰ I adjust the figure to ensure the properties $KP \parallel AB$, $MO \parallel BP$, $LM \perp KM$ and $BR \perp MO$. I add the angle of incidence i and the angle of refraction r .

³¹ Kepler, *Ad Vitellionem Paralipomena*, 111.

³² Gérard Simon, *Kepler, rénovateur de l'optique* (Paris: Garnier, 2019), 100–101, note 28.

³³ Kepler, *Ad Vitellionem Paralipomena*, 115.

Cornelis de Waard proposed to recognize the relation $\alpha = m_1 \gamma + m_2 \sec \gamma$, in which α notes the angle of incidence, γ the deviation, $\alpha - \gamma$ the angle of refraction.³⁴ Applying usual notations, this relationship can be rewritten $i = k_1 (i - r) + k_2 \sec (i - r)$. Accordingly, Kepler would have imagined an additive combination of factors.

Jean Itard³⁵ refutes the idea that Kepler's idea can be written as $i - r = mi + n \sec r$ and shows that it is in fact $i = r + ki / \cos r$.

Buchdahl, Simon, Dijksterhuis and Cardona argue that the formula is of the type $i - r = ki \sec r$. Kepler would have used a multiplicative combination of factors.³⁶

Cardona also claims that Itard is mistaken: "M. J. Itard summarizes in this way the Kepler conjecture: $d = mi + n \sec(r)$, with m and n constants of proportionality (1957: 65). I intend to show that the composition that Kepler has in mind is not additive as Itard suggests, but that is of the form: $d = ki \sec(r)$, with k a constant of proportionality."³⁷

However, in his article on refraction, Itard cites the additive formula only to refute it, and also defends the idea that Kepler had in mind a multiplicative composition of factors. The confusion arises from the fact that Itard expresses this relationship by isolating i . All you have to do is pass r into the other member of the equation to recover the multiplicative form: if $i - r = ki / \cos r$ and $\sec a = 1 / \cos a$, then $i - r = ki / \cos r$ is equivalent to $i - r = ki \sec r$. So, despite criticism, Cardona uses the same formula as Itard.

In the end, only two interpretations remain: De Waard's (1933) and Itard's (1959), expressed in different notations. These interpretations can be decided by comparing the angular values appearing in Kepler's refraction table.³⁸

Let us note i the angle of incidence (*distantia radiantis à vertice*), i' the complementary angle, r the angle of refraction, $d = i - r$ the angle of deviation (*angulus refractionis*) and k some constant. The use of secants³⁹ is justified by the fact that $BM = RM \sec r$.

³⁴ *Correspondance du P. Marin Mersenne*, ed. Cornelis de Waard et al. (Paris: Beauchesne, 1933), vol. 1, 357, 432.

³⁵ Jean Itard, "Les lois de la réfraction de la lumière chez Kepler," *Revue d'histoire des sciences* 10 (1957): 59–68, 65.

³⁶ Gerd Buchdahl, "Methodological aspects of Kepler's theory of refraction," *Studies in History and Philosophy of Science* 3 (1972): 265–298, 291; Simon, *Kepler*, 122; Fokko Jan Dijksterhuis, *Lenses and Waves. Christiaan Huygens and the Mathematical Science of Optics in the Seventeenth Century* (Dordrecht: Kluwer, 2004), 121; Cardona, "Kepler," 34.

³⁷ Cardona, "Kepler," 32.

³⁸ Kepler, *Ad Vitellionem Paralipomena*, 115. Remember that Kepler's goal was to correct Witelo's inaccurate refraction tables.

³⁹ The use of secants is again recommended in IV, 6: "It must therefore be known that the secants of the angles of incidence which are established at the surface of the densest medium contribute to the measurement of refractions" *sciendum igitur, eorum angulorum incidentiae secantes concurrere ad mensuram refractionum, qui constituuntur ad superficiem in medio densiori*, Kepler, *Ad Vitellionem Paralipomena*, 113.

We know that:

$\angle LMB = i$
 $\angle MBR = r$

Here is the air-water refraction table given in Proposition IV, 8:

<i>Distance from source to zenith in rare medium i</i>	<i>Deviation propor- tional to incidences d_1</i>	<i>Complement to secants d_2</i>	<i>Total deviation demonstrated $d = d_1 + d_2$</i>	<i>Results of Wite- lo experiments d_w</i>	<i>Difference $d - d_w$</i>
10°	2;25	0;01	2;26	2;15	– 0;11
20°	4;49	0;10	4;59	4;30	– 0;29
30°	7;14	0;35	7;49	7;30	– 0;19
40°	9;39 ⁴⁰	1;23	11;02	11;00	– 0;02
50°	12;04	2;42	14;46	15;00	+ 0;14
60°	14;28	4;40	19;08	19;30	+ 0;22
70°	16;52	7;19	24;11	24;30	+ 0;19
80°	19;17	10;43	30;00	30;00	0;00
90°	21;43	14;47	36;30		

KEPLER'S REFRACTION TABLE.

These angular values can be tested by calculating the ki and $\sec r$ terms separately. The calculated values and the Kepler values are adjusted by the least-squares method. The best fit is given by $k = 0.2411$ (column 2).

Similarly, the values of the refraction angles are calculated using the law of sines, in order to minimize the resulting total deviation. If we apply the least-squares method again, we see that the values show significant deviations at 80° and 90° of incidence (difficult to measure). If we adjust the other values, we find the best fit for $n_1 / n_2 = 3 : 4$, a common value for air-water refraction. We deduce from the table $d = i - r = ki \sec r$ (column 4), then $d_2 = d - d_1$ which is the value of the “complement” produced by the secants (column 3).

If we now compare columns 2, 3 and 4 from the two tables, we can see that the reconstructed values correspond fairly well to Kepler’s values, except at high incidences. Kepler seems to have used a multiplicative combination of factors, before obtaining the complement by subtraction – hence the table’s additive appearance, which deceived De Waard. We can now decide between De Waard’s and Itard’s interpretations: there is little doubt that Kepler used Itard’s formula.

⁴⁰ 9;39] 3;39! misprint.

Incidence relative to zenith i	First part of the deviation $d_1 = ki$	Complement to secants $d_2 = d - d_1$	Kepler total deviation $d = ki \sec r d_1$	Actual total deviation d	Refraction for ratio 3:4 r
10°	2;25	0;01	2;26	2;31	7;29
20°	4;49	0;10	4;59	5;08	14;52
30°	7;14	0;34	7;48	7;59	22;01
40°	9;39	1;22	11;00	11;11	28;49
50°	12;03	2;40	14;44	14;56	35;04
60°	14;28	4;34	19;02	19;30	40;30
70°	16;53	6;55	23;47	25;11	44;49
80°	19;17	9;19	28;37	32;23	47;37
90°	21;42	11;06	32;48	41;25	48;35

RECONSTRUCTED REFRACTION TABLE.

A comparison of columns 4 and 5 shows that Kepler's values for total deviations deviate progressively from the actual values calculated from the law of refraction. The deviation is greater than 1° at 70°, 4° at 80° and 8° at 90°. It follows that, even neglecting calculation errors, the deviation values proposed by Kepler are not those actually observed. The formula $i - r = ki \sec r$ does not correspond to reality.

From proposition IV, 3, Kepler derived the *refractio simplex*, or Kepler's law of small angles, described in *Dioptrice* (1611).

Science historians have attempted to elucidate the historical significance of this proposition by returning to the inaccuracy of refraction angles. For Lohne, the absence of experimental research on the subject would have led Kepler to record erroneous angular values, from which it was impossible for him to discover the law of refraction.⁴¹ It can be argued that Kepler's corrected values were sufficient to formulate a relationship between angle of incidence and angle of refraction, since they diverge from reality at large angles. Heeffer considers that Kepler was in possession of an adequate representation of the refraction problem.⁴² He would have been the first to conceive of the existence of a relationship between certain geometric lines and the index of refraction, but would have missed the right relationship. This interpretation also raises a difficulty: as the concept of refractive index appeared at the beginning of the 19th century, it cannot be used to explain a text from 1604.

One thing is certain: Kepler used trigonometry and was intellectually prepared to express the statement on refraction in sine form. In fact, there is direct evidence of this

⁴¹ Johannes Lohne, "Zur Geschichte des Brechungsgesetzes," *Sudhoffs Archiv* 47 (1963): 152–172.

⁴² Albert Heeffer, "Data-driven induction in scientific discovery: a critical assessment based on Kepler's discoveries," E. Weber et al., eds., *Logic, Reasoning and Rationality* (Dordrecht: Springer, 2014), 59–76, 70.

possibility. In a passage from the *Paralipomena*, Kepler calculates the position of the stars on the horizon using a sine ratio identical to that on which the modern law of refraction is based:

$$\sin \angle FCE : \sin \angle FE :: \sin \angle CFE : \sin \angle CE$$

*Ut sinus CFE ad sinum CE sic sinus FCE ad sinum FE*⁴³

The obstacle that prevented Kepler from formulating the law of refraction was not intellectual. He probably went astray in thinking that refraction had to be explained by a combination of factors.

Because of its independence of view from ancient optics, *Ad Vitellionem Paralipomena* is Kepler's great book on optics. His later research in optics was more dependent on his exchanges with Galileo.⁴⁴

3. Kepler, *Dioptrice*, 1611

Kepler was promptly informed of Galileo's discoveries with the telescope. Galileo sent him a copy of *Sidereus Nuncius* (1610) as soon as it was published. Immediately realizing that the study of refraction could improve the telescope's lenses, Kepler set to work hard, and *Dioptrice* was published less than a year later by Franck in Augsburg (1611).

Dioptrice comes in an *more geometrico* altered form. The propositions do refer to definitions, axioms and postulates, but these are not grouped together in a preamble, as tradition dictates. Kepler distributes them according to need, so that the text includes a definition, two axioms, two problems, six axioms, four propositions, a sensible axiom, two propositions, then an axiom again, etc.⁴⁵ This organization reflects both the influence of the classical model and the relaxation of the Euclidean norm. The most important for the theoretical study of refraction are axioms 7 and 8 and proposition 12:

Axiom 7: Up to an incidence⁴⁶ of 30°, deviations⁴⁷ in the glass are roughly proportional to the inclinations

*Crystalli refractiones usque ad tricesimum inclinationis, sunt ad sensum proportionales inclinationibus.*⁴⁸

⁴³ Kepler, *Ad Vitellionem Paralipomena*, 131.

⁴⁴ M. Walter Burke-Gaffney, *Kepler and the Jesuits* (Milwaukee: Bruce Publishing Co, 1944), 60–70; Simon, *Kepler*, 163–164.

⁴⁵ Kepler, *Dioptrice* (Augsburg: D. Franck, 1611), 1–9.

⁴⁶ inclinatio] incidence

⁴⁷ refractio] deviation

⁴⁸ Kepler, *Dioptrice*, 3.

For $i \leq 30^\circ$, $d = k i$.

Axiom 8: Up to this limit, the deviation in the glass is about one third of the [angle] of incidence in air

*Angulus refractionis in Crystallo est usque ad dictum terminum, quàm proximè tertia pars inclinationis in aëre.*⁴⁹

For $i \leq 30^\circ$, $d = \frac{1}{3} i$ ($r = \frac{2}{3} i$).

Proposition 12: Carefully evaluated, the deviations are not proportional to the incidences in the air

*Refractiones exquisitè pensitatae non sunt proportionales inclinationibus in aëre.*⁵⁰

Kepler justifies this assertion by noting the inconsistency: according to axiom 8, an incidence of 90° should correspond to a deviation of 30° ($d = \frac{1}{3} i$), yet experience gives a deviation of 48° (i.e. a refraction angle of 42°).

The main purpose of Proposition 12 is to draw attention on the approximate nature of axioms 7 and 8. These axioms constitute Kepler's law for small angles, or *refractio simplex*. The simplification $i \leq 30^\circ$, $r = k i$ will be much used in later literature. It is worth noting that, whatever Kepler's solution, the three fundamental statements of *Dioptrice* do not seek a law, but a proportion linking incidences and refractions.

If we now turn to Kepler's use of proportion, we see that his conception of proportion in general is not strictly mathematical, as was the case with Maurolico. According to Kepler, proportional relationships do not describe the physical world; they carry a physical and cosmological meaning in themselves. This conception is clearly apparent in the epigraph to the *Harmonices Mundi* of 1619, where Kepler quotes a passage from Proclus' *Commentary on Euclid Book I*:

Mathematics also makes contributions of the very greatest value to physical science [i.e. the study of Nature]. It reveals the orderliness of the ratios according to which the Universe is constructed and the proportion that binds things together in the cosmos."⁵¹

Kepler was not interested in the strictly mathematical significance of proportions, but in the fact that they enable us to unravel the mysteries of the universe, being themselves the principles on which the world is built. This concept underpins all Keplerian astronomy,

⁴⁹ Ibid., 4.

⁵⁰ Ibid., 4.

⁵¹ Kepler, *Harmonice mundi libri V* (Linz: G. Tamach, 1619), 1; Proclus, *A Commentary in the First Book of Euclid's Elements*, trad. Glenn R. Morrow (Princeton: Princeton University Press, 1970), 19; Judith V. Field, *Kepler's Geometrical Cosmology* (London: Bloomsbury, 1988), 99.

in particular the idea that planetary orbs follow the series of regular polyhedra inscribed in the sphere.⁵²

4. *Descartes, La Dioptrique, 1637*

Another milestone in refraction theory can be found in the *Dioptrique*, one of the three treatises illustrating Descartes' *Discours de la Méthode*, published by Maire in Leiden (1637). In the First Discourse, the behavior of light is described by a mechanical analogy: light would behave like a ball thrown against a wall (reflection) or against a piece of water (refraction). As has been noted, Descartes' constant use of analogies raises more difficulties than it resolves, as it vitiates the meaning of physical theory.⁵³ The mechanical analogies of the ball and the stick introduce an inconsistency between the thesis of instantaneous propagation and that of temporal propagation of light, while at the same time it leads to the counterintuitive thesis that the speed of light is greater in dense media than in sparse media, an idea that many of Descartes' predecessors and contemporaries disputed.

The purpose of the Second Discourse is to determine the measure of refractions.⁵⁴ In it we find the formulation that resembles the modern law of refraction. Descartes starts from the mechanical analogy of the First Discourse: "Si une balle rencontre la surface d'un corps liquide, elle se détourne et change son cours en y entrant..." and retraces the same figure in a refined form. He then shows that "la force ou la facilité" with which the ray moves in the second medium relative to the first is like the line AH to GI, and then generalizes, showing that this relationship is the same, whatever the incident angle:

Comme, par exemple, s'il passe un rayon dans l'air d'A, vers B, qui rencontrant au point B la superficie du verre CBR, se détourne vers I dans ce verre; & qu'il en vienne un autre de K vers B, qui se détourne vers L; & un autre de P vers R, qui se détourne vers S; il doit y avoir mesme proportion entre les lignes KM & LN, ou PQ & ST, qu'entre AH & IG... Si bien que vous voyés maintenant en quelle sorte se doivent mesurer les refractions; & encores que pour determiner leur quantité, en tant qu'elle dépend de la Nature particuliere des cors où elles se font, il soit besoin d'en venir à l'experience, on ne laisse pas de le pouvoir faire assés certainement & aysement, depuis qu'elles sont ainsi toutes reduites sous une mesme mesure⁵⁵ (Figure 3).

⁵² Bruce Stephenson, *The Music of the Heavens. Kepler's Harmonic Astronomy* (Princeton, NJ: Princeton University Press, 1994), 4.

⁵³ Peter Galison, "Descartes's comparisons: from the visible to the invisible," *Isis* 75 (1984): 311–326; Gideon Manning, "Analogy and falsification in Descartes' physics," *Studies in History and Philosophy of Science* 43 (2012): 402–411.

⁵⁴ René Descartes, *Dioptrique* (Leiden: I. Maire, 1637), 13–25.

⁵⁵ *Ibid.*, 21–22.

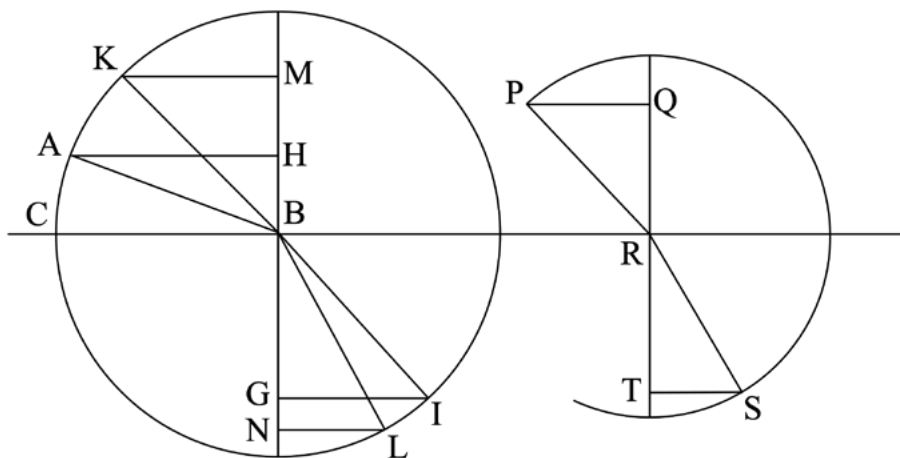


Fig. 3 – Descartes' diagram of refraction.

By considering several rays AB, KB falling on the point of incidence B of the dioptré CBR, Descartes finds a relationship between the projections of the points of the circle on the normal:

$$AH : GI :: KM : LN :: PQ : ST \text{ (= constant)}$$

This relationship does not involve sines, a fact that few authors have noticed apart from Eduard Dijksterhuis: “It is typical of the geometrical orientation still dominating mathematical thought that neither Snellius nor Descartes formulated the law as a constant ratio of sines.”⁵⁶

Consequently, when we invoke the “law of sines” in *Dioptrique*, it is a translation; and this translation needs to be made explicit.

If we equate the KACLI circle with the trigonometric circle:

$$\begin{aligned} AH &= \sin \angle ABH = \sin i_1 \\ GI &= \sin \angle IBG = \sin r_1 \\ KM &= \sin \angle KBM = \sin i_2 \\ LN &= \sin \angle LBN = \sin r_2 \end{aligned}$$

⁵⁶ Eduard Jan Dijksterhuis, *The Mechanization of the World Picture* (Oxford: Clarendon Press, 1961), 390.

Proportion $KM : LN :: AH : GI$ can then be rewritten as $\sin i_1 / \sin r_1 = \sin i_2 / \sin r_2$ (= constant). The equivalence of the two formulations can be invoked. However, the rewriting exercise underlines the existence of a translation process and, in doing so, reveals a gap between Descartes' original formulation: "Il doit y avoir mesme proportion entre les lignes KM & LN , ou PQ & ST , qu'entre AH & IG ,"⁵⁷ and the law of refraction expressed in terms of sines.

Since historians of science and Cartesians have attributed the law of sines to Descartes, and have multiplied evidence, we can only shed full light on the subject once these witnesses have been analyzed. For the time being, we must suspend all judgment. Let us say from the outset that the problem is not about attribution, but rather about the nature of knowledge and the administration of proof. Ten documents mark this history, and none of them suggests that Descartes formulated a true law of refraction.

Descartes and Mydorge, 1626

Among Father Mersenne's papers is a treatise by the mathematician Claude Mydorge, *L'inclination et la refraction de quelconque rayon* (Paris, Bibliothèque nationale de France, naf 5176, fols. 15r-21r). Since the text was published in Mersenne's correspondence,⁵⁸ it has often been mistakenly assumed to be a letter from Mydorge to Mersenne, whereas it is in fact a treatise of its own.

Mydorge's treatise consists of two parts. Propositions 1 and 2, deal with refraction in general and date from the collaboration between Mydorge and Descartes (1626). Propositions 3 to 5, devoted to the study of anaclastic curves (hyperbola, ellipse), belong to Mydorge alone (1626/9).⁵⁹ Proposition 1 states the fundamental relationship:

⁵⁷ Ibid., 21–22.

⁵⁸ *Correspondance de Mersenne*, vol. 1, 404–415.

⁵⁹ Although this treatise bears no direct trace of Descartes' intervention, the argument of an interrupted collaboration between Mydorge and Descartes suggests a two-phase redaction: propositions 1–2 by Mydorge and Descartes; propositions 3–5 by Mydorge alone, Robert Goulding, "The harvest of optics: Descartes, Mydorge, and their paths to a theory of refraction," *Annals of Science* 79 (2022): 164–214, 178. This division respects historical attributions. In the *Examen du livre des recreations mathematiques*, Mydorge admits Descartes' contribution: "Ce noble subject de refractions dont la nature n'a point esté cogneuë ny aux anciens, ny aux modernes Philosophes & Mathematiciens iusques à present, doitb maintenant l'honneur de sa decouverte à un brave Gentilhomme de nos amis," Claude Mydorge, *Examen du livre des recreations mathematiques* (Paris: R. Boutonné, 1630), 139. At the same time Ferrier credits Mydorge with the discovery of anaclastic solutions: "Monsieur Mydorge propose un moyen qu'il a de tracer la ligne necessaire pour brûler à un point qu'il determinera à tout verre donné..." (October 26, 1629), *Correspondance de Descartes*, vol. 1, 68.

We draw IG // CE
 Thus IG meets EG at point G
 We describe circle LDG with center E and aperture EG
 Thus LDG cuts the extended diameter CE in D
 And LDG cuts the extended diameter BE in L
 We draw HM // BA
 We draw MN // CD
 Thus MN intersects circle LDG at point N
 Thus HE is refracted into EN, forming an angle $\angle$ DEN.

Mydorge and Descartes conclude: “On trouvera de la mesme sorte la refraction de tous les autres rayons.”⁶² This geometric construction involves neither law nor sine. The authors speak in terms of “la raison de la droite GE à la droite EF” or “la raison de la refraction”⁶³ – raison being the ancient term for the ratio involved in a proportion.

Descartes, 1628

A reference to the law of refraction has also been recognized in *Règles pour la direction de l'esprit* (*Regulae ad directionem ingenii*). This uncompleted work was written in the winter of 1628, before Descartes' final departure for the United Provinces in the spring of 1629. The manuscript, identified by the letter F in the inventory drawn up in Stockholm on February 14, 1650, following Descartes' death, was passed on to Clerselier. The text was published for the first time in 1701.

Rule VIII (according to which the mind must never examine the consequences of a difficulty without having elucidated the difficulty itself) is illustrated by an example taken from optics:

If, in a word,⁶⁴ someone who has only studied Mathematics asks⁶⁵ about this line, which in Dioptrics is called Anaclastic, i.e. the one where parallel rays refract, so that after refraction they all intersect at a single point, he will no doubt easily notice, following the fifth and sixth rules, that the determination of this line depends on the proportion observed between the angles of refraction and the angles of incidence; but because he will not be able to pursue the search, since it does concern Mathesis, but Physics, he will be forced to stop at the threshold ...

Si, verbi gratia, quaerat aliquis solius Mathematicae studiosus lineam illam, quam in Dioptrica

⁶² Paris, BnF, naf 5176, fol. 15r, *Correspondance de Mersenne*, vol. 1, 405.

⁶³ Ibid., 407.

⁶⁴ Verbi gratia] lit. for example

⁶⁵ quaerat] asks, i.e. wonders about the nature of

*Anaclasticam vocant, in qua scilicet radij paralleli ita refringantur, ut omnes post refractionem se in uno puncto intersecent, facile quidem animadvertet, juxta Regulas Sam & 6am, hujus lineae determinationem pendere a proportionem, quam servant anguli refractionis ad angulos incidentiae; sed quia hujus indagandae non erit capax, cum non ad Mathesin pertineat, sed ad Physicam, hic sistere cogetur in limine.*⁶⁶

Because of the philosophical issues it raises, this passage has given rise to a number of interpretations – some of which are properly apologetic.

Costabel writes about the proportion between the angles of refraction and incidence: “Il s’agit évidemment de la loi des sinus.”⁶⁷ This interpretation goes far beyond the text. Descartes states the proportion observed between the angles of refraction and incidence, without employing any trigonometric concept. Rule VIII therefore does not state a “law of sines.” Descartes is merely formulating a proportion between geometric magnitudes.

Costabel adds: “Descartes entend déjà ici par angle de réfraction un angle compté comme l’angle d’incidence par rapport à la normale à la surface de séparation des deux milieux en présence” and: “Descartes considère les angles d’incidence et de réfraction comme nous les entendons aujourd’hui.”⁶⁸ This interpretation is also questionable, as Descartes employs three different sets of definitions of angles in *Dioptrique*, see below: *Definition of angles*.

Anaclastic is often taken as a marker of Descartes’ advanced research in optics. Only recently, it has been written that “Rule 8 is arguably the most important document detailing Descartes’s reflections on how to discover the law of refraction and the shape of the anaclastic lens.”⁶⁹ This judgment ignores all the work of Descartes’ predecessors on the subject.⁷⁰ Furthermore, in this passage, Descartes does not state the law of refraction in ex-

⁶⁶ refringantur] refringuntur ; admavertet] admavertat ; etiam] enim MS Cambridge. Descartes, *Règles utiles et claires pour la direction de l’esprit en la recherche de la vérité*, trad. J.L. Marion, notes mathématiques de P. Costabel (La Haye: M. Nijhoff, 1977), 27; Descartes, *Regulae ad directionem ingenii, An Early Manuscript Version*, ed. Richard Serjeantson and Michael Edwards (Oxford: Oxford University Press, 2023), 196–197.

⁶⁷ Descartes, *Règles*, 198.

⁶⁸ Ibid., 196, 198.

⁶⁹ Tarek R. Dika, *Descartes’s Method. The Formation of the Subject of Science* (Oxford: Oxford University Press, 2023), 276.

⁷⁰ Around 984, Ibn Sahl showed that the hyperbolic plano-convex lens is anaclastic for a light source at infinity, and that the hyperbolic bi-convex lens is anaclastic for a source at finite distance (Rashed, *Geometry and Dioptrics in Classical Islam* (London: al-Furqān, 2005), 108–109, 138–141). In 1554, Maurolico stated the anaclastic problem in clear terms (*Diaphana*, 80). In 1604, Kepler took up the anaclastic problem and wondered whether the solution was a parabola, a hyperbola or an ellipsis (*Ad Vitellionem Paralipomena* V, 19, 194). In 1611, Kepler wrote that the hyperbolic lens is anaclastic, justifying this with arguments, but no mathematical demonstration

plicit terms; he does not say how the anaclastic line is deduced from the law of refraction; he mentions “this line” in the singular, whereas for a bundle of parallel rays, two solutions already exist (hyperbola, ellipse). Ultimately, the passage is purely allusive, and the formulation so vague as to be compatible with any state of knowledge of the anaclastic problem prior to the 17th century.

Beeckman, 1628/9

Descartes’ work on refraction is also mentioned in Beeckman’s Journal, or *Loci communes*, the manuscript of which is preserved in Middelburg, ZB Bibliotheek van Zeeland, MS 6471, parts I-IV. Beeckman’s Journal was edited by Cornelis de Waard in four volumes between 1931 and 1953.

The refraction research is attached to the date October 8, 1628, and the solutions to the anaclastic problem to February 1, 1629.

Beeckman writes in his Journal: “[Descartes], I say, came to see me in Dordrecht on October 8, 1628” *Is, inquam, die 8^o mensis octobris 1628, ad me visendum venit Dordrechtum*. Refraction is one of the subjects covered. Beeckman begins by reporting the measurement of refraction angles on a right prism, a passage similar to Proposition 2 of Mydorge’s treatise, *L’inclination et la refraction de quelconque rayon* (1626).

He then presents a figure derived from proposition 1 of the same treatise:

One angle of refraction being known, we deduce the others from the sine of the angles because, it is said, CD is to IF as AB is to HG. We consider that there is water under ST, that the rays are AEG and CEF, and that they seem to undergo the same [cause] as the equal arms of a balance, at the ends of which weights are suspended: the one in the water is lighter, and raises the arm. We then look for numerous points such as R, and around them we trace a hyperbola through which all the incident parallel rays converge at a single point. This would be the best glass for making telescopes ... He says it worked perfectly

Cognito uno angulo refractionis, deducit inde reliquos secundum angulorum sinus: ut enim, inquit AB ad HG, ita CD ad IF. Considerat enim sub ST esse aquam, radios esse AEG, CEF, idemque videntur ipsi pati quod brachia aequalia bilancis quo finibus appensa sunt pondera, quorum id quod in aqua est levius est et brachium attollit. Tandem quaerit multa puncta qualia est R ac circa

(Kepler, *Dioptrice*, 21–22). In the same year, his pupil Ambrosius Rhodius disseminated the terms *anaclasticis* and *anaclastica* (Ambrosius Rhodius, *Optica Ambrosii Rhodii, Kembergensis* (Wittenberg: L. Seuberlich, 1611), B4v. In 1623, Mersenne took up the problem again, highlighting the difficulties of creating an anaclastic lens (Marin Mersenne, *Quaestiones celeberrimae in Genesim* (Paris: S. Cramoisy, 1623), 163). In 1626/9, Mydorge demonstrated mathematically that the hyperbola and ellipse are anaclastic curves for a source at infinity, i.e. parallel beam (Mydorge, *L’inclination et la refraction de quelconque rayon*, *Correspondance de Mersenne*, 404–415).

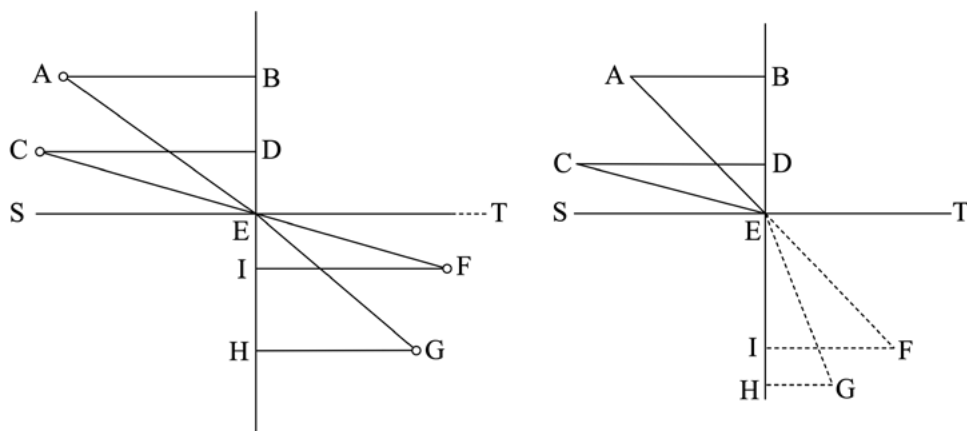


Fig. 5AB – Beekman's diagram of refraction⁷¹.

*illa hyperbolam ducit per quam omnes radii paralleli incidentes concurrunt in unum punctum. Quod vitrum optimum foret ad faciendos tubos oculares... Ipse dicit sibi perfecte successisse*⁷² (Fig. 5AB).

This time, the relationship is written:

$$CD : IF :: AB : HG (= \text{constant})$$

The important sentence is obviously: “One angle of refraction being known, we deduce the others from the sine of the angles.” It corresponds to the problem posed in Proposition 1 of Mydorge's treatise.⁷³

Beekman would later return to the anaclastic problem. On February 1, 1629, he found a demonstration for the hyperbola, which Descartes had not found⁷⁴. Descartes thanked him for it: “Tu mérites de grands éloges pour l'hyperbole que tu m'as apprise.”⁷⁵

⁷¹ On the left, I reproduce the figure from MS 6471, fol. 333v. The straight line ST is extended by a third on the side of T. I would like to thank Mrs. Sanne van Splunter, curator of the ZB Bibliotheek van Zeeland, who provided me with a photograph of the Beekman manuscript. The figure on the right is the one that appears in *Correspondance de Descartes*, vol. 10, 336.

⁷² *Correspondance de Descartes*, vol. 10, 336; *Journal tenu par Isaac Beekman de 1604 à 1634*, ed. by C. de Waard (Den Haag: M. Nijhoff, 1939), vol. 1, 97.

⁷³ Mydorge, Paris, BnF, naf 5176, fol. 15r; *Correspondance de Mersenne*, vol. 1, 404–415, 404.

⁷⁴ *Journal tenu par Isaac Beekman*, vol. 3, 109.

⁷⁵ *Correspondance de Descartes*, vol. 1, 163.

Beeckman will further reproduce Descartes' alternative proofs,⁷⁶ which will be adapted and reproduced in *Dioptrique's* Discourse VIII – without calling on the sines.⁷⁷

These fragments of Beeckman's Journal have often been used as evidence that Descartes had discovered the law of refraction and the solution to the anaclastic problem before 1628. Even quite recently, it has been written: "By the time Descartes met with Beeckman in October 1628, he not only already had the sine law in hand, but had also already deduced the shape of the anaclastic in the case of the ellipse."⁷⁸

Here are the limits of this judgment:

1. As most commentators access this text through Adam-Tannery's reference edition *Oeuvres de Descartes*, no one seems to have noticed that Beeckman's text and figure are wrong. Comparison of the handwritten figure (Figure 5A) and the edited figure (Figure 5B) shows that the edited figure has been rectified: the rays AEG and CEF in the manuscript have been intentionally broken at point E and deflected to reveal refractions EF and EG; the origin-points of the incident rays and the end-points of the refracted rays, which have any position in the manuscript, have been placed on a trigonometric circle ACSTFG to justify the relationship of sines. These properties – absent from Beeckman's Journal – are taken from *Dioptrique*, which dates from ten years later. This anachronistic editorial choice forces continuity between the two texts. On the handwritten figure, the segments AB and HG (CD and IF) are derived from the rectilinear rays AEG (CEF) passing through the point of incidence E. Therefore, the sine relationship only appears on the codicologically false figure edited by Adam and Tannery.
2. As the relation of sines, quoted in Beeckman's Journal, does not appear in proposition 1 of the 1626 treatise by Mydorge, nor in later Cartesian texts, we do not know to whom the sines should be attributed: is it Descartes who uses it before later abandoning it, but for what reason would he have discarded a clear trigonometric notion? Was it Beeckman who interpreted in trigonometric terms the figure presented to him by Descartes?
3. The dating is debatable. Beeckman wrote his notes from day to day, before compiling them in his Journal. Several passages show that he did not keep to the chronology of his notes. For example, Gassendi's visit to Beeckman in 1629 precedes a note dated 1627.⁷⁹ The anachronisms are numerous⁸⁰ and appear even in the passage considered here. The date "8 oktober 1628" to which the passage refers is the date of the meeting between Descartes and Beeckman. Later in the text,

⁷⁶ Ibid., 338–340; *Journal tenu par Isaac Beeckman*, vol. 4, 135–136.

⁷⁷ Descartes, *Dioptrique*, 94, 103.

⁷⁸ Dika, *Descartes's Method*, 312.

⁷⁹ *Journal tenu par Isaac Beeckman*, vol. 1, xxxi note 1.

⁸⁰ Ibid., vol. 1, xxviii, xxxviii, 354; 1942: 173; vol. 2, 173; vol. 3, i–ii, 335, 360, 443, etc.

Beeckman writes: “By meditating until the age of 33, he seems to have found more perfectly than the others what he was looking for” *Usque ad 33 aetatis suae annum meditando, eam rem quam quaesivit, perfectius quàm reliqui invenisse videtur*.⁸¹ Descartes’ 33rd birthday provides the *terminus a quo* March 31, 1629. Consequently, the passage on refraction was written well after Descartes’ visit on October 8, 1628. In any case, the content has been altered and reworked.

Mersenne, 1629

Mersenne, who corresponded extensively with Descartes, discusses refraction in his major work *Harmonie universelle*, published by Sébastien Cramoisy in 1636. The printing of this large work was so delayed that it is preferable to refer the text to the ecclesiastical approval of 1629.

Proposition I, 29 reads: “Mais puis que l’un des plus excellens esprits de ce siecle a trouvé la vraye proportion des refractions aux inclinations, ie veux icy en remarquer l’analogie... Il y a mesme raison du sinus GO, de l’angle d’incidence GNO, au sinus PE, de l’angle d’incidence PNE, que du sinus IR, de l’angle rompu NIR, au sinus KQ, de l’angle rompu NKQ, ce qu’il démontrera dans sa Dioptrique, quand il luy plaira.”⁸²

This passage raises two distinct problems: the identification of “l’un des plus excellens esprits de ce siecle” and the form in which the relation of refraction is expressed. Let us leave aside the problem of identification (Descartes) and concentrate on the form of the relation. Mersenne states the “vraye proportion des refractions” $\sin \angle GNO : \sin \angle PNE :: \sin \angle RNI : \sin \angle QNK$ without explanation. Knowing that sines do not appear in Descartes’ texts on refraction, one wonders: who owns the expression of sines in the *Harmonie universelle*? Is Mersenne reproducing a statement by Descartes, but why would Descartes then delete the sines? Or is Mersenne translating the proportion of refractions into his own language? Remember that, like Kepler, the Minime was familiar with trigonometric methods. As early as 1623, he calculated the distance from the center of the solar orb to the center of the world in this way: *Proportio IA ad IK est sinus anguli IAK ad totum sinum... Proportio verò GI ad IK sicut sinus totius ad sinum anguli HGI*.⁸³

Descartes to Ferrier, 1629

There is no autograph of this letter, written from Amsterdam on November 13, 1629. Descartes suggests to Ferrier that he draw the template for the lens according to the proportion set out in Proposition 1 of the treatise on refraction: “Que si vous en voulez tracer une [ligne] qui brûle à une plus grande ou moindre distance, par exemple, à la distance de

⁸¹ Ibid., vol. 3, 95.

⁸² Marin Mersenne, *Harmonie universelle* (Paris: S. Cramoisy, 1636), 65.

⁸³ Mersenne, *Quaestiones in Genesim*, 170.

DE, cherchez EF, qui soit à DE comme BC est à AB... c'est à dire que si vous avez une fois la proportion qui est entre les lignes AB & BC, par le moyen de vostre quadran, elle vous servira pour tous les verres d'un mesme diaphane."⁸⁴ Note again that Descartes expresses a proportion $EF : DE :: BC : AB$, and not the sine relation.

Descartes to Golius, February 2, 1632

The fifth document showing similarities with the modern law of refraction appears in the letter Descartes sent to Golius in Amsterdam on February 2, 1632. It has been claimed that, in this letter, Descartes aimed to inform his correspondent of “sa découverte de la loi des réfractions.”⁸⁵ But Descartes determines the angle of refraction by the proportion $hi : ef :: ab : cd$ (= constant). The text makes no mention of sines. The absence of sines in the *Dioptrique* may well be the result of a pedagogical choice on Descartes' part.⁸⁶ But what can be said about their absence in the letter to Golius? Why would Descartes, a promoter of mathematical clarity, make a clear trigonometric concept disappear in his exchanges with a mathematician?

Descartes to Mersenne, June 1632

This letter has no date. The editors place it in June 1632. In this letter, which deals with the parallels between acoustics and optics, Descartes argues that the refraction of light and sound do not follow the same rules. It is in this context that he recalls: “Pour la façon de mesurer les refractions de la lumiere, j'établis une comparaison entre les sinus des angles d'incidence et des angles de réfraction” *instituo comparationem inter sinus angulorum incidentiae & angulorum refractorum*.⁸⁷ This is Descartes' only document employing the concept of sine. The statement refers to the existence of a proportion between the sines of the angles of incidence and refraction. This letter was included in the reference edition without being checked against the autograph. The text appears only in Clerselier's edition,⁸⁸ whose editorial methods will be examined below.

⁸⁴ *Correspondance de Descartes*, vol. 1, 64–65.

⁸⁵ Gaston Milhaud, “Descartes et la loi des sinus,” *Revue générale des sciences pures et appliquées* 18 (1907): 223–228, 224.

⁸⁶ Descartes writes in the Avertissement to the *Géométrie*: “Iusques icy i'ay tasché de me rendre intelligible a tout le monde, mais pour ce traité ie crains qu'il ne pourra estre lu que par ceux, qui sçavent desia ce qui est dans les livres de Geometrie” (*Géométrie*, 296). Descartes' rhetorical and pedagogical intentions are analyzed in the literature (John A. Schuster, *Descartes and the Scientific Revolution: 1618–1634*, Doctoral Dissertation (Princeton: Princeton University, 1977), 327; Bruce S. Eastwood, “Descartes on refraction,” *Isis* 75 (1984): 481–502, 498).

⁸⁷ *Correspondance de Descartes*, vol. 1, 255.

⁸⁸ *Lettres de M. Descartes*, ed. Claude Clerselier, 3 vols. (Paris: Angot, 1657–1659), 344–345.

Descartes, *Géométrie* 1637

Refraction, the central subject of *Dioptrique*, reappears in *Géométrie* published alongside *Discours de la méthode* (1637). In Book II, Descartes studies the anaclastic problem, which consists in determining the shape of a lens that focuses rays on a point.⁸⁹

Given that sines are used by Mydorge to study anaclastic curves in the 1626/9 treatise *L'inclination et la réfraction de quelconque rayon*, it would have been logical for this relationship to appear in the 1637 *Géométrie*. Unexpectedly, Descartes' ovals are described only in terms of the proportion stated in proposition 1 of the 1626 treatise, with no mention of sines. The reason for this is probably that the demonstrations of propositions 3 to 5 of the treatise were written after Descartes left for the United Provinces in the spring of 1629.

In his *Géométrie*, Descartes explains the construction of anaclastic curves using the same proportion of refractions as in his *Dioptrique*: “Je tire la ligne droite 56, qui coupe l'autre au point 6, en sorte que A6 soynt moindre que A5 selon telle *proportion* donnée qu'on veut...” and similarly: “en sorte que FL soynt à LG comme A5 à A6, c'est à dire qu'elles ayent la *proportion*, qui mesure les refractions”; “suivant ce qui a esté dit en la *Dioptrique*, elles peuvent toutes estre mesurées par la *proportion* qui est entre les lignes A5 & A6.”⁹⁰

Géométrie makes no mention of a sine ratio that would justify the study of anaclastic curves.

Descartes, *Météores* 1637

A final mention of the relationship of refractions can be found in the *Météores*, the last treatise illustrating Descartes' *Discours de la méthode*, published by Ian Maire (1637). Discourse VIII of *Météores* deals with the rainbow. Here is the central passage, which explains the calculation of refracted angles in the raindrop: “Je connois de ce qui a esté dit en la *Dioptrique*, qu'AE ou HF, & CI, ont entre elles la *proportion* par laquelle la refraction de l'eau se mesure. De façon que si HF contient 8000 parties, telles qu'AB en contient 10000, CI en contiendra environ de 5984...”⁹¹

The proportion of refractions serves as the basis for applying a rule of three. From the relationship $CI : HF :: 187 : 250$, Descartes derives $CI = 187 HF / 250 = 5984$. Two points deserve our attention. Firstly, the proportion of refractions has a calculatory function: “Et calculant en mesme façon tous les autres rayons...” Secondly, contrary to a modern reading that would associate the numerator and denominator each with a refractive medium – say 187 with air, 250 with water – in *Météores*, as in all Descartes' texts, the entire ratio is attached to a single medium: water. Descartes did not use refractive indices.

⁸⁹ Descartes, *Géométrie*, 352.

⁹⁰ Ibid., 353, 356 and 357, italics mine.

⁹¹ Descartes, *Météores*, 262–263.

Discussion

This overview of sources shows that, beyond their particularities, the Cartesian texts written between 1626 and 1637 do not deal with the law of refraction as we understand it today (i.e. sines and refractive indices inserted into a statement with the scope of a physical law), but are oriented by the theory of proportions.

We can now develop a critical analysis of each of these points.

1. Sine relationship. – Descartes is credited with a “law of refraction” based on the sines of the angles of incidence and refraction. Let us leave aside the status of “law,” which we shall come back to, and call the residue: “relation of sines.”

Many philosophers and historians of science credit Descartes with the discovery of the sine relationship⁹² sometimes a relation of cosines⁹³ or a relation of cosecants, which was then transformed into the relation of sines we know today.⁹⁴

A synthesis of the ten documents analyzed above reveals a clear division between two conceptions of refraction.

Seven texts mention a proportion of the type $KM : LN :: AH : GI$, in which the sines (cosines, cosecants) of the angles of incidence and refraction are not involved at all. This conclusion also applies to Proposition 1 of the 1626 treatise *L'inclination et la réfraction de quelconque rayon*; to the 1628 *Règles pour la direction de l'esprit*, to the 1629 letter to Ferrier, to the 1632 letter to Golius, and to the 1637 *Dioptrique*, anaclastic section of the *Géométrie*, and *Météores*.

Only three texts mention sines. None of these is an autograph by Descartes: two are written by his correspondents (Beeckman 1628 and Mersenne 1629); the third is an un-

⁹² For volumetric reasons, I cannot give a complete list of references; See for instance: Montucla, *Histoire des mathématiques*, 1802: 248; Arago, “Descartes,” 1855: 303; Kramer, “Descartes und das Brechungsgesetz des Lichtes,” 1882: 258; Milhaud, “Descartes et la loi des sinus,” 1907: 223; Descartes, *Discours de la méthode*, ed. Gilson 1925: 270; Mach, *Principles of Physical Optics*, 1925: 35; Crombie, *Robert Grosseteste*, 1953: 282; Rodis-Lewis, *L'œuvre de Descartes*, 1971: 85; Ziggelaar, *Le physicien Ignace Gaston Pardies*, 1971: 174; Mahoney, “Descartes,” *Dictionary of Scientific Biography*, 1971: 59; Costabel, *Démarches originales de Descartes savant*, 1982: 57; Shea, *The Magic of Numbers and Motion*, 1991: 155; Jullien, *Descartes, La Géométrie*, 1996: 106; Horstmann, “Hobbes und das Sinusgesetz der Refraktion,” 2000: 417; Heefer, “Descartes’ Theory of Explanation in Natural Philosophy,” 2002: 11; Antoine-Mahut, *Encyclopedia of Early Modern Philosophy*, 2022: 436; Bellis, *Encyclopedia of Early Modern Philosophy*, 2022: 1116; McDonough, *A Miracle Creed*, 2022: 11; Dika, *Descartes’s Method*, 2023: 279, 283, etc.

⁹³ A. Mark Smith, “Descartes’s theory of light and refraction,” *Transactions of the American Philosophical Society* 77 (1987): 1–92, 64.

⁹⁴ John A. Schuster, “Descartes opticien,” Stephen Gaukroger et al., eds., *Descartes’ Natural Philosophy* (London; New York: Routledge, 2000), 258–312, 265; Schuster, *Descartes–Agonistes: Physico-mathematics, Method and Corpuscular-Mechanism 1618–33* (Dordrecht: Springer, 2013), 186.

dated letter from Descartes to Mersenne, known only from Clerselier's edition. A study of Clerselier's editorial methods shows that he intervened massively in edited texts, to the point of fabricating them from scratch to vindicate Descartes against his opponents. Here is a letter from Descartes to Mersenne dated December 18, 1629, in autograph form (Paris, Bibliothèque de l'Institut, MS 2001, fol. 2r) and as edited by Clerselier.⁹⁵

Descartes

Il ne reste plus que quelque chose touchant la vitesse du mouvemant que vous dite que le Sr Becman vous a mande, ce qui viendra mieus en respondant a vostre derniere

Clerselier

[I 617] Il ne reste plus que quelque chose, touchant la vitesse du mouvemant que vous dites que Monsieur Beecman vous a mandé, mais cela viendra mieus en répondant à vostre derniere. *Pour la proportion de vitesse selon laquelle descendent les pois, ie vous en ay écrit ce que i'en sçavois en la précédente, saltem in vacuo, sed in aëre, ce que vous a mandé Monsieur Beecman est veritable, pourvû que vous suposiez que plus le pois descend viste, plus l'air luy resiste; car si cela est, dequoy ie ne suis pas encore du tout assuré, enfin il arrivera que l'air empeschera iustement autant, que la pesanteur ajouteroit de vitesse au mouvement in vacuo, & cela estant, le mouvement demeurera tou-jours égal; mais cela ne se peut [I 618] déterminer que de la pensée; car en pratique il ne le faut pas esperer. Et pour vos experiences, qu'un pois, descendant de cinquante piés, employe autant de tems à parcourir les vingt-cinq derniers que les premiers, salvâ pace, ie ne me sçaurois persuader qu'elles soient iustes: car in vacuo, ie trouve qu'il ne mettra que le tiers du tems à parcourir les vingt-cing derniers, & ie ne puis croire que l'empeschement de l'air soit si notable, qu'il rende cette difference-là imperceptible. Je suis, &c.*

⁹⁵ Left: Paris, Bibliothèque de l'Institut, MS 2001, fol. 2r. Right: *Lettres de M. Descartes*, ed. C. Clerselier, Paris: Angot, 1657–1667, vol. I: 617–618, vol. II: 482–483.

in qua 1° petis quare dicam celeritatem imprimi ut unum primo momento a gravitate et ut duo 2° momento etc. Respondeo, salva pace me non ita intellexisse sed celeritatem imprimi ut unum primo momento a gravitate et rursus ut unum 2° momento ab eadem gravitate etc. unum autem 1i momenti et unum 2i faciunt duo, et unum 3ii faciunt tria atque ita crescit in arithmetica proportione.

Nam si supponeremus exempli causa plumbi massam deorsum delabentem vi gravitatis et postquam per primum momentum labi coepit,

Deum tollere omnem gravitatem ex plumbo, adeo ut postea massa plumbi non sit magis gravis quam si esset aer aut pluma, perget nihilominus descendere ista massa saltem in vacuo, quoniam et coepit moveri et nulla potest afferri ratio cur desinat;⁹⁶

sed non augebitur eius celeritas atqui si post aliquod tempus restituat Deus gravitatem isti plumbo ad momentum temporis tantum quo elapso rursus eandem subtrahat nunquid secundo isto momento vis gravitatis tantundem impellet plumbum quantum fecerat 1° momento ac proinde duplicabitur celeritas motus? Idem de reliquis momentis dicere licet.

[II 482] Mais permettez-moy de vous répondre que ie ne l'ay pas ainsi entendu; mais bien ay-je dit, que la force de la vitesse s'imprime comme un au premier moment par la pesanteur, & derechef comme un au second moment, & ainsi de suite comme un au troisiemes, &c. mais l'un du premier moment, & l'un du second, font deux momens, & c'est ainsi que croist la proportion Arithmetique.

Ainsi par exemple, si nous supposons qu'une masse de plomb par la force de sa pesanteur tombe en bas, & que si-tost qu'au premier moment elle a commencé à descendre, Dieu luy oste toute sa pesanteur, en sorte que cette masse de plomb ne soit pas plus pesante que l'air, ou qu'une plume. Cette masse ne laissera pas pour cela de continuer à descendre dans le vuide, puis qu'elle a une fois commencé à se mouvoir, & qu'on ne sçauroit donner de raison pourquoy elle dust cesser; (*car il faut se ressouvenir que ie suppose que ce qui a une fois commencé à se mouvoir dans le vuide, continue tousiours à se mouvoir, & i'espere le demonstrier en Physique*) mais sa vitesse ne sera point augmentée: Et si quelque temps après, Dieu vient à rendre pour un moment à cette masse de plomb, [II 483] toute la pesanteur qu'elle avoit auparavant, & qu'un moment après il la luy oste derechef, ne voit-on pas qu'en ce second moment la force de la pesanteur doit pousser autant cette masse de plomb, qu'elle avoit fait au premier moment; & par consequent son mouvement sera augmenté de moitié, & le mesme arrivera aux troisiemes, quatrieme & cinquiemes momens, &c.

⁹⁶ desinat] *add. marg.* a Oportet meminisse nos supponere illud quod semel motū est in vacuo semper moveri et in meo tractatu demonstrare conabor.

dou il suit certainement que si vous laissiés tomber une boule *in spatio plane vacuo* de 50 pieds de hault que de quelle matiere quelle pust estre elle employeroit tousiours iustemant trois fois autant de tans aus 25 premiers pieds quelle feroit aus 25 derniers.

[II 483] D'où il suit certainement, que si vous laissez tomber une boule *dans une espace tout à fait vuide* de cinquante pieds de haut, que de *quelque* matiere qu'elle puisse estre, elle employeroit tousiours iustemant trois fois *autant* de temps à *descendre les vingt-cinq* premiers pieds, *que les vingt-cinq* derniers:

Mais dedans laer cest tout autre chose et pour revenir au Sr Beecman, encore que ce quil vous a mande soit fauls a scavoir quil y ait un lieu auquel un poids qui descend estant parvenu pour-suit par apprés tousiours desgale vitesse, toutes-fois il est vray quapprés certain espace cette vitesse s'augmente de si peu quelle peut estre iugée insensible. Et ie men vois vous expliquer ce quil veult dire, car nous en avons autrefois parle ensemble.

mais dans l'air c'est toute autre chose. Et pour revenir au Sieur N. encore que ce qu'il vous a mandé soit faux, à sçavoir qu'il y ait un lieu auquel *une pierre* qui descend estant parvenuë, elle *descendra* par après d'égale vitesse. Toutefois il est vray que *cette augmentation* de vitesse est si *petite après certain espace*, qu'elle peut estre *estimée* insensible, & ie m'en vais vous expliquer ce qu'il faut dire, car nous en avons autresfois parlé ensemble, & ie vous diray *après enquoy il se méprend*.

This excerpt shows that the changes introduced by Clerselier range from simple rephrasing to the dismantling of the text, including significant omissions and additions, and biased rewritings that attribute to Descartes ideas belonging to others.⁹⁷

In such conditions, only Descartes's autographs are reliable, and knowledge of the law of refraction cannot be attributed to Descartes based solely on a single text published by Clerselier ten years after Descartes's death.

This leaves the accounts of Beeckman and Mersenne, two authors who used trigonometry in their own work, and to whom we must leave the authorship of the texts they wrote, in which they set out, perhaps in their own words, what they understood of Descartes' research. In any case, *Descartes never publicly claimed the law of sines*.

The conclusion that Descartes did not formulate a sine-based law of refraction rests primarily on the absence of any autograph documents whatsoever attesting to the use of sines in his writings.

⁹⁷ Giulia Belgioioso, "Un faux de Clerselier," *Bulletin cartésien* 33 (2005): iv (unpaginated); Giulia Belgioioso, Vincent Carraud, *Des éditions du XVIIe siècle aux éditions modernes* (Lecce: Centro dipartimentale di studi su Descartes "Ettore Lojacono," 2024), <http://www.cartesius.net>.

2. *Definition of the angles.* – There is another proof that Descartes was indifferent to writing the refraction relation in sine form, but this evidence is invisibilized by Costabel's opinion that "Descartes considère les angles d'incidence et de réfraction comme nous les entendons aujourd'hui."⁹⁸ This statement is false, as can be demonstrated by examining how Descartes defines the angles of incidence and refraction.

Sines can only be used in a mathematical expression if the angles to which they refer are clearly and uniquely defined. However, in *Dioptrique*, the angles of incidence and refraction are subject to varying definitions:

Definition 1. Sometimes angles are measured in relation to the interface, in the manner of the Ancients: "Vous voyés facilement, comment se fait la reflexion, à sçavoir selon un angle toujours esgal à celuy qu'on nomme l'angle d'incidence... en sorte que l'angle de la reflexion FBE, n'est ne plus ne moins grand que celuy de l'incidence ABC."⁹⁹

Definition 2. Sometimes, the angle of refraction means the deviation $r - i$: "Cette inclination se doit mesurer par la quantité des lignes droites... non par celle des angles... semblables à DBI, qu'on nomme les angles de Refraction."¹⁰⁰

Definition 3. Sometimes, angles of incidence and refraction are measured in relation to the normal, in the manner of the Moderns: "Il doit avoir même proportion entre les lignes KM et LN... mais non pas la même entre les angles KBM et LBN, ou PRQ et SRT, qu'entre ABH et IBG."¹⁰¹

Descartes thus uses mutually exclusive definitions of the angles of incidence and refraction just a few pages apart. This refutes Costabel's opinion.

But the multitude of definitions of angles has implications for the very nature of the law. Indeed, each definition leads to a different "law." Definition 1 (angles of incidence i and refraction r measured relative to the diopter) gives law (1). Definition 2 (angle of deviation $\delta = i - r$) gives law (2). Only definition 3 (angles i and r measured with respect to the normal) gives the modern sine law (3):

$$\cos i_1 / \cos r_1 = \cos i_2 / \cos r_2 (= \text{constant}) \quad (1)$$

$$\sin i_1 / \sin (i - \delta)_1 = \sin i_2 / \sin (i - \delta)_2 (= \text{constant}) \quad (2)$$

$$\sin i_1 / \sin r_1 = \sin i_2 / \sin r_2 (= \text{constant}) \quad (3)$$

While Descartes' text is immediately compatible with an approach based on proportions (which requires no definition of the angles of incidence and refraction whatsoever), a law of refraction based on sines does require such definitions. The lack of stable definitions of

⁹⁸ Descartes, *Règles*, 198.

⁹⁹ Descartes, *Dioptrique*, 16.

¹⁰⁰ *Ibid.*, 21.

¹⁰¹ *Ibid.*, 21–22.

the angles has never been pointed out, nor have the implications been drawn: the inconsistent definitions of the angles of incidence and refraction preclude the identification of a sine law, even in implicit form, in Descartes' text.

3. *Refractive indices*. – Another point of contention is the omnipresence of refractive indices in philosophical and historical commentaries on Descartes' optics¹⁰². None of the texts just reviewed – not even the *Dioptrique*, which is the most accomplished – mentions refractive indices. Descartes simply speaks of “air” and “eau” or, to generalize a little, of “corps transparents” that light “pénètre plus ou moins facilement.” The refrangibility of bodies is neither conceptualized, nor quantified.

This absence shifts the focus of the investigation and raises the question of when the refractive indices were incorporated into the law of refraction.

Taking up Newton's idea¹⁰³ of measuring the *refractive power* of different media by the ratio of the sines $n_i : n_r = \sin r : \sin i$, Wollaston (1766–1828) presented a paper to the Royal Society in June 1802, in which he explained how to determine refractive indices using a right-angled prism. His work is compiled in a table of refractive indices for 83 bodies, ordered from the most refractive (diamond 2.44) to the least refractive (atmospheric air 1.00032).¹⁰⁴

1807 saw the publication of the *Lectures* by Thomas Young (1773–1829), a collection of lectures on natural philosophy given at the Royal Institution between 1801 and 1803. The lectures are undated, but Lecture xxxv, *On the theory of optics*, is later than 1802, as it mentions Wollaston's measurement of refractive indices. Although Young's terminology varies,¹⁰⁵ he is credited with establishing the current term *refractive index* (*index of refraction*). He defined it as follows: “The ratio of the angles of refraction and incidence ... in all

¹⁰² I cannot provide full references. See for instance: Rogers, “Genius and Writings of Descartes,” 1852: 12; Kramer, “Descartes und das Brechungsgesetz des Lichtes,” 1882: 239; Peddie, *A Manual of Physics*, 1896: 216; Ball, *A Short Account of the History of Mathematics*, 1901: 286; Preston, *The Theory of Light*, 1901: 128; Ball, *A Short Account of the History of Mathematics*, 1908: 277; Mach, *Principles of Physical Optics*, 1925: 4; Wootton, *The Physical Work of Descartes*, 1927: 473; Scott, *The Scientific Work of Descartes*, 1951: 110; Cohen, *Notes and Records of the Royal Society*, 1964: 135; Sirven, *Les années d'apprentissage de Descartes*, 1987: 328; Rodis-Lewis, *La science chez Descartes*, 1987: 69; Hatfield, “Science, Certainty, and Descartes,” 1988: 258; Thomson, *On Descartes*, 2000: 29; Burnett, *Descartes and the Hyperbolic Quest*, 2005: 62; Smith, *From Sight to Light*, 2014: 395; Watson, *Light*, 2016: 90; etc.

¹⁰³ “Having made these observations, I first computed from them the refractive power of that glass, and found it measured by the ratio of the sines, 20 to 31,” Isaac Newton, “A new theory about light and colors,” *Philosophical Transactions* 80 (1672): 3075–3087, 3077.

¹⁰⁴ William H. Wollaston, “A method of examining refractive and dispersive powers by prismatic reflection,” *Philosophical Transactions of the Royal Society* 92 (1802): 365–380, 570–571, Table 1.

¹⁰⁵ Rarity, density, refractive power, refractive density...

cases, being expressed by the ratio of 1 to a certain number, which is called the index of the refractive density of the medium.” And later on “The index of refraction at the common surface of any two mediums is the quotient of their respective indices.”¹⁰⁶ In his lecture, Young does not attempt, as Wollaston did, to determine the refractive indices of an entire series of refracting substances. He provides only the indices for water (4 : 3), crown glass (3 : 2), and flint glass (8 : 6).

In 1814, Joseph Fraunhofer (1787–1826) resumed the measurement of refractive indices of optical glasses, with the aim of perfecting achromatic lenses.¹⁰⁷ Knowing that chromatic aberration in lenses can be compensated for by combining flint and crown glass (more and less refractive), determining the refractive index of glass more accurately made it possible to better adjust the lenses of an achromatic doublet. Fraunhofer renamed the refractive index “exponent of the refractive ratio” (*Exponent des Brechungsverhältnisses*) and determined the refractive indices of twelve optical lenses to the sixth decimal place.¹⁰⁸

4. Law. – Even more numerous are those who credit Descartes with a “law of refraction.”¹⁰⁹ In certain circumstances, this expression may have been used inadvertently. When Arago

¹⁰⁶ Thomas Young, *A Course of Lectures on Natural Philosophy and the Mechanical Arts* (London: J. Johnson, 1807), 413.

¹⁰⁷ Joseph Fraunhofer, “Bestimmung des Brechungs- und Farbenzerstreuungs- Vermögens verschiedener Glasarten, in Bezug auf die Vervollkommnung achromatischer Fernrohre,” *Denkschriften der königlichen Akademie der Wissenschaften zu München* 5 (1814): 193–226.

¹⁰⁸ *Ibid.*, 225, Tabelle III.

¹⁰⁹ Again, I cannot provide full references. These are for information only: Montucla, *Histoire des mathématiques*, 1802: 248; Arago, “Descartes,” 1855: 303; Airy, *Geometrical Optics*, 1870: 43; Kramer, “Descartes und das Brechungsgesetz des Lichtes,” 1882: 236; Adam and Tannery, *Correspondance de Descartes*, 1897: 241; Milhaud, “Descartes et la loi des sinus,” 1907: 223; Mach, *Principles of Physical Optics*, 1925: 32; Descartes, *Discours de la méthode*, ed. Gilson 1925: 270; Papanastassiou, *Les théories sur la nature de la lumière*, 1935: 26; Crombie, *Robert Grosseteste*, 1953: 283; Descartes, *Discours de la méthode*, ed. Alquié 1963: 673; Rodis-Lewis, *L’oeuvre de Descartes*, 1971: 85; Ziggelaar, *Le physicien Ignace Gaston Pardies*, 1971: 170; Mahoney, “Descartes,” *Dictionary of Scientific Biography*, 1971: 59; Costabel, *Démarches originales de Descartes savant*, 1982: 66; Lindberg, *Studies in the History of Medieval Optics*, 1983: 23; Smith, *Descartes’s Theory of Light and Refraction*, 1987: 4–5, 29, 81; Shea, *The Magic of Numbers and Motion*, 1991: 241; Jullien, *Descartes, La Géométrie*, 1996: 105; Schuster, “Descartes opticien,” 2000: 258; Blay, *Penser avec l’infini*, 2010: 55; Schuster, *Descartes-Agonistes*, 2013: 187; McDonough, *The Cambridge Descartes Lexicon*, 2016: 552–553; Schuster, *The Cambridge Descartes Lexicon*, 2016: 455; Bellis, *Descartes en dialogue*, 2019: 192; Antoine-Mahut, *Encyclopedia of Early Modern Philosophy*, 2022: 436; Bellis, *Encyclopedia of Early Modern Philosophy*, 2022: 1116; McDonough, *A Miracle Creed*, 2022: 11; Schuster, *Descartes’ mechanichal philosophy*, 2022: 448; Dika, *Descartes’s Method*, 2023: 270, etc.

wrote that Descartes was the first to “communiquer la loi des sinus,”¹¹⁰ he was primarily seeking to establish Descartes’ priority over Snel. As we know, this priority has been contested because Snel’s discovery predates Descartes.¹¹¹ When Gaston Milhaud wrote that Descartes gave a “démonstration théorique de la loi des sinus,”¹¹² his aim was to contrast the contributions of Descartes and Fermat. This interpretation has also been debated: Descartes engages more in an exercise of rhetorical persuasion than in a demonstration.¹¹³ In both cases, the focus is on the relationship between Descartes and his peers. Neither Arago nor Milhaud were interested in the *epistemological status* of the refraction relationship.

If we refer to the texts, Descartes does not claim any “law of refraction.” Even when he wrote that “la proportion [AH : GI :: KM : LN :: PQ : ST] demeure la mesme en toutes les refractions qui sont causées par les mesmes cors,”¹¹⁴ his aim is not to state a physical law, but to calculate angles, freeing himself from the need to measure them empirically.

The absence of a law in the strict sense raises secondary questions, and shifts the focus of the investigation.

And first of all, why did Descartes not consider the ratio of refractions to be a law? We know, for one thing, that Descartes conceived of laws of nature. As he wrote in a letter to Mersenne in April 1630: “C’est Dieu qui a établi ces lois en la nature ainsi qu’un Roi établit des lois en son Royaume.”¹¹⁵ We also know that Descartes accepted the existence of “laws of motion” which in fact constitute the foundational hypothesis of *Dioptrique*: “Car il est bien aysé à croire que l’action ou inclination à se mouvoir, que i’ay dit devoir estre prise pour la lumiere, doit suivre en cecy les mesmes loys que le mouvement.”¹¹⁶ Why did these favorable conditions not lead to a law of refraction? As far as I know, Descartes’s attempt to reduce optics to mechanics ended in failure. When comparing the motions of a ball and a ray of light, Descartes had to conclude that they move in different directions relative to the extended incident ray: one toward the surface, the other toward the normal. To explain the opposite trajectories of light and the ball, Descartes assumes that light propagates faster in dense bodies than in rare bodies, which is contrary to experience and to the common opinion that a medium offers resistance proportional to its density.¹¹⁷ Consequently, the

¹¹⁰ François Arago, “Descartes,” *Oeuvres de François Arago*, ed. by Jean-Augustin Barral (Paris: Baudry, 1855), vol. 3, 297–309, 303.

¹¹¹ See Vollgraff, “Snellius’ notes on the reflection and refraction of rays,” 718–725; Hentschel, “Das Brechungsgesetz in der Fassung von Snellius,” 297–344.

¹¹² Gaston Milhaud, “Descartes et la loi des sinus,” 223.

¹¹³ Eastwood, “Descartes on refraction,” 501; Antoni Malet, “Gregorie, Descartes, Kepler, and the law of refraction,” *Archives internationales d’histoire des sciences* 40 (1990): 278–304, 300.

¹¹⁴ Descartes, *Dioptrique*, 21.

¹¹⁵ *Correspondance de Descartes*, vol. 1, 145.

¹¹⁶ Descartes, *Dioptrique*, 8.

¹¹⁷ Descartes’ physics is also self-contradictory: when he claims that light travels faster in a dense body, he is advocating the temporal propagation of light (finite speed); when he says that dense

mechanical analogy could not serve as the basis for the law of refraction. Because of this lack of a strict equivalence between mechanics and optics, Descartes was unable to apply the laws of motion to optics and, consequently, was unable to formulate optical laws such as the law of refraction.

Next, one is also led to wonder when the Snel-Descartes relationship came to be considered a physical law.

It should be noted in advance that ancient texts refer to the law of refraction sometimes in the singular and sometimes in the plural. Since the law of sines is unique, we must first question the use of the plural.

When Mersenne mentions the “laws of refraction” in *Harmonie universelle*, he is not referring to the modern law of sines, but to the ancient rules of refraction set forth by Alhacen, *Optics* VII, 2.77, rewritten by Risner, according to which the ray “s’esloigne” or “s’approche de la perpendiculaire.”¹¹⁸ Let us then exclude occurrences of the laws of refraction in the plural. We can see, by contrast, that the phrase “law of refraction” in the singular only came into common usage from the 1690s onwards, as evidenced by the following expressions:

- loix¹¹⁹ de la refraction (Huygens 1690)
- loi des sinus (Leibniz 1693)
- veritable loy des refractions (Leibniz 1693)
- loi de la refraction (Mariotte 1717)
- loi constante de la refraction (De Montmor 1718)
- loi connuë de la réfraction (Maraldi 1723)
- loi ordinaire des réfractions (Bouguer 1729)
- loi de la refraction (Mairan 1730)
- loi de la réfraction (Clairaut 1741)
- loi de la réfraction (Maupertuis 1748)
- etc.

It is anachronistic to look for a law of sines before the end of the 17th century. A good connoisseur of 17th-century optics might still object that Fermat already speaks of a “law of refractions” in *Synthèse pour les réfractions* (1662). This is an artifact: Tannery forced the translation to include the word law, which does not appear once in the Latin text: *In ipsam tamen Cartesii proportionem incidimus* should be translated as “we nevertheless arrive at the same proportion as Descartes” and not “nous sommes retombés précisément sur la loi

bodies do not resist the passage of light, he is defending the instantaneous propagation of light (infinite speed). Light can have a finite or infinite speed, not both at the same time.

¹¹⁸ Mersenne, *Harmonie universelle*, 64.

¹¹⁹ The plural here refers to all laws of refraction, including the law of sines, Christiaan Huygens, *Oeuvres complètes de Christiaan Huygens*, ed. by Diderik J. Korteweg (La Haye: M. Nijhoff, 1916), 741, 755. These notes, which supplement the *Traité de la lumière*, were written after 1690.

que Descartes a énoncée.”¹²⁰ It is symptomatic that Tannery’s translation error consisted in substituting Descartes’ own word *proportio* with the modern concept of law. But here, as elsewhere, the theory of proportions is the only one historically documented.

5. *Proportions*. – What remains of Descartes’ account of refraction once the modern garb of sines, refractive indices and physical laws has been removed?

What remains is a proportion of the type $AH : GI :: KM : LN$ defined between geometric magnitudes, which requires clarification of Descartes’ conception of proportional relationships.

Unlike Maurolico’s and Kepler’s texts, which are composed *more geometrico*, Descartes’ texts are written freely, making it more difficult to discern the influence of mathematical axioms. And yet, many Cartesian problems boil down to the manipulation of proportions. Here are a few examples. In Book II of *Géométrie*, Descartes introduces an instrument, similar to Eratosthenes’ mesolab, which can be used to find proportional means. The lengths of the instrument’s rules obey the proportion $YC : YB :: YD : YC :: YE : YD :: YF : YE = \mu$.¹²¹ In a letter dated September 12, 1638, he explained the nature of a spiral to Mersenne. The (logarithmic) spiral conforms to the proportion $OB : OA :: OC : OB :: OD : OC :: OE : OD :: OF : OE \dots$.¹²² In another letter dated June 1645, Descartes mentions that Florimond de Beaune proposed a problem to him five or six years ago, which led him to define a (logarithmic) curve by the proportion $BC : CL :: N : BI$.¹²³

The Cartesian idea of proportions is neither *purist* – Descartes often confuses proportion with identity of ratios¹²⁴ – nor truly *new* – many subjects, such as Pappus’ problem, are posed in the terms of the Ancients. The novelty of the Cartesian project lies instead in the role he assigns to proportions in the development of science. Hence their lack of visibility. This role can, however, be documented based on certain passages from *Discours de la méthode* and *Règles pour la direction de l’esprit*.

In *Discours de la méthode*, Descartes stresses the fundamental role of proportions. By questioning the relationships between the mathematical sciences (including optics), he manages to reduce the core of these sciences to proportions. Descartes states: “encore que leurs obiets soient differens, elles ne laissent pas de s’accorder toutes, en ce qu’elles n’y considerent autre chose que les divers rapports ou proportions qui s’y trouvent.”¹²⁵

Further references to the role of proportions appear in Descartes’ *Règles pour la direction*

¹²⁰ Pierre de Fermat, *Synthesis ad refractiones* (1662), *Oeuvres de Fermat*, ed. by Paul Tannery and Charles Henry (Paris: Gauthier-Villars, 1896), vol. 3: 151–152.

¹²¹ Descartes, *Geometrie*, 317.

¹²² *Correspondance de Descartes*, vol. 2, 360–361.

¹²³ *Ibid.*, vol. 4, 229–230.

¹²⁴ Descartes, *Météores*, 262–263.

¹²⁵ Descartes, *Discours de la méthode*, 21.

de l'esprit. In Rule VI, the theory of proportions is the central idea, the “secret” from which numbers can be arranged into continuous proportion. In Rule XIV, Descartes goes further when he aims at the “réduction de ces proportions jusqu'à ce que l'égalité entre la ‘chose’ demandée et quelque ‘chose’ connue soit vue clairement” or when he proposes to “réduire les proportions si emmêlées qu'elles soient au point de trouver ce qui est inconnu en égalité avec quelque autre connu”¹²⁶ (Marion 1977: 62). Here, proportions take on a new dimension. They are no longer merely a tool for calculation; they embody an ambitious project of knowledge aimed at unifying the sciences – the *Mathesis universalis*. The reason why it transcends the divisions between arithmetic, geometry, astronomy, mechanics, optics ... is precisely that proportions form the underlying foundation of each of these sciences.

Commenting on the relationship between these texts and geometry, Vuillemin considers that the theory of proportions is more central than analytical geometry in Descartes' work: “Aux yeux du philosophe, l'invention de la géométrie analytique paraît secondaire par rapport à l'invention d'une méthode universelle de pensée, contenue ... dans la théorie générale des proportions.”¹²⁷ Descartes' ambition seems to have been to renew geometry, replacing the consideration of “figures” with that of “lines” between which relationships and proportions can be defined. In this respect, one can even view proportions as a tool for organizing mathematical problems.¹²⁸

Descartes' theory of proportions is the very counterexample to the common idea – which originated with Kuhn – that scientific theories are subject to a law of replacement: as soon as a new paradigm appears, it replaces the previous one.¹²⁹ However, just as Canguilhem was right to denounce the continuous view of the history of science based on the unclear notion of precursor, so too could one challenge the discontinuous view based on the idealization of the notions of theory and paradigm. As Duhem showed with regard to optics, “une théorie qui disparaît ne disparaît jamais tout entière.”¹³⁰ This maxim applies perfectly to Descartes' work, which contains within itself a historical stratification of mathematical principles from different periods. The author of a radical innovation is not entirely new. As the inventor of analytical geometry, Descartes nevertheless made extensive use of the theory of proportions, which guided both his geometric research and his theory of refraction.

Returning now to the theory of refraction, once we set aside the anachronisms represented by the sines, refractive index and physical law, which gradually developed between

¹²⁶ Descartes, *Règles*, 62, 67.

¹²⁷ Jules Vuillemin, *Mathématiques et métaphysique chez Descartes* (Paris: PUF, 1960), 10.

¹²⁸ Vincent Jullien, *Descartes, La Géométrie de 1637* (Paris, PUF, 1996), 27.

¹²⁹ “These men ... are particularly likely to see that those rules no longer define a playable game and to conceive another set that can *replace* them. The resulting transition to a new paradigm is scientific revolution,” Thomas Kuhn, *The Structure of Scientific Revolutions* (Chicago: University of Chicago Press, 1962), 90, and the countless occurrences of the verb *replace* in *Structure*.

¹³⁰ Pierre Duhem, “Les théories de l'optique,” *Revue des deux mondes* 123 (1894): 94–125, 123.

the late 17th century and the 19th century, what remains is an attempt to express a proportion. This observation extends far beyond the authors discussed in this article. This becomes clear when we examine the expressions used at the time:

- sunt proportionales (Maurolico 1567)
- proportion (Harriot av. 1605)
- non sunt proportionales (Kepler 1611)
- ratio (Snel 1621)
- raison de la refraction (Mydorge 1626)
- vraye proportion des refractions (Mersenne 1636)
- proportion (Hérigone 1637)
- mesme proportion (Descartes 1637)
- raison de refraction (Roberval av. 1651)

In the end, even though Maurolico, Kepler, Mydorge, Descartes and others came up with different theories on refraction, they were all firmly rooted in the classical theory of proportions. Incidentally, the influence of the theory of proportions did not disappear in the mid-17th century. Its mark is still visible in the works of Huygens and Newton,¹³¹ but that is another matter, which is beyond the scope of this article.

¹³¹ Niccolò Guicciardini, *Reading the Principia. The debate on Newton's mathematical methods for natural philosophy from 1687 to 1736* (Cambridge: Cambridge University Press, 1999), 118–135.

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— ESSAYS —





An outline of Celestino Cominale's (1722–1785) critique of Isaac Newton's natural philosophical method

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Abstract

This essay examines Celestino Cominale's (1722–1785) self-proclaimed 'anti-Newtonianism'. Between 1754 and 1770, Cominale published four volumes under the title of *Anti-Newtonianismi*, in which he launched a sustained attack on Newton's natural philosophy. Despite a modest resurgence of interest in his work, Cominale's critique has largely been overlooked and is often dismissed as an isolated, provincial, and misguided attack on Newton's theories. This article seeks to offer a comprehensive account of Cominale's critique of Newton's natural philosophy, with a particular focus on the opening chapters of the second volume of *Anti-Newtonianismi*. It will be shown that Cominale was deeply engaged with contemporary debates surrounding Newton's natural philosophy and, at times, advanced original and insightful criticisms of Newton's natural philosophical method.

Keywords

Celestino Cominale, Isaac Newton, *Anti-Newtonianismi*, 'anti-Newtonianism', Italian 'Newtonianism'

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Introduction: Celestino Cominale and “the first detailed anti-Newtonian study”¹

The dissemination of Isaac Newton's ideas in Italy has been the focus of numerous studies, which often emphasize how the geopolitical landscape of the peninsula, consisting of numerous kingdoms and city-states, significantly shaped their acceptance and spread.² Although scientific debate was thriving across Italy in the early eighteenth century, the distinct character of academic institutions played a key role in shaping the diffusion and teaching of Newton's ideas. A key factor to consider, for instance, is the diverging approaches taken by northern and southern Italy in embracing Newton's ideas. By the 1710s and 1720s, universities in the north had begun incorporating Newton's teachings into their curricula. In contrast, the south appeared slower to adopt them.³ In the north, many scholars engaged with both the mathematical and experimental aspects of Newton's theories and they selectively integrated them with pre-existing practices.⁴ Meanwhile, in the south, where most university professors were also part of the clergy, the apologetic features of Newton's natural philosophy were emphasized and its philosophical and epistemological implications were explored.⁵ Although Newton's ideas were incorporated into the academic curriculum, their adoption often relied on the initiative of individual professors.⁶ In Rome and Naples, Newton's ideas featured prominently in scientific discourse, prompting many scholars and intellectuals to engage in deeper study of his natural philosophy, contributing to its widespread acceptance by the 1740s.⁷

After the 1740s, critiques of Newton's natural philosophy became increasingly rare. When they did surface, they were typically dismissed, marginalised, and in some cases, nearly forgotten. A particularly intriguing example of a self-proclaimed ‘anti-Newtonian’⁸

¹ Celestino Cominale, *Anti-Newtonianismi pars secunda, in qua rejectis methodo, et philosophandi regulis Newtonianis, totiusque sectae principiis indicatis, evertitur prae caeteris, argumentis ut plurimum ex adversariorum penu depromptis, vacuum, vis inertiae, mutua gravitas et attractio* (Naples: Ex typographia Benedicti Gessari, 1756), [av]: “primam Anti-newtonianam elucubrationem.”

² On the spread of Newton's ideas in Italy, see, for instance, Paolo Casini, *Newton e la coscienza Europea* (Bologna: il Mulino, 1983), 173–227; Vincenzo Ferrone, *Scienza, natura, religione. Mondo newtoniano e cultura italiana nel primo Settecento* (Naples: Jovene Editore, 1982); and Massimo Mazzotti, “Newton in Italy,” in *The Reception of Isaac Newton in Europe*, ed. by Helmut Pulte and Scott Mandelbrote, vol. 1 (London: Bloomsbury, 2019), 159–178.

³ Mazzotti, “Newton in Italy,” 164–165.

⁴ Ibid., 177.

⁵ Ibid.

⁶ Ibid., 173–174.

⁷ On the spread of Newton's theories in Rome and Naples, see, for instance, Casini, *Newton e la coscienza Europea*, 191–194.

⁸ On the topic of ‘anti-Newtonianism’, see the essays in Philippe Hamou and Neil Ribe, eds., “Figures de l’antinewtonianisme/Faces of Anti-Newtonianism (1672–1832),” *Archives Internationales d’Histoire des Sciences* 53 (2003): 115–271.

active in the post-1740s period is Celestino Cominale (1722–1785), who published four volumes of his monumental work, *Anti-Newtonianismi*, between 1754 and 1770.⁹ Very little is known about Cominale, and only a handful of accounts of his life have survived.¹⁰ According to the available sources, Cominale was born on 29 October 1722 in Uggiano la Chiesa, located in what is now Apulia. His education seems to have been an excellent one. He first studied in Lecce and later at the University in Naples, where he pursued physics, mathematics, and sciences under the mentorship of some notable scholars of his time, including Michelangelo De Rubertis (1713–1776), Francesco Serao (1702–1783), Felice Rossetti (?–?), Mario Lama (?–?), and two brothers who ardently supported Newton's natural philosophy, Pietro (1707–1746) and Nicola Di Martino (1701–1769).¹¹ These formative years laid the foundation for his reputation as a respected physician. A significant chapter in Cominale's life was marked by his travels across Italy, during which he visited numerous Italian universities and engaged in intellectual exchanges with leading natural philosophers. Among those he met were the chemist Jacopo Bartolomeo Beccari (1682–1766) in Bologna, and the two Minim friars, Thomas Le Seur (1703–1770) and

⁹ Celestino Cominale, *Anti-Newtonianismi* (4 vols.) (Naples: Ex typographia Benedicti Gessari, 1754, 1756; Ex typographia Francesci Morelli, 1769, 1770).

¹⁰ See Giovanni Battista Lezzi, "Vite degli scrittori salentini," Ms. D/5, Biblioteca Pubblica Arcivescovile di Brindisi 'A. De Leo', 1787–1807, 308v–310v; Giovanni Battista Lezzi, "Vite de' letterati salentini," Biblioteca Provinciale di Lecce 'Nicola Bernardini', Ms. n. 52, n.d., 563–565; Luigi Maggiulli, "Dizionario biografico degli uomini chiari di Terra d'Otranto," State Archive of Lecce, Ms. n.C 47, vol. 2, n. 209; Luigi Giuseppe De Simone, "Dizionario biografico salentino," Biblioteca Provinciale di Lecce 'Nicola Bernardini', Ms. n. 249, n.d., vol. 1, n. 199; Giovanni Battista De Tomasi, "Celestino Cominale," in [various authors], *Biografia degli uomini illustri del Regno di Napoli ornata de loro rispettivi ritratti, compilata da diversi letterati nazionali, dedicata a S. E. RMA. Monsig. D. Gabriele M. Gravina, Arcivescovo di Militene Cappellano Maggiore di S. M. il Re del Regno delle due Sicilie*, vol. 9 (Naples: Presso Nicola Gervasi, 1822), 196–200.

¹¹ Not much is known about Cominale's teachers. Rossetti exchanged letters with Antonio Vallisnieri (1661–1730), published as "Sistema nuovo intorno all'Anima pensante, e alla Circolazione degli Spiriti animali," in *Raccolta d'opuscoli scientifici e filologici*, ed. by Angelo Calogerà, vol. 5 (Venice: Appresso Cristoforo Zane, 1721), 265–316. Serao was a follower of Celestino Galiani (1682–1753) (Ferrone, *Scienza, natura, religione*, 512). A biography was written about him in 1784 by Michele Arcangelo Lupoli, *De vita muniis et scriptis Francisci Serai Philosophi et Medici Neapolitani clarissimi. Commentarius* (Naples: Typographia Simoniana, 1784). Regarding De Rubertis, see Giulia Belgioioso, *Cultura a Napoli e cartesianesimo. Scritti su G. Gimma, M. Doria, C. Cominale* (Galatina: Congedo Editore, 1992), 400. De Rubertis was a physician who became famous for curing the Bishop Giuseppe Orlandi (1713–1776). Orlandi put De Rubertis in contact with notable people of the city and helped him obtaining the chair of medicine of the University of Naples in 1753. For more on Mario Lama and the brothers Di Martino, see Ferrone, *Scienza, natura, religione*, 498–502, 514–517 and Claudia Addabbo, "The *Philosophiae Naturalis Principia Mathematica* in Naples," in *Reading Newton in Early Modern Europe*, ed. by Elizabethanne Boran and Mordechai Feingold (Leiden: Brill, 2017), 23–63, 32–42.

François Jacquier (1711–1788) in Rome – collaborators on the Geneva edition of Newton's *Principia*, alongside the Genevan Professor of Mathematics, Jean-Louis Calandrini (1703–1758).¹² Upon returning to Naples, Cominale continued his medical practice while also founding and directing his own school, where students were taught a broad range of scientific disciplines.¹³ He also held a position at the University of Naples. His commitment to science and education remained steadfast even after he returned to Ugento in 1770, where he continued his work until his death fifteen years later. Beyond his medical practice, Cominale devoted himself to natural philosophical research, often exploring its philosophical implications. During his time in Naples, he authored several works, though unfortunately, many of these works seem to have been lost.¹⁴

In recent decades, there has been a modest resurgence of interest in Cominale's critique of Newton's natural philosophy, particularly among Italian scholars. This renewed attention has led to the publication of several essays.¹⁵ As we will see, Cominale's *Anti-Newtonianismi* is often portrayed as an isolated, provincial, and misguided attempt to challenge Newton's theories.

Giulia Belgioioso, focusing primarily on the first volume of Cominale's *Anti-Newtonianismi* – a volume devoted to an extended critique of Newton's optics – claims that it reveals Cominale's "backwardness, combined with arrogance and stubbornness."¹⁶ She writes: "The topic of the four volumes of his muddled work [are] [...] the scientific theories of *Opticks* and the *Principia*. [...] It is a work where a hypothetical and speculative science is proposed, [...] The outcome is a confused argument which is not always easy to follow."¹⁷ Belgioioso further characterizes *Anti-Newtonianismi* as "a late [and hence be-

¹² Cominale describes his travels in *Praelectio Academica de lentibus quae congregant, et segregant luminis radios, earumque effectibus habita in Regio Archigymnasio Neapolitano a Caelestino Cominale Physices, et Matheseos extra ordinem Professore. In petitione cathedrae physices experimentalis IV. Idus Septembris Ann. MDCCCLIX* (Naples: Typis Francisci Morelli, 1759), iv. On the Geneva edition of the *Principia*, see e.g. Niccolò Guicciardini, "Editing Newton in Geneva and Rome: The Annotated Edition of the *Principia* by Calandrini, Le Seur and Jacquier," *Annals of Science* 72 (2015): 337–380.

¹³ Lezzi, "Vite de' letterati salentini," 563.

¹⁴ A complete list of Cominale's works can be found in Lezzi, "Vite de' letterati salentini," 565; Maggiulli, "Dizionario biografico degli uomini chiari di Terra d'Otranto," n. 209.

¹⁵ Belgioioso, *Cultura a Napoli e cartesianesimo*, 399–441; Gabriella Guerrieri, "Newtonianesimo e antinewtonianesimo in Terra d'Otranto nella seconda metà del '700. Note in margine a un manoscritto inedito di Oronzo Amorosi," *Physis* 37 (2000): 535–542; and, Irene Giannì, "Le aberrazioni della *libertas philosophandi*: Celestino Cominale contra Isaac Newton," in *Filosofia e scienza nel Salento dell'età moderna*, ed. by Adele Spedicati (Lecce: Pensa multimedia, 2021), 63–121.

¹⁶ Belgioioso, *Cultura a Napoli e cartesianesimo*, 5–6: "ne svelano impietosamente tutta l'arretratezza, non disgiunta da presunzione e pertinacia."

¹⁷ *Ibid.*, 403: "Contenuto dei quattro tomi della sua farraginosa opera [sono] [...] le teorie scien-

nighted] intervention,” appearing at a time when Newton’s ideas had already gained significant traction within Italian scientific and academic circles.¹⁸ She concludes that both the title and the objective of the work – namely, to refute Newton’s natural philosophy – are “pretentious.”¹⁹

Gabriella Guerrieri cautions that “it would be misleading [...] to consider Celestino Cominale an exemplary and characteristic figure of Apulian Newtonianism; as his position appears antithetical not only compared to the Neapolitan atmosphere, but also, more specifically, to that in Salento.”²⁰ Furthermore, she argued that although Cominale “believes he is providing indisputable proof of the weakness of this [i.e., Newton’s] universally accepted theory,”²¹ his critique is marred by fundamental misconceptions and a lack of acuity in both physics and mathematics.

Irene Gianni, on the other hand, challenges such dismissive and anachronistic readings of Cominale’s *Anti-Newtonianismi*. She argues that Cominale’s work should be understood as the product of a scholar operating within a broadly speaking Descartes-inspired framework, who employed “an adversarial method” in his critique of Newton’s *Principia* and *Opticks* – an interpretation with which we concur.²² She maintains that Cominale grounded his critique of Newton’s natural philosophy in the principle of *libertas philosophandi*, which he invoked to challenge what he perceived as the uncritical acceptance of Newton’s doctrines.²³ Cominale’s critique of Newton’s natural philosophy is indeed anchored in two guiding principles: *libertas philosophandi* and epistemological humility. The latter stems from the recognition that “everyone could discover a few things in physics but not fully examine everything.” For this reason, Cominale found it reasonable “not to acquiesce entirely to the opinions of any single philosopher, but to go through them collectively; and to select from all what seemed true and more probable, and to put together for myself an eclectic physics.”²⁴ He further remarked: “I judged that authorities should not be trusted

tifiche dell’*Optica* e dei *Principia*. [...] Si tratta di un’opera dove viene proposta una scienza che ha carattere ipotetico e congetturale, [...] Ne viene fuori un discorso confuso che non è sempre facile seguire.”

¹⁸ Ibid., 403.

¹⁹ Ibid., 399.

²⁰ Guerrieri, “Newtonianesimo e antinewtonianesimo in Terra d’Otranto,” 537: “Sarebbe fuorviante [...] considerare Celestino Cominale figura esemplare e caratteristica del newtonianesimo pugliese; giacché la sua posizione risulta antitetica non solo all’ambiente napoletano, ma anche specificatamente a quello salentino.”

²¹ Ibid., 538: “crede di fornire le prove inoppugnabili della debolezza di questa teoria universalmente accettata.”

²² Gianni, “Le aberrazioni della *libertas philosophandi*,” 76–77.

²³ Ibid., 88–89.

²⁴ Cominale, *Anti-Newtonianismi pars secunda*, 30: “Quare, cum nonnulla potuerit quisque in physicis invenire, non omnia vero perlustrare; ideo placuit mihi, nullius peculiaris philosophi sen-

too much; and that, to the contrary, one should not fight for them [...], as the Newtonians fight for Newton today.”²⁵

At the beginning of the eighteenth century, Descartes's theories gained wide-spread acceptance in southern Italy, particularly in the Neapolitan area, which soon emerged as a major intellectual hub for the study of Descartes's philosophy in the period thereafter,²⁶ when “his [i.e., Descartes's] fortune varied according to the different moments and contexts of the Neapolitan culture.”²⁷ Despite the inclusion of Descartes's works in the *Index librorum prohibitorum* in 1663, various schools of thought and interpretation of Descartes's doctrines flourished. Descartes's philosophy became a vehicle to criticize Aristotle's thought and the Catholic Church in the name of *libertas philosophandi*.²⁸ However, with the advent of Newton's theories and the ensuing debates, Descartes's philosophy (and its many branches) fell out of favor. By the middle of the century, “throughout Europe, the discussion between Newton's followers and Descartes's epigones was rapidly drawing to a close, with the Newtonian synthesis emerging victorious.”²⁹ When Cominale published the first volumes of *Anti-Newtonianismi*, Descartes's ideas were mostly abandoned in favor of Newton's in the Neapolitan area.

With regard to Cominale's critique of Newton's natural philosophy, scholarly attention has largely focused on his objections to specific concepts, arguments and experiments within Newton's natural philosophy.³⁰ What remains absent, however, is a comprehensive account that outlines the principal lines of Cominale's critique of Newton's natural philosophy. To address this, we turn to the opening chapters of the second volume of Cominale's *Anti-Newtonianismi*, where he articulates his principal objections to Newton's natural philosophy. In these opening chapters, Cominale criticized the foundational aspects of Newton's natural philosophical method. Cominale contended that Newton's natural philosophical method – particularly his mathematization of nature – is fundamentally flawed. He also presents a pointed critique of Newton's renowned *regulae philosophandi*, arguing

tentiae ex toto acquiescere, sed cunctos percurrere; & ab omnibus, quae vera, & probabiliora videbantur, seligere, & Physicam Eclecticam mihi comparare, [...].”

²⁵ Ibid., 30: “Auctoritatibus non nimis fidendum esse censui; & pro iis non secus [...] pugnare, ut hodie pro Newtono pugnant Newtoniani; [...].”

²⁶ Ferrone, *Scienza, natura, religione*, 465; Casini, *Newton e la coscienza europea*, 191.

²⁷ Maria Teresa Marcialis, “Il ‘cogito’ e la coscienza. Letture cartesiane nella Napoli settecentesca,” *Rivista di Storia della Filosofia* 51 (1996): 581–612, 581: “la sua fortuna variò in momenti e contesti diversi della cultura napoletana.”

²⁸ Ferrone, *Scienza, natura, religione*, 465.

²⁹ Belgioioso, *Cultura a Napoli e Cartesanesimo*, 167: “in tutt’Europa la disputa fra i seguaci di Newton e gli epigoni di Cartesio si avviava a rapida conclusione con la vittoria della sintesi newtoniana.”

³⁰ Belgioioso, *Cultura a Napoli e Cartesanesimo*, 406–440; Guerrieri, “Newtonianesimo e anti-newtonianesimo in Terra d’Otranto,” 538–542.

that they are inadequate as guiding principles for natural philosophical inquiry. The next section examines Cominale's critique of Newton's mathematization of nature, while the subsequent section explores his critique of the *regulae philosophandi*.

On the progress of physics and its obstacles

In the first chapter of *Anti-Newtonianismi pars secunda*,³¹ Cominale set out show that “the progress of physics has been impeded, and that this is above the result of the abuse of geometry” – a trend he attributed to Newton and his followers. This statement, he acknowledged, contradicts the “vulgar opinion,” that “physics has been brought to its highest perfection and peak of its evidence chiefly by the help of geometry.”³² He noted that the “Aristotelians, atomists, and Cartesians” all claimed to have perfected physics, yet

³¹ Most contemporary assessments of the second part of Cominale's *Anti-Newtonianismi* were unfavorable. One notable example is Oronzo Amorosi, *Gara letteraria inedita tra i Signori Oronzo Amorosi di Galatone e Celestino Cominale di Uggiano della Chiesa. Copiata dall'autografo di esso Amorosi da Vincenzo Lillo, 1821. Introduzione, trascrizione e note a cura di Gabriella Guerrieri* (Lecce: Conte, 1999). The letters exchanged between Amorosi and Cominale in 1765 remained unpublished for over a century until they were published in the *Gara letteraria*. Initially, Amorosi pointed out several errors in Cominale's *Anti-Newtonianismi*, but the correspondence quickly developed into a heated exchange of accusations and insults. In a particularly scathing presentation letter, Amorosi wrote (p. 53): “What is the purpose of embellishing so many pieces of paper to create such a long and varied mess, if with many deceits and things he haphazardly collects, he does not untangle anything of this feigned, awful, and inappropriate learning? [Ma a che scicchierar tanti fogli a formar sì lungo, e sì svariato ciloma, se con tante anfanie e con tante cose che v'affastella d'un erudizione [sic] affettata, indigesta e fuor di proposito che non ne discioglie egli alcun nodo?].” Another critical voice appeared in [anon.], “Italy,” in *The Library: Or, Moral, and Critical Magazine, for the year MDCCLXII. By a Society of Gentlemen*, ed. by Andrew Kippis, vol. 2 (London: printed for R. Griffiths in the Strand, and C. Henderson under the Royal-Exchange, 1762), 273–276, at 275, where the reviewer asserted that Cominale's work “pleads against him.” A particularly pointed assessment appeared in a review of *Anti-Newtonianismi pars secunda*, in Pierre Rousseau, ed., *Journal encyclopédique*, vol. 4, pt. 3 (Bouillon: De l'Imprimerie du Journal, 15 June 1763), 137–138, where the reviewer wrote: “Il y a de la témérité à vouloir attaquer Newton. Ce nouveau Physicien veut absolument renverser le système de Philosophe Anglois; il ne se propose rien moins que de flétrir la gloire de cet homme immortel. Il veut combattre le système des couleurs de Newton; nous doutons qu'il y réussisse.” A rare positive appraisal, however, can be found in Abate Giovanni Antonio Battarra, “Articolo d'una Lettera scritta dal sig. Abate Gio. Antonio Battarra di Rimino ad un suo Amico di Firenze il dì 22. Luglio 1760,” in *Novelle letterarie pubblicate in Firenze*, ed. by Giovanni Lami, vol. 21 (Firenze: nella stamperia di Gaetano Albizzini, 1760), 570–573.

³² Cominale, *Anti-Newtonianismi pars secunda*, 1: “Physices [...] progressus esse praepeditos; & id potissimum, Geometriae abusu, factum”; “Physicam [...] Geometriae potissimum auxilio ad summum perfectionis, ac evidetiae suae apicem esse perductam.”

their views have been discarded for their lack of substance. Cominale argued that “today’s [physics]” [i.e., that of Newton and its followers] will “similarly be abandoned, growing old, and time, the impartial detector of truth, will expose its errors to all.”³³

Although Cominale asserted that physics had not made significant progress, he was acutely aware of advancements in certain areas. To address this, he introduced a distinction between historical physics (*physica historica*) and theoretical and speculative physics (*physica theórica & speculativa*). He acknowledged significant advancement in historical physics, highlighting notable advancements in chemistry and the natural history of insects, referencing the pioneering work of Francesco Redi (1626–1697), Jan Swammerdam (1637–1680), Antonio Vallisneri (1661–1730) and René-Antoine Ferchault de Réaumur (1683–1757). He also noted progress in the natural history of plants, citing the work of Robert Morison (1620–1683, Paul Hermann (1646–1695), Augustus Quirinus Rivinus (1652–1723) and John Ray (1627–1705). Further, he highlighted the growth of in physico-theology through the writings of William Derham (1657–1735) and Bernard Nieuwentijt (1654–1718), as well as notable progress in anatomy.³⁴

In contrast, Cominale asserted that “speculative physics, or that [part of physics] that strives to give the causes of phenomena, has not only failed to make progress since its inception, but it seems to have collapsed into the futile solutions of the ancients.”³⁵ He illustrated this with examples from astronomy and optics, specifically referencing the explanations he believed Newton had offered in the *Principia* and the *Opticks*. Regarding astronomy, he acknowledged that the history of the heavens (*caelestis historia*) has progressed, discovering “that the heavens are not solid, as the ancients feigned; that the Earth is not the center of our system, and that the planets rather revolve around the Sun.”³⁶ However, he argued that “when theoretical [physics] [i.e., Newton’s celestial mechanics] advances to explain the cause of the motions of the celestial bodies, it is observed to be more confused than the ancient [theoretical physics], with its epicycles and solid rotating heavens.”³⁷ Cominale argued that Newton’s theory of the motion of the celestial bodies is inconsistent, because it “contends that matter is inert and lacking any intrinsic active prin-

³³ Ibid., 1: “hodierna consenescent, & tempus, aequus veri detector, omnibus errores suos ostendens, defectura pariter erit.”

³⁴ Ibid., 2.

³⁵ Ibid.: “Physica speculativa, sive quae phaenomenorum rationem reddere nititur, non modo novos progressus, ut incepterat, minime fecit; sed in vanas, ac fuitiles Antiquorum resolutiones collapsa videtur.”

³⁶ Ibid.: “Coelos minime solidos esse, ut finxerunt Antiqui: Terram non esse systematis nostri centrum, & Planetas potius circa solem revolvi.”

³⁷ Ibid.: “quum Theorica motuum corporum Coelestium explicare aggreditur causam confusa magis, quam Antiqua, cum epicyclis, suisque solidis rotantibus Coelis observatur.”

ciple; and then wants to give this same matter an intrinsic attractive force.”³⁸ It is important to note that Newton himself did not endorse the view that gravity is an intrinsic force of matter. In Query 28, he explicitly suggested that gravity should be attributed to “some other Cause than dense Matter,” and in the advertisement to the 1717 edition of the *Opticks* he denied having ever regarded gravity as “an essential Property of Bodies.”³⁹

Newton’s *Opticks* provides a similar case. While it, according to Cominale, correctly concluded that the sun’s refracted light produces seven colors through a prism, it “falsely teaches that the same light is composed of seven different types of rays,” failing to explain it “according to the true mechanical laws, by which nature operates.” Instead, it posits “some instinct, or innate attractive or repulsive force, or any other inclination created as necessity urges.”⁴⁰ Cominale concluded: “Is this, I ask, a way of clarifying and mechanically explaining the actions of nature, or rather rejecting nothing beneath [a veil of] arcane and empty words? Therefore, it is very true that in these times, theoretical physics has relapsed into the vain absurdities of the ancients [namely, the reintroduction of occult qualities].”⁴¹

In the third chapter of *Anti-Newtonianismi pars secunda*, Cominale commented on Newton’s natural philosophical explanations. He criticized what he saw as a fundamental inconsistency: on the one hand, Newton and his followers “despise primary causes and the first principles of things,” as evidenced by Newton’s self-proclaimed ignorance of the cause of gravity in the *Principia*.⁴² On the other hand, they seem to elevate effects into causes.⁴³ In support of this latter point, Cominale cited Louis-Bertrand Castel’s (1688–1757) critique of Newton’s theory of the heterogeneity of white light, as presented in his *Optique des couleurs* (1740):

³⁸ Ibid., 3: “Decernit materiam inertem esse, omnique intrinseco activo principio destitutam; & dein hanc eandem materiam vi intrinseca attrahente donari vult.” Cf. ibid., 27–28.

³⁹ Isaac Newton, *Opticks or a Treatise of the Reflections, Refractions, Inflexions & Colours of Light* (New York: Dover, 1952), 369, cxxiii. For further discussion, see Steffen Ducheyne, “Newton on Action at a Distance,” *Journal of the History of Philosophy* 52 (2014): 675–702.

⁴⁰ Cominale, *Anti-Newtonianismi pars secunda*, 3: “Veridica demonstrat historica, Solis lucem prismate refractam septem colores producere. At falsò nimis docet Theorica, ex septem diversis radiorum speciebus eandem esse compositam. [...] non idem speculativa juxta verae mechanices leges, quibus operatur natura, explanare aggreditur; sed statim illud, aut instinctu quodam, aut vi innata attrahente vel repellente, aut alia quavis inclinatione, ut necessitas urget, creata explicat.”

⁴¹ Ibid., 3: “Est ne quaeso hicce modus enucleandi, ac mechanice explicandi naturae actiones, aut potius sub arcanis, vacuisque vocibus nihil explodere? Verissimum ergo est, hisce temporibus, Physicam Theoricam in vanas Antiquorum ineptias esse relapsam.”

⁴² Isaac Newton, *The Principia: Mathematical Principles of Natural Philosophy*, transl. I. Bernard Cohen and Anne Whitman (Berkeley (CA): University of California Press, 1999), 943.

⁴³ Cominale, *Anti-Newtonianismi pars secunda*, 26: “Spernunt Newtoniani, a Magistro edocti, causas primas, primaque rerum principia [...]”; 29: “quod erigit effectus in causas.”

It is a known fact that he [i.e., Newton] always stops at the phenomena, turns them over, notes them, enriches them with a thousand parallel, investigated, curious, and marvelous phenomena, which he immediately erects into causes and principles; refraction into refrangibility, color into colorability, redness into rubrification, gravity into gravitation, the central into centripetal, the accelerated into accelerative, tendency into attraction, and here, specifically, the spectrum into reality.⁴⁴

At the heart of Cominale's criticism of Newton's natural philosophy lies his analysis of how Newton, in his view, misused mathematics in theoretical physics. Cominale readily embraced the usefulness of mathematics in advancing physics. "Geometry," he observed, "which considers all extension and compares and estimates magnitudes under various figures, unfolds easy ways to achieve knowledge of various proportions that exist between different parts of matter, and to measure various magnitudes."⁴⁵ However, he remarked that physicists nowadays "seek to derive physics from geometry and not at all urge geometry to serve physics, as things should be."⁴⁶ In this context, he claimed that it is highly ironic that in his *An Account of Sir Isaac Newton's Philosophical Discoveries* (1748) the Scottish mathematician Colin Maclaurin (1698–1746), "Newton's pupil" and his ardent defender, cited Francis Bacon's (1561–1626) perceptive claim that "*mathematics must round off natural philosophy, and not generate or create it*," failing to observe "that Newton himself had forced geometry to serve natural philosophy."⁴⁷ Despite the usefulness of mathematics in

⁴⁴ Ibid., 26–27, quoting Louis-Bertrand Castel, *L'optique des couleurs* (Paris: Chez Briasson, 1740), 381: "Il s'arrête toujours, c'est un fait connu, aux phaenomenes, les retourne, les constate, les enrichit de mille phaenomenes paralleles, recherchés, curieux, merveilleux, qu'il erige tout de suite en causes, & en principes; la réfraction en réfrangibilité, la couleur en colorabilité, la rougeur en rubrification, la gravité en gravitation, le central en centripete, l'acceleré en acceleratif, la tendance en attraction, & ici nommement, le *spectre* en réalité." On the French Jesuit Castel, who was involved in the *Mémoires de Trévoux*, see Jean-Olivier Richard, "The Art of Making Rain and Fair Weather: The Life and World System of Louis-Bertrand Castel, SJ (1688–1757)" (PhD diss., The John Hopkins University, 2015).

⁴⁵ Ibid., 3: "Geometria enim, quae extensum omne considerat, & magnitudines sub variis figuris comparat, ac metitur, facillimas pandit vias ad varias proportionones dignoscendas, quae inter diversas materiae partes existunt, variasque magnitudines dimetiendas."

⁴⁶ Ibid., 4: "ex geometria physicam educere contendunt; & minime physicae geometriam inservire cogunt, ut revera se res habere deberet."

⁴⁷ Colin Maclaurin, *An Account of Sir Isaac Newton's Philosophical Discoveries in Four Books* (London: Printed for the Author's Children, 1748), 35: "*Mathesin philosophiam naturalem terminare debere, non generare aut procreare*"; Francis Bacon, *The Oxford Francis Bacon*, ed. by Graham Rees with Maria Wakely, vol. 11 (Oxford: Clarendon Press, 2012), 155; Cominale, *Anti-Newtonianismi pars secunda*, 4: "non observabat, Newtonum ipsum geometriae philosophiam naturalem inservire coegisse." For a discussion of Bacon's view on the role of mathematics in physics, see e.g. Giuliano Mori, "Mathematical Subtleties and Scientific Knowledge: Francis

advancing physics, Cominale warned that “where it is poorly applied, it has the power to produce the most fertile errors, as Louis-Bertrand Castel cautions.”⁴⁸ Cominale stated that the application of mathematics to nature, “due to the solidity and evidence of its demonstrations,” seems to be a privileged and certain method in physics. However, he continued, “its abuse has reached the point where it has lost most of its evidence,” resulting in “mathematicians being no less entangled in their demonstrations than physicists are regarding natural philosophical questions.”⁴⁹

To distinguish between proper and incorrect applications of mathematics in physics, Cominale, indebted to the Dutch scholar Nieuwentijt, differentiated between pure mathematics and mixed mathematics.⁵⁰ While Nieuwentijt himself regarded Newton’s natural philosophical work as exemplary cases of mixed mathematics, Cominale viewed it instead as exercises in pure mathematics. Pure mathematics, Cominale explained, builds on abstract ideas, that “not only never existed outside the intelligible world or the world of ideas, but also will never exist.” Mixed mathematics, on the other hand, is based on ideas “that really exist outside the intelligible or intellectual world.”⁵¹ Regarding pure mathematics, Cominale wrote:

Bacon and Mathematics, at the Crossings of Two Traditions,” *The British Journal for the History of Science* 50 (2017): 1–21.

⁴⁸ Cominale, *Anti-Newtonianismi pars secunda*, 4: “ubi male applicatur errores foecundissimos progignere valet, ut admonet Benedictus Castellus.” For relevant excerpts, see Castel, *L’optique des couleurs*, 370–407, esp. 386–387: “Car voila l’inconvenient de la géometrie & de la justesse du raisonnement, de rendre l’erreur féconde & systematique.”

⁴⁹ Cominale, *Anti-Newtonianismi pars secunda*, 4: “Mathemata ob soliditatem ac evidentiam suarum demonstrationum prae caeteris facultatibus scientiae nomen sibi vindicasse videntur. At nostra hac tempestate eo devenit abusus, ut evidentiae suae maximam partem amiserint. Quoniam saepe saepius mathematicos in suis demonstrationibus implexos videmus non minus, quam physicos in naturalibus quaestionibus.”

⁵⁰ He referenced the French translation of Nieuwentijt’s, *Het regt gebruik der werelt beschouwingen, ter overtuiging van ongodisten en ongelovigen* (Amsterdam: By de Wed. J. Wolters, en J. Pauli, 1715). For a discussion of Nieuwentijt’s distinction between pure and mixed mathematics, see Steffen Ducheyne, *Physics in Minerva’s Academy. Early to Mid-Eighteenth-Century Appropriations of Isaac Newton’s Natural Philosophy at the University of Leiden and in the Dutch Republic at Large, 1687–c.1750* (Leiden: Brill, 2025), 125–132. The terms ‘pure mathematics’ and ‘mixed mathematics’ were introduced previously by Willem Jacob ’s Gravesande in 1719 (*ibid.*, 126). It should be noted that Bacon, whom Cominale, as we have seen, cited, already distinguished between pure and mixed mathematics (for a discussion, see Gary I. Brown, “The Evolution of the Term ‘Mixed Mathematics,’” *Journal for the History of Ideas* 52 (1991): 81–102, 81–83).

⁵¹ Cominale, *Anti-Newtonianismi pars secunda*, 4: “Alterum ideæ constituunt, quas ipsi ut meras ideas abstractas considerant, quaeque non modo nunquam extra Mundum intelligibilem, sive Idearum extiterunt: sed nec etiam existent. Alterum obiectum constituunt ideae, quae realiter extra Mundum intelligibilem sive intellectualem existunt.” The terms ‘pure mathematics’ and ‘mixed mathematics’ are Nieuwentijt’s, not Cominale’s.

The first ideal and abstract mode of philosophizing is observed in speculative geometry, of which Euclid's *Elements*, Archimedes's *The Method of Mechanical Theorems*, Theodosius of Bithynia's *Spherics*, and Apollonius' of Perga's *Conics* are examples, Algebra, and other similar [branches of mathematics]. In these, a point is conceived as a thing without parts, a line as a length without breadth, and a solid as penetrable; and in algebra, magnitudes or solids bestowed with more than three dimensions are proposed and examined. Yet these are abstract modes of conceiving things that never existed in the corporeal world. However, they are granted to geometers to solve and demonstrate their wonderful problems and theorems.⁵²

To illustrate the difference between pure and mixed mathematics, Cominale referenced a quotation drawn from Pierre Gassendi's (1592–1655) *Syntagma philosophicum* (1658), according to which in pure geometry mathematicians have created “a kind of most free kingdom for themselves” by “abstracting quantity from matter” and supposing lines created by the ‘flowing’ of a point, lines lacking length and breadth, and the like.⁵³ In an embellishment that appears like it is part of the quotation from Gassendi but is not, Cominale added that while such assumptions is perfectly acceptable for pure mathematicians operating “in their kingdom of abstraction (*regno abstractionis*),” they are not permissible for physicists, who operate “in kingdom of matter (*regno materiae*).”⁵⁴ In the 1740s, Castel, to whom Cominale is indebted, contended that Newton mistakenly mixed mathematics

⁵² Ibid., 4–5: “Primus idealis, & abstractus philosophandi modus observatur in Geometria speculativa, cujus generis sunt, Elementa Euclidea, Theoremata Archimedis, Sphaerica Theodosii doctrina, Sectiones conicae Apollonii Pergaei, Algebra, aliaque similia; In quibus concipitur punctum tamquam res, cujus nullae sunt partes, linea, ut longitudo latitudinis expers, & solidum penetrabile; & in Algebra proponuntur & examinantur magnitudines, sive solida pluribus, quam tribus dimensionibus donata. Verum hi modi sunt abstracti concipiendi res, quae in Mundo corporeo nunquam extiterunt. Attamen Geometris conceduntur, ut mira problemata, & Theoremata sua resolvant, & demonstrent.”

⁵³ Pierre Gassendi, *Opera omnia*, vol. 1 (Lyon: Sumptibus Laurentii Anisson, & Ioan. Bapt. Devenet, 1658), 264b: “inprimisque Geometrae, quantitatem abstrahentes à materia quoddam quasi regnum sibi ex ea fecerunt quam-liberrimum, quippe nullo facto à materiae crassitie, pertinaçiaeque impedimento. Quare & supposuere inprimis in ea sic abstracta eiusmodi dimensiones, ut punctum, quod foret prorsus immune partibus fluendo lineam, longitudinemve latitudinis expertem crearet; [...]. [...] Atque istae quidem suppositiones sunt, ex quibus mathematici intra purae, abstractaeve Geometriae cancellos, & quasi regnum consistentes suas illas praeclaras demonstrationes texunt, quasdamve adeo mirabiles, ut fidem omnem alioquin excessurae sint, ut nuper viri praeclari *Cavalierius*, & *Torricellius* ostenderunt de acuto quodam solido infinite longo, & cuiuspiam tamen parallelepipedo cylindrove finito aequali.”

⁵⁴ Cominale, *Anti-Newtonianismi pars secunda*, 5: “uno igitur verbo Mathematici sunt, qui in suo illo abstractionis regno ea indivisibilia supponunt, quae sine partibus, sine longitudine, sine latitudine sint, ac eam multitudinem divisionemque partium, quae ad finem nunquam perveniat. Non item vero physici, quibus in regno materiae versantibus tale nihil licet.”

and physics, thereby blurring the line between mathematical and physical explanations.⁵⁵ According to Cominale, Newton's mathematical program is misguided because it mixes "the kingdom of abstraction" and "the kingdom of matter":

Let the moderns ponder this question and let them see how well they align with Newton's *Principia* and how this method inadequately blends ideal entities with real ones; [...]. For whoever wishes to philosophize in physics in this manner, is one not easily led to praise ideal entities for real ones? [...] In the meantime, who does not see that philosophizing with these ideal principles is the same as wrongly and excessively introducing ideal entities into real physics, which do not exist in nature?⁵⁶

By way of example, Cominale mentioned that followers of Newton, such as the Scottish mathematician and natural philosopher John Keill (1671–1721), have argued that matter is infinitely divisible based on purely mathematical arguments.⁵⁷ This conclusion was unacceptable for Cominale and he stated that this would imply, as Antonio Genovesi (1712–1769), "the glory of our city [i.e., Naples]" and a Naples-based follower of Newton, has observed, that "bodies [...] would be unions of modes without substance."⁵⁸

Next, Cominale presented a number of detailed arguments highlighting the shortcomings of Newton's mathematical approach to physics. He criticized Newton because

⁵⁵ For a discussion of Castel's critique of Newton's *Principia*, see Yves Gingras, "What did Mathematics do to Physics?," *History of Science* 39 (2001): 383–416, 398–403.

⁵⁶ Cominale, *Anti-Newtonianismi pars secunda*, 7–8: "Perpendant quaeso haec Recentiores, videantque, quam bene hisce consonent Principia Mathematica Philosophiae naturalis Newtoni; & quomodo methodus haec male entia idealia cum realibus miscet; [...] qui enim hac in physicis philosophari voluerit, nonne facile entia idealia pro realibus venditare inducitur? [...] Interim quis non videt, quod idealibus hisce principiis philosophari idem sit, ac in physica reali male nimis entia idealia introducere, quae in natura minime existunt?"

⁵⁷ John Keill, *Introductio ad veram physicam seu lectiones physicae* (Oxford: Impensis Thomae Bennet, 1702), 17–27.

⁵⁸ Cominale, *Anti-Newtonianismi pars secunda*, 9: "quod corpora, ut urbis nostrae decus Antonius Genuensis docte nimis admonet, essent uniones modorum absque substantia." The original statement occurs in Antonio Genovesi, *Disciplinarum metaphysicarum elementa, Mathematicum in morem adornata*, vol. 1 (Venice: Ex typographia Remondiniana, 1764), 183: "Si corpus quodlibet est physicòs in absolutum infinitum divisibile, corpus quodlibet est unio modorum sine prima substantia." On Genovesi, see e.g. Paola Zambelli, "Antonio Genovesi and Eighteenth-Century Empiricism in Italy," *Journal of the History of Philosophy*, 16 (1978): 195–208; Ferrone, *Scienza, natura, religione*, 609–641 and Addabbo, "The *Philosophiae Naturalis Principia Mathematica* in Naples," 43–45. On his physical work, see Immacolata Bergamasco and Maria Lippiello, "Il libro di Antonio Genovesi sulla fisica," in *Atti del XXIII Nazionale di storia della fisica e dell'astronomia, Università degli Studi di Bari, Dipartimento di Fisica, Bari, 5–7 giugno 2003*, ed. by Pasquale Tucci, Augusto Garuccio and Maria Nigro (Bari: Progedit, 2004), 61–82.

he “abstracts forces from acting bodies, or considers them merely mathematically, since he desires to perfect mathematics alone.”⁵⁹ To support his critique, Cominale selectively quoted several isolated passages from the *Principia*.⁶⁰ He referenced Newton’s statement in the *Praefatio*, where Newton asserted that “it seemed best in this treatise to concentrate on *mathematics* as it relates to natural philosophy.”⁶¹ Additionally, Cominale quoted comment in the scholium to Section 11 of Book I, which states that “Mathematics requires an investigation of those quantities of forces and their proportions that follow from any conditions that may be supposed.”⁶² To further illustrate Newton’s disregard for physical causes, Cominale cited Definition 5, which claims that “*Centripetal force is the force by which bodies are drawn from all sides; are impelled, or in any way tend, toward some point as to a center.*”⁶³ He also referenced the introductory text to Section 11 of Book I, where Newton writes: “For this reason I now go on to set forth the motion of bodies that attract one another, considering centripetal forces as attractions, although perhaps – if we speak in the language of physics – they might be more truly called impulses. For here we are concerned with mathematics; and therefore, putting aside any debates concerning physics, we are using familiar language so as to be more easily understood by mathematical readers.”⁶⁴ Finally, he quoted the opening section to Book III, where Newton stated: “In the preceding books I have presented principles of philosophy that are not, however, philosophical but strictly mathematical – that is, those on which the study of philosophy can be based.”⁶⁵ From these quotations, Cominale arrived at the following two conclusions:

From the principles of Newton just explained, two things can be concluded. First, that Newton delivers the mathematical or abstract principles of natural philosophy rather than the real ones. Therefore, Newtonian physics will be merely intellectual or ideal. Furthermore, by this method, Newton compels geometry to serve physics in a repugnant manner. And since from ideal principles only ideal things and conclusions may flow, Newtonian physics will be an empty phantom, which perhaps will not exist outside the intelligible world: and thus, under Newtonian mathematics, by the abuse of geometry, ideal physics has been restored, akin to [its state] under the Peripatetic metaphysics in the past. In this [respect] in particular, if my opinion does not fail me, physicists should strive to form the intelligible world from observations of and the most exact experiments on bodies; certainly not

⁵⁹ Cominale, *Anti-Newtonianismi pars secunda*, 9: “Abstrahit Newtonus ab agentibus corporibus vires, sive mathematicae tantum illas considerat, quoniam mathesin tantum excolere cupit.”

⁶⁰ *Ibid.*, 9.

⁶¹ Newton, *The Principia*, 381.

⁶² *Ibid.*, 588.

⁶³ *Ibid.*, 405.

⁶⁴ *Ibid.*, 561.

⁶⁵ *Ibid.*, 793.

to erect the corporeal from the intelligible, as the Newtonians do, whilst they try to erect physics from the bosom of ideal geometry. And that these things are very true, I observe secondly, that Newtonians everywhere change abstract terms into real entities. They use the term ‘attraction’ abstractly, as the effort by which bodies tend either towards each other or towards some center; nor do they investigate or determine whether this effect depends on an extrinsic impelling force or on an intrinsic force. Subsequently, they exalt those abstract forces or effects into causes and assert that bodies truly have an innate attractive force, which are rather forces of impulses, as Newton himself teaches.⁶⁶

According to Cominale, this represents a reversal of the proper method in physics, wherein historical physics, i.e. the exact description of phenomena, should precede theoretical physics, which provides the causes of these phenomena.⁶⁷ Cominale was not the first to argue that Newton’s *Principia* is ideal rather than physical. Castel – whom Cominale cited – similarly observed that Newton relied on “purely ideal, abstract, and mathematical causes, that have nothing to do with the idea of cause and physical, effective, and operative influence.”⁶⁸

To elaborate on his critique of Newton’s mathematized physics, Cominale discussed how the renowned Cantabrigian inadequately applied the notion of ‘common center of

⁶⁶ Cominale, *Anti-Newtonianismi pars secunda*, 9–10: “Ab expositis modo Newtoni placitis duo colligere datur. Primo Newtonum principia mathematica, sive abstracta philosophiae naturalis tradere potius, quam realia. Erit igitur Physica Newtoniana mere intellectualis, sive idealis. Insuper Newtonus hac methodo Geometriae physicam turpe inservire cogit. Et cum ex principiis idealibus nonnisi res, & conclusiones ideales profluere possint; erit physica Newtoniana inane Phantasma, quod extra mundum intelligibilem forsan non existet: & en quomodo sub mathematicis Newtonianis, abusu Geometriae, physica idealis reddita est, non secus ac sub methaphysicis Peripateticis fuit olim. In eo praecipue, ni mea me fallit opinio, incumbere debent physici, ut mundum intelligibilem observationibus, ac experimentis exactissimis ex corporeo efformarent; minime vero ex intelligibili corporeum educere, ut Newtoniani faciunt, dum è sinu idealis Geometriae physicam educere contendunt. Et quod verissima haec sint, observo secundo, Newtonianos passim voces abstractas in entia realia commutare. Vocem attractionis usurpant abstracte, ut conatum illum, quo corpora, vel ad se invicem, vel ad aliquod centrum, tendunt; neque quaerunt, neque decernunt, num effectus ille dependeat a vi extrinseca impellente, an vero a vi intrinseca. Deinde abstractas vires illas sive effectus exaltant in causas, & asserunt, corpora revera vim innatam attrahentem habere, quae vires potius impulsus sunt, ut Newtonus ipse docet.”

⁶⁷ Cf. Cominale, *Anti-Newtonianismi pars secunda*, 31: “In philosophando, physicam historicam, sive exactam phaenomenorum descriptionem Physicae Theoricae, quae phaenomenorum omnium rationem reddere, & causas exponere studet, praecedere feci.”

⁶⁸ Louis-Bertrand Castel, *Le vrai système de physique générale de M. Isaac Newton* (Paris: Chez Claude-François Simon, 1743), 99: “*raisons purement idéales, abstraites & mathématiques* qui ne portent avec elles aucune idée de Cause & d’influence physique, effective, opérative.”

gravity' to celestial bodies. Cominale observed that, when used legitimately, the notion of 'common center of gravity' between two bodies refers to the point on the lever (or any other medium) around which the two bodies equally exert their forces, when the centers of the two bodies are suspended from the ends of a lever (or are connected by any medium). He explained that, although both bodies are separate from each other, they will balance around a certain point as parts of a whole, forming a single body united by the lever (or another medium).⁶⁹ Newton and his followers misuse this concept when they mathematically consider the centers of gravity of, for example, the Earth and Moon. They "connect these centers with an ideal, namely mathematical line; and on this ideal line, they seek a point, found by the mind and calculation, which they commend as real, around which they want to balance the Earth and the Moon."⁷⁰ Furthermore, Cominale noted that "the Earth and the Moon, and the other planets, are highly distinct bodies, separate from each other, and united by no medium; because they float, as Newtonians please, in an immense vacuum; and so, they can in no way exert their forces around any common point."⁷¹

Newton's rules of philosophizing

In the second chapter of *Anti-Newtonianismi pars secunda*, Cominale offers a critique of Newton's first three *regulae philosophandi* – the foundational rules that underpin the theory of universal gravitation presented in Book III of the *Principia*.⁷² Newton's first rule and its commentary read: "No more causes of natural things should be admitted than are both true and sufficient to explain their phenomena. As the philosophers say: Nature does nothing in vain, and more causes are in vain when fewer suffice. For nature is simple and does not indulge in the luxury of superfluous causes."⁷³ Cominale observed that this rule comprises two distinct parts. The first aims to prevent "philosophy from transforming into vain inventions of opinions" by avoiding hypotheses.⁷⁴ Here Cominale referenced

⁶⁹ Cominale, *Anti-Newtonianismi pars secunda*, 10.

⁷⁰ Ibid., 11: "linea ideali, mathematica nempe, eadem centra conjungunt; & in hac linea ideali quaerunt punctum, & mente, calculoque inventum, tamquam reale, venditant; circa quod aequilibrari volunt Terram & Lunam."

⁷¹ Ibid., 11: "Lunam & Terram, caeterosque Planetas corpora esse distinctissima, & a se invicem disjuncta, nulloque medio unita; innatant enim, ut placet Newtonianis, in vacuo immenso; adeoque nullo pacto posse vires suas circa commune aliquod punctum exercere."

⁷² Because, as Cominale noted, the fourth rule is often omitted by Newton's followers, he chose to mention it without offering further discussion (ibid., 25).

⁷³ Newton, *The Principia*, 794.

⁷⁴ Cominale, *Anti-Newtonianismi pars secunda*, 12: "ne Philosophia in vana abeat opinionum commenta." Although Cominale did explicitly state it, this is a direct quotation taken from an annotation by Jean-Louis Calandrini in Isaac Newton, *Philosophiae naturalis principia mathematica* [...]. *Perpetuis commentariis illustrate, communi studio Thomae Le Seur & Francisci Jacquier*, vol.

an annotation to Newton's first rule by Calandrini, included in the Geneva edition of the *Principia* (1739–1742) that was produced by Le Seur and Jacquier. Calandrini wrote: “if we wish to philosophize with evidence and certainty, we should neglect all hypotheses; for a hypothesis, if legitimate, indeed provides the possibility of a cause, but not at all its existence, since the same effect can be produced in many ways.”⁷⁵ Cominale also referenced similar remarks by the newly appointed Leiden professor Pieter van Musschenbroek (1696–1761) in the French edition of his *Beginsels der natuurkunde* (1739),⁷⁶ as well as by Giovanni Maria Della Torre (1713–1782), a Naples-based follower of Newton.⁷⁷ Returning to Calandrini's annotation to the first *regula philosophandi*, Cominale highlights the second part of Newton's first rule, which prescribes that “no more causes of natural things are to be admitted than suffice to explain their phenomena.” This, Calandrini argued, is obvious, “since when the true cause of an effect has once been found through experience, and especially when it has been demonstrated with the help of mathematics that that cause has that power which is sufficient to produce the effect, it is clear, however, that any other cause is useless.”⁷⁸

Cominale's first objection to the first rule is that nature is not, in fact, simple and often produces similar effects through different causes. To substantiate his claim, he drew on examples from the generation of animals and plants. Regarding animal generation, he cited Herman Boerhaave's (1668–1738) *Praelectiones academicae*, which outlines three distinct modes of animal reproduction: through the union of male and female specimen; via a

3, pt. 1, 2 (Geneva: Typis Barrollet & Filii, 1739–1742; henceforth: *Principia*_{Geneva}), note (a).

⁷⁵ Cominale, *Anti-Newtonianismi pars secunda*, 13, quoting Newton, *Principia*_{Geneva}, vol. 3, pt. 1, 2, note (a): “si velimus cum evidentia ac certitudine philosophari; omnes hypotheses negligenda nobis sunt; hypothesis enim si legitima est, causae quidem possibilitatem, minimè verò existentiam adstruit, cum effectus idem pluribus modis produci possit.”

⁷⁶ Pieter van Musschenbroek, *Beginsels der natuurkunde beschreeven ten dienste der landgenooten* (Leiden: Samuel Luchtmans, 1739); Id., *Essai de physique*, transl. Pierre Massuet, vol. 1 (Leiden: Samuel Luchtmans, 1739), 14: “C'est pourquoi on ne doit pas raisonner par Suppositions; car il est clair, que dès que l'on suppose une Cause, on reconnoit n'être pas au fait de la véritable, puisqu'autrement il ne seroit pas nécessaire de faire des Suppositions. Toute conclusion, que l'on tire d'une Supposition, est bien éloigné de pouvoir être regardée comme Preuve, & nous laisse dans la même incertitude où nous étions auparavant.”

⁷⁷ Giovanni Maria Della Torre, *Scienza della natura*, vol. 1 (Naples: Appresso Raffaele Gessari, 1748), 4: “E per vero dire tolta di mezzo questa regola, dobbiamo introdurre immediatamente i possibili, già esclusi dalla Fisica.” On Della Torre, see Ugo Baldini, “Della Torre, Giovanni Maria,” in [various authors], *Dizionario biografico degli italiani*, vol. 37 (Rome: Istituto delle enciclopedia italiana), 573–577.

⁷⁸ Cominale, *Anti-Newtonianismi pars secunda*, 13, quoting Newton, *Principia*_{Geneva}, vol. 3, pt. 1, 2, note (a): “nam cum vera effectus causa per experientiam semel inventa est, & matheseos ope praesertim demonstratum est causae illius eam vim quae ad effectum producendum sufficiat, liquet aliam quamlibet causam esse inutilem.”

specimen possessing both male and female organs, which interact with the corresponding organs of another such specimen; and through hermaphroditic specimens capable of self-fertilization.⁷⁹ Reflecting on the third case, Cominale posed a rhetorical question, “Now, if the axiom were true, namely that more is in vain when less will do, would not human generation be deemed superfluous? For it could have been accomplished by lesser means.”⁸⁰ Cominale extended this argument to the plant kingdom, citing the reproduction of palms and acorns. He challenged Newton’s notion of the simplicity of nature: “If Newton understands nature herself as the totality of things,” he stated, “I deny that it is simple; is there anyone among mortals who could never perceive its diversity? Who could wish for its simplicity? Does not its variety delight, please, and demonstrate the power of the supreme deity?”⁸¹ Returning to Calandrini’s abovementioned annotation – which asserts that once a sufficient cause has been identified, all other causes are rendered useless – Cominale offered a pointed critique:

For has he [i.e., Calandrini] not taught himself that the same effect can be produced in many ways? And can it not be arisen from very different, even opposite causes, as we will see below?⁸² Why, therefore, if a single cause of a given effect is found, is the inquiry into others useless? It was first required to demonstrate that the same identical effect can be produced by only one and the same cause; then, certainly, the inquiry into other causes was useless, but the first part of this proposition indeed has not thus far been demonstrated; and I do not know whether Calandrini can [prove] it hereafter.⁸³

⁷⁹ Cominale, *Anti-Newtonianismi pars secunda*, 14; Herman Boerhaave, *Praelectiones academicae in proprias institutiones rei medicae*, ed. by Albrecht von Haller, vol. 6 (Venice: Apud Simonem Occhi sub insigno Italiae, 1743–1745), 111–112.

⁸⁰ Cominale, *Anti-Newtonianismi pars secunda*, 14: “Nunc si verum esset axioma, frustra nempe fieri per plura, quae fieri possunt per pauciora; nonne frustranea dicenda foret humana generatio? fieri enim poterat per pauciora.”

⁸¹ *Ibid.*, 15: “si per Naturam intelligat Newtonus ipsam rerum universitatem, nego, simplicem illam esse; ecquis Mortalium unquam ejus diversitatem percipere potuit? Ecquis ejusdem simplicitatem peroptare posset? Nonne ejus varietas delectat, juvat; & supremi Numinis potentiam demonstrat?”

⁸² A couple of pages further, Cominale observed, for instance, that plants can be dried by both excessive heat and excessive cold (*ibid.*, 20).

⁸³ *Ibid.*, 15–16: “Nam, nonne ipse docuit, idem effectum pluribus modis produci posse? Nonne & a diversissimis causis, imo, & ab oppositis, ut videbimus infra, produci potest? Cur ergo dati effectus, inventa unica causa, inutilis est inquisitio aliarum? Demonstrare prius oportebat, idem identice effectum non nisi ab una eademque causa produci posse; tunc certe inutilis foret aliarum inquisitio: verum propositionis hujus pars prior adhuc demonstrata non est; & nescio, an id possit imposterum Calandrinus.”

It is worth noting that Cominale was not the first to critique Newton's *regulae*, drawing on the complexity of the animal and plant kingdoms. As recent scholarship has shown, Van Musschenbroek advanced similar criticism independently.⁸⁴ However, Van Musschenbroek's objections surfaced only after the publication of *Beginfels der natuurrkunde*, on which the French translation – used by Cominale – was based. Musschenbroek's objections were first made public in his *Introductio ad philosophia naturalem* (1762).⁸⁵ It appears therefore that Cominale critique – that Newton *regulae philosophandi* mistakenly presuppose the simplicity of nature – is an original one.

In his critique of the first *regula philosophandi*, Cominale further emphasized the irony that Newton, “who is proclaimed the greatest enemy of hypotheses,” has himself introduced hypotheses.⁸⁶ To illustrate this, he cited a speculative passage from Query 31 of the *Opticks* (first edition: 1704), in which Newton proposed that “God in the Beginning form'd Matter in solid, massy, hard, impenetrable, moveable Particles, of such Sizes and Figures, and with such other Properties, and in such Proportions to Space, as most conduced to the End for which he form'd them; [...]. [...] Now by the help of these Principles [i.e., active principles such as gravity and the cause of fermentation and cohesion] all material Things seem to have been composed of the hard and solid Particles above-mention'd, variously associated in the first Creation by the Counsel of an intelligent Agent.”⁸⁷ Cominale argued that this contention is no less speculative than the hypotheses of the ancient atomists, Democritus and Epicurus, which Newton “poorly adapted.”⁸⁸ It can be argued, however, that this line of criticism fails to do justice to Newton's natural philosophy. Newton himself clearly distinguished between the demonstrative part of natural philosophy and the speculative part of natural philosophy. The demonstrative part is embodied in the mathematical demonstrations and experimental investigations in the *Principia* and the *Opticks*. The speculative part of natural philosophy, explicitly framed as exploratory, is to be found in the Queries added to the *Opticks*.⁸⁹ Cominale concluded his discussion by contending that hypotheses cannot – and should not – be entirely expelled from physics. To support this claim, he once more turned to Calandrini's abovementioned annotation which states:

Nevertheless, where the hope of obtaining certainty from experiments and from there proceeding mathematically does not appear, it is permissible to use certain particular hypothe-

⁸⁴ Ducheyne, *Physics in Minerva's Academy*, 359–364.

⁸⁵ Pieter van Musschenbroek, *Introductio ad philosophiam naturalem*, vol. 1 (Leiden: Apud Sam. et Joh. Luchtmans, 1762), 15–16.

⁸⁶ Cominale, *Anti-Newtonianismi pars secunda*, 16: “qui[...] hypothesium inimicissimus depredicatur.”

⁸⁷ Cominale, *Anti-Newtonianismi pars secunda*, 16–17; Newton, *Opticks*, 400–402.

⁸⁸ Cominale, *Anti-Newtonianismi pars secunda*, 17.

⁸⁹ For a discussion, see Ducheyne, “Newton on Action at a Distance.”

ses to investigate the truth with new experiments, just as astronomers have applied various hypotheses to predict and more accurately observe celestial phenomena, and thus to investigate their true causes by conjecturing.⁹⁰

Cominale then shifted attention to Newton's second rule of philosophizing, which states: "Therefore, the causes assigned to natural effects of the same kind must be, so far as possible, the same."⁹¹ Cominale argued that because this rule is a corollary of the first,⁹² it too must be rejected in light of his critique of the first rule. He added that "in fact, Newtonians abuse this [rule] more than Newton himself; for he wisely qualifies it with the expression, *so far as possible*; and in his illustration, he limits himself to examples such as the respiration in man and in beast, the descent of stones in Europe and in America, [and] light in kitchen fire and the sun."⁹³ Newton added the phrase "*quatenus fieri potest*" in the second edition of the *Principia*,⁹⁴ a modification that can be interpreted as an implicit acknowledgement that the second rule, although effective in the context of the argument for universal gravitation, is not universally applicable.

Next on Cominale's list is Newton's third rule of philosophizing, which states: "Those qualities of bodies that cannot be intended and remitted [...] and that belong to all bodies on which experiments can be made should be taken as qualities of all bodies universally."⁹⁵ According to Cominale, if this rule is true, then "the most famous system of mutual gravity of all bodies collapses, and attraction – adorned with so many praises and loaded with so many honors, to which all the phenomena of nature are attributed today – vanishes very much like the wind."⁹⁶ To support this provocative claim, Cominale pointed out: "Because, ac-

⁹⁰ Cominale, *Anti-Newtonianismi pars secunda*, 19, quoting Newton, *Principia*_{Geneva}, vol. 3, pt. 1, 2, note (a): "Verumtamen ubi certitudinis obtinendæ ab Experimentis & indè Mathematicâ via procedendo spes non affulget hypothesibus quibusdam particularibus uti licet ad veritatem novis experimentis indagandam, quemadmodum Astronomi varias adhibuerunt hypotheses ut phænomena cælestia prædicere & accuratius observare, atquè ità veras eorum causas conjectando investigare possent."

⁹¹ Newton, *The Principia*, 795.

⁹² Cf. Della Torre, *Scienza della natura*, vol. 1, 4.

⁹³ Cominale, *Anti-Newtonianismi pars secunda*, 20: "Revera Newtoniani magis, quam Newtonus hac abutuntur; nam ipse sapienter advertit, *quatenus fieri potest*; & in illustratione affert tantum exemplum Respirationis in homine, & in bestia; Descensus lapidum in Europa, & in America; Lucis in igne culinari, & in Sole."

⁹⁴ Alexandre Koyré, I. Bernard Cohen, and Anne Whitman, eds., *Isaac Newton's 'Philosophiæ Naturalis Principia Mathematica': The third edition (1726) with variant readings*, vol. 2 (Cambridge: Cambridge University Press, 1972), 550–551.

⁹⁵ Cominale, *Anti-Newtonianismi pars secunda*, 22; Newton, *The Principia*, 795.

⁹⁶ Cominale, *Anti-Newtonianismi pars secunda*, 22: "corrui celeberrimum de mutua corporum omnium Gravitate systema, & vento simillima evanescit tot laudibus condecorata, tantisque muneribus onerata, Attractio, cui hodie omnia naturæ phaenomena accepta referuntur."

cording to Newton and all the Newtonians, gravity and attraction are one and the same; and both are not at all constant in bodies, since they can be remitted and intended in the same bodies, solely by varying the distance from the center of the Earth.”⁹⁷ To show that this interpretation aligns with Newton’s doctrine and that of his followers, Cominale cited Newton’s own comment on the third rule, where he states that “Gravity is diminished [diminuitur] as bodies recede from the earth.”⁹⁸ He also referenced a similar observation by the Leiden professor Willem Jacob’s Gravesande (1688–1742).⁹⁹ Now, if it is true that gravity and attraction can be intended and remitted by varying the distance, then they cannot be regarded as universal qualities of bodies, according to Cominale. As he argued: “Thus, the distance of a body from the center of the Earth, or of bodies mutually attracting or gravitating, could increase so that their gravity completely vanishes, and then the bodies would no longer attract each other or else fall back towards the Earth, because they are devoid of all centripetal force.”¹⁰⁰ Although Newton clearly stated that gravity diminishes as the distance increases according to the inverse-square law, he did not support the notion that gravity could be wholly nullified as the distance increases.¹⁰¹

Conclusion

Cominale regarded Newton’s natural philosophy (and that of his followers) as a popular yet ultimately transient school of natural philosophical thought. Central to his critique is his assertion that Newton’s natural philosophy hinders the progress of physics through its misuse of the mathematical method. Cominale argued that Newton’s *Principia* exemplifies an inversion of the proper natural philosophical method: rather than beginning with the analysis of phenomena and building toward theoretical physics, Newton attempted to

⁹⁷ Ibid., 22: “Quia, juxta Newtonum, Newtonianosque omnes, Gravitatio, & Attractio unum idemque sunt; & ambae minime in corporibus constantes sunt, quoniam remitti & intendi possunt in iisdem corporibus, sola variata distantia a centro Terrae.”

⁹⁸ Cominale, *Anti-Newtonianismi pars secunda*, 23; Newton, *The Principia*, 796.

⁹⁹ Cominale, *Anti-Newtonianismi pars secunda*, 23; Willem Jacob’s Gravesande, *Physices elementa mathematica, experimentis confirmata. Sive introductio ad philosophiam Newtonianam*, vol. 2 (Leiden: Apud Johannem Arnoldum Langerak, Johannem et Hermannum Verbeek, 1742), 988: “[gravitas] immutabilis est in singulis Particulis; ad eandem Distantiam semper eadem; aucta autem Distantiâ decrescit Vis, ut Quadratum Distantiæ augetur.”

¹⁰⁰ Cominale, *Anti-Newtonianismi pars secunda*, 23: “Si vera, inquam sunt haec, vi expositae regulae, Gravitas, & Attractio minime ut qualitates corporum omnium accipi possunt. Nam eo increscere poterit distantia corporis a centro Terrae, sive corporum se mutuo attrahentium, aut gravitantium, ut gravitas eorum omnino evanescat; & tunc corpora non amplius se attraherent, aut versus Terram relaberentur, quia omni vi centripeta destituta.”

¹⁰¹ For a discussion, see Steffen Ducheyne, “The main Business of Natural Philosophy:” Isaac Newton’s Natural Philosophical Methodology (Dordrecht: Springer, 2012), 114–118.

derive physics from mathematics. Drawing upon 's Gravesande's and Nieuwentijt's terminology, Cominale viewed Newton's *Principia* as an exercise in pure mathematics, rooted in abstract ideas that lack correspondence with physical reality. Consequently, Newton's *magnum opus*, in Cominale's view, remains an idealized construct, one that conflates mathematical explanations with physical explanations, rather than a genuine contribution to physics. As we have seen, Cominale's critique of Newton's mathematization of nature was influenced by the ideas of the Descartes-inspired Jesuit Castel.

While Cominale's critique of Newton's third *regula philosophandi* may fall short, his analysis of the first and second *regulae* is original, anticipating similar comments later made by Van Musschenbroek. As we have seen, Cominale challenged the principle of nature's simplicity underlying the first and second *regulae* by appealing to the generation of animals and plants. These phenomena, he argued, reveal nature's inherent diversity and show that similar effects can arise from different causes. To strengthen his critique, Cominale implied that even Newton acknowledged the second *regula* is not universally applicable, by adding the words "*quatenus fieri potest*."

Cominale was well-read in Newton's *Opticks* and *Principia*, and he demonstrated a broad familiarity with the works of several prominent proponents of Newton's natural philosophy, including Calandrini's annotations in the Geneva edition of the *Principia*, Maclaurin's *An Account of Sir Isaac Newton's Philosophical Discoveries*, Keill's *Introductio ad veram physicam seu lectiones physicae*, 's Gravesande's *Physices elementa mathematica, experimentis confirmata*, the French translation of Van Musschenbroek's *Beginnels der natuurkunde*, Genovesi's *Disciplinarum metaphysicarum elementa*, and Della Torre's *Scienza della natura*. Intellectually speaking, Cominale was far from the isolated and provincial figure he is often taken to be. As shown, Cominale was deeply engaged with contemporary debates surrounding Newton's natural philosophy. Moreover, he drew upon the arguments and conceptual frameworks of a wide range of authors – not only figures such as Gassendi and Castel, but also scholars like Nieuwentijt, Boerhaave and Van Musschenbroek, who are traditionally regarded as supporters of Newton's project – and, in doing so, he simultaneously contributed his own counterarguments to the ongoing debate. Although certain aspects of his critique may fall short, Cominale offered a distinctive and original critique of Newton's first *regula philosophandi*. It is our hope that this article will encourage further research into Cominale's work and its place within the broader landscape of eighteenth-century critiques of Newton's natural philosophy.¹⁰²

¹⁰² Centre Leo Apostel, Vrije Universiteit Brussel, Pleinlaan 2, B-1050, Brussels, Belgium. Research for this article was funded by the Special Research Fund of the Vrije Universiteit Brussel (SRP100: 'Contra Isaac Newton. British (Natural) Philosophical and Poetic Criticism of Newton's Natural Philosophy and Natural Philosophical Methods, 1672–c.1750'). The order of the authorship reflects the relative extent of each contributor's involvement in the development of this article. We are indebted to the referees of this journal for their valuable feedback.

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Galileo tra gli Scolopi: l'esperimento della Scuola di Matematica a Firenze

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English title

Galileo among Piarists: The experiment of the School of Mathematics in Florence

Abstract

This essay reconstructs the exceptional educational experiment carried out by the Piarists in Florence during the early 17th century with the foundation of a mathematics school inspired by Galilean thought. In a cultural context dominated by Aristotelianism and strong ecclesiastical resistance, the Calasancian order promoted a groundbreaking scientific education based on mathematics, astronomy, and experimental science. Central to this effort was Famiano Michelini, closely linked to Galileo and his circle, supported by other Piarists such as Clemente Settimi and Angelo Morelli. Despite pressure and denunciations from the Inquisition, the school educated prominent intellectuals, including Vincenzo Viviani, and played a key role in the founding of the Accademia del Cimento. The text also examines the evolution of the Galilean legacy between the 17th and 18th centuries, showing how the Florentine school left a lasting mark on the Piarist educational identity and contributed significantly to the scientific culture of the Grand Duchy of Tuscany and beyond.

Keywords

Galileism, Piarists, School of Mathematics, Florence

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Il 22 maggio 1630, dopo due anni di intense trattative con la corte granducale, gli scolopi ottennero l'autorizzazione ad aprire le loro scuole a Firenze, in via de' Cimatori. L'operazione fu resa possibile grazie al sacerdote fiorentino Giovanni Francesco Fiammelli, che, all'età di settantacinque anni, decise di ritirarsi cedendo il proprio istituto all'ordine calasanziano. La scuola, attiva dal 1619, contava già circa cinquecento studenti, numero che rimase stabile anche dopo il passaggio agli scolopi. Vi si insegnavano leggere, scrivere, abaco, grammatica, umanità, retorica e matematica.

Completato l'insegnamento di base, gli studenti potevano scegliere se proseguire con lo studio del latino o, per chi puntava a un inserimento rapido nel mondo del lavoro, frequentare la classe di abaco. Qui si imparava a "far di conto": nozioni essenziali di aritmetica commerciale, tra cui l'uso dei numeri decimali, il calcolo degli interessi, e le quattro operazioni fondamentali, competenze considerate indispensabili per futuri mercanti o artigiani.

La scuola di matematica o algebra, invece, era separata da questo percorso e si distingueva per le materie trattate, il profilo sociale degli studenti, l'identità dei docenti e l'originalità dei metodi didattici. A differenza delle altre classi, frequentate per lo più da ragazzi tra i dieci e i quattordici anni appartenenti al ceto medio-basso, la scuola di matematica attirava inizialmente quasi esclusivamente adulti, per lo più membri del patriziato cittadino.

Tra gli allievi più illustri figurano Vincenzo Viviani, celebre matematico, ultimo discepolo di Galileo e membro delle principali accademie scientifiche del Seicento; Carlo Dati (1619–1676), anch'egli allievo galileiano, maestro di greco e latino e accademico della Crusca; Valerio Chimentelli (1620–1668), docente di greco all'Università di Pisa e precettore dell'erede del granduca Ferdinando II; i fratelli Candido (1618–1676) e Paolo Del Buono (1625–1659), scienziati tra i primi collaboratori dell'Accademia del Cimento; i fiorentini Simone e Filippo Paganucci.¹

La selettività di questa scuola era legata alla natura delle discipline insegnate: algebra, geometria e scienze sperimentali, materie che richiedevano una formazione avanzata e non adatte a studenti privi di una solida base culturale.

Come per le altre classi, il modello di riferimento era quello gesuita. Nelle scuole fiorentine della Compagnia di Gesù era previsto un insegnamento specifico di matematica per gli studenti di fisica, che comprendeva geometria euclidea, geografia e astronomia. Questo orientamento risentiva dell'influenza dell'Accademia Superiore di Matematiche, fondata al Collegio Romano nel 1593, che offriva a un'élite selezionata un ciclo di formazione in geometria, aritmetica, astronomia e algebra, situato tra gli studi filosofici e quelli teologici.²

¹ Archivio della Provincia Italiana dei P. P. Scolopi a Firenze (da ora APF), Reg. prov. 61–8, Prefettura delle Scuole Pie di Firenze. Scolari celebri delle Scuole Pie, ff. 1–56.

² Sull'insegnamento delle matematiche nei collegi gesuiti cfr.: Ugo Baldini, *Legem impone subactis. Studi su filosofia e scienza dei Gesuiti in Italia: 1540-1632* (Roma: Bulzoni, 1992); Id., ed., *Christoph Clavius e l'attività scientifica dei gesuiti nell'età di Galileo. Atti del Convegno Internazio-*

Ciò che distingueva radicalmente la scuola calasanziana erano i suoi metodi didattici, basati sulla cosmografia copernicana e sulle teorie galileiane del moto e della meccanica, in aperto contrasto con la visione aristotelico-tolemaica ancora dominante. Un simile orientamento era eccezionale: non solo i gesuiti e altre istituzioni rifiutavano tali contenuti, ma essi erano anche formalmente proibiti dalle autorità ecclesiastiche dopo la condanna dell'eliocentrismo.

Che un ordine religioso nato in piena Controriforma, con l'obiettivo di rafforzare la fede cattolica contro la Riforma protestante, potesse insegnare dottrine considerate eretiche è un fatto che merita particolare attenzione. Come ricorda Mauro Pesce, il dibattito sul copernicanesimo non implicava una revisione dei fondamenti dogmatici della teologia cristiana, ma metteva comunque in crisi il sistema aristotelico e il suo legame con la dottrina cattolica. Sostenere il nuovo modello cosmologico significava dunque rompere con un impianto filosofico consolidato nei secoli, un atto consapevole per pensatori come Campanella, ma imposto dalle circostanze per Galileo.³

Dopo il Concilio di Trento, gli ordini religiosi assunsero un ruolo centrale nella repressione dell'eterodossia, e Firenze non fece eccezione. Pochi anni prima dell'arrivo degli scolopi, furono soprattutto i domenicani – in particolare Tommaso Caccini e Niccolò Lorini – a spingere il Sant'Uffizio a condannare Galileo e a porre all'indice il *De revolutionibus orbium coelestium* di Copernico (5 marzo 1616). La loro opposizione non si limitò all'eliocentrismo: essi arrivarono a mettere in discussione la stessa matematica, accusandola di costituire una scienza autonoma dalla teologia, quindi sospetta e potenzialmente eretica.⁴

nale: Chieti, 28-30 aprile 1993 (Roma: Bulzoni, 1995); Id., "The Academy of Mathematics of the Collegio Romano from 1553 to 1612," in *Jesuit Science and the Republic of Letters*, ed. di Mordechai Feingold (Cambridge-London: MIT, 2002), 47-98; Romano Gatto, "L'insegnamento delle nuove scienze nei collegi gesuitici italiani," *Annali di storia dell'educazione e delle istituzioni scolastiche*, 3 (1996): 53-72; Antonella Romano, *La Contre-réforme mathématique. Constitution et diffusion d'une culture mathématique jésuite à la Renaissance* (Roma: École française de Rome, 1999); Federica Favino, "Scienza ed erudizione nei collegi degli ordini religiosi a Roma tra Sei e Settecento," *Cheiron*, 43-44 (2005): 331-370; Ead., "Ordini religiosi e Scienza," in *Enciclopedia Italiana – Il contributo italiano alla storia del pensiero. Le scienze – VIII appendice*, eds. di Antonio Clericuzio e Saverio Ricci (Roma: Istituto dell'Enciclopedia Italiana, 2013), 173-180.

³ Mauro Pesce, *L'ermeneutica biblica di Galileo e le due strade della teologia cristiana* (Roma: Edizioni di Storia e Letteratura, 2005); Id., "Il Copernicanesimo e la teologia. Perché il caso galileiano non è chiuso," in *Il Caso Galileo. Una rilettura storica, filosofica, teologica. Atti del Convegno internazionale di studi* (Firenze, 26-30 maggio 2009), eds. di Massimo Bucciantini, Michele Camerota, e Franco Giudice (Firenze: Olschki, 2011), 33-46.

⁴ Cfr. Paolo Cristofolini, "Caccini, Tommaso," *DBI* XVI (1973): 35-37; Annibale Fantoli, *Galileo: per il copernicanesimo e per la Chiesa* (Città del Vaticano: Specola Vaticana-Libreria Editrice, 1993); Massimo Bucciantini, *Contro Galileo. Alle origini dell'Affaire* (Firenze: Olschki, 1995); Sergio Pagano, *I documenti del processo di Galileo Galilei (1611-1741). Nuova edizione accresciuta, rivista e annotata* (Città del Vaticano: Archivio segreto Vaticano, 2009); Adriano Prosperi,

Anche i gesuiti presero progressivamente le distanze dalle posizioni galileiane. Cristoforo Clavio, figura eminente della Compagnia e sostenitore dell'autonomia metodologica della matematica, respinse le nuove scoperte giudicandole incompatibili con la dottrina cristiana. Emblematica è la lettera inviata nel 1610 da quattro matematici del Collegio Romano, tra cui lo stesso Clavio, al cardinale Bellarmino, in cui si contestavano le osservazioni del *Sidereus Nuncius*, in particolare l'idea di una superficie lunare imperfetta, la cui accettazione avrebbe richiesto una revisione teologica troppo profonda.⁵

Dopo la morte di Clavio (1612), la resistenza interna alla Compagnia si accentuò. La disputa sulle comete – che portò Galileo a polemizzare nel *Saggiatore* (1623) contro il gesuita Orazio Grassi – segnò il punto di rottura. Nel 1633, Melchior Inchofer partecipò al processo inquisitoriale redigendo il *Tractatus syllepticus*, apertamente ostile al copernicanesimo.⁶

Tuttavia, come hanno mostrato gli studi di Antonella Romano e Ugo Baldini, la Compagnia non fu un blocco monolitico. Al suo interno coesistevano posizioni diverse e talvolta aperturiste.⁷ Rivka Feldhay ha evidenziato l'ambiguità del cardinale Bellarmino, gesuita: pur difendendo i confini tomisti tra filosofia e teologia, egli incoraggiava gli astronomi dell'ordine a proseguire il dialogo con il copernicanesimo. I membri della Compagnia costituivano un'élite intellettuale nuova, alternativa alla rigidità domenicana, impegnata a conciliare sapere scientifico e ortodossia. Il confronto tra Galileo e studiosi come Christoph Scheiner esprime bene questa tensione: mentre Galileo sfidava apertamente i limiti disciplinari fissati dalla *Ratio studiorum* (1599), i gesuiti cercavano di preservare il proprio ruolo educativo senza esporsi all'accusa di eterodossia. Alla fine, le pressioni domenicane li spinsero verso un progressivo allineamento alle posizioni più conservatrici.⁸

Per Paula Findlen, tale evoluzione rappresenta la crisi di una rete di relazioni intellettuali. Fino al processo, molti gesuiti furono interlocutori e collaboratori di Galileo; ma con l'intensificarsi delle tensioni dottrinali e politiche, quella rete si disgregò. Dopo il 1633, la rottura del "Roman network" segnò la fine del dialogo tra scienza e teologia: la Compagnia assunse un atteggiamento più prudente, volto a tutelare l'autorità ecclesiastica. Il

"L'Inquisizione fiorentina al tempo di Galileo," in *Inquisizioni*, ed. Id. (Macerata: Quodlibet, 2023), 41–58.

⁵ Cfr. Baldini, "The Academy," 47–98; Gatto, "L'insegnamento," 53–72; Romano, *La Contre-réforme mathématique*, 85–179; Rivka Feldhay, "The Jesuits transmitters of the new sciences," in *Il caso Galileo*, eds. Bucciantini, Camerota e Giudice, 47–74.

⁶ Melchior Inchofer, *Tractatus syllepticus, in quo quid de Terrae Solisque motu vel statione secundum Saam Scripturam et Sanctos Patres sentiendum, quae certitudine alterutra sententia tenenda sit breviter ostenditur* (Romae: excudebat Ludovicus Perignanus, 1633).

⁷ Baldini, *Legem*; Id., "The Academy," 47–98; Romano, *La Contre-réforme mathématique*.

⁸ Rivka Feldhay, *Galileo and the Church. Political Inquisition or Critical Dialogue?* (Cambridge: Cambridge University Press, 1995); Ead., "Recent Narratives on Galileo and the Church: or The Three Dogmas of the Counter-Reformation," *Science in Context* 13, 3–4 (2000): 489–507.

caso Galileo divenne così una crisi di comunità e comunicazione, più che una semplice opposizione ideologica.⁹

Inoltre, diversi gesuiti – come Biancani, Cabeo, Zurchi e Grimaldi – offrirono contributi significativi alla scienza moderna. Romano ha documentato la promozione della matematica nelle università francesi del XVI e XVII secolo,¹⁰ mentre Denise Arigò ha analizzato l'attività sperimentale della scuola di Santa Lucia a Bologna, diretta da Giovanni Battista Riccioli, nota per la precisione delle sue osservazioni.¹¹ Anche fuori dall'Europa, l'ordine contribuì alla diffusione delle nuove teorie, come dimostrano le missioni in Persia studiate da Christian Windler.¹² Secondo Pierre Chaunu, nel Settecento i gesuiti francesi avevano ormai abbandonato l'aristotelismo, manifestando una graduale apertura verso la scienza moderna.¹³

Questa trasformazione interna riflette la complessità della Compagnia di Gesù, attraversata da tensioni e continui adattamenti nel difficile rapporto tra ortodossia teologica e nuove scoperte scientifiche.

L'atteggiamento della comunità calasanziana nei confronti della scienza galileiana resta invece, ancora oggi, un ambito poco esplorato dagli storici. Nonostante i contributi recenti di studiosi come Massimo Bucciantini,¹⁴ Federica Favino¹⁵ e Stefania Montacutelli,¹⁶ molte questioni rimangono irrisolte, in gran parte per la scarsità di fonti dirette sull'argomento. Una risorsa utile – pur con evidenti limiti di imparzialità e rigore critico – è rappre-

⁹ Paula Findlen, "Rethinking 1633: Writing the life of Galileo after the trial," in *Nature Engaged: Science in Practice from the Renaissance to the Present*, eds. di Mario Biagioli e Jessica Riskin (London: Palgrave Macmillan, 2012), 205–226; Paula Findlen e Hannah Marcus, "The breakdown of Galileo's Roman network: Crisis and community, ca. 1633," *Social Studies of Science* 47, 3 (2017): 326–352.

¹⁰ Romano, *La Contre-réforme mathématique*.

¹¹ Denise Arigò, "Les 'yeux d'Argos', et les 'étoiles d'Astrée' pour mesurer l'univers. Les jésuites et la science nouvelle," *Revue de Synthèse* 120 (1999): 285–303.

¹² Christian Windler, *Missionaries in Persia: Cultural Diversity and Competing Norms in Global Catholicism* (London: I. B. Tauris, 2024).

¹³ Pierre Chaunu, *La civiltà dell'Europa dei lumi* (Bologna: il Mulino, 1987), 86–90.

¹⁴ Massimo Bucciantini, "Eredità galileiana e politica culturale medicea: il caso degli scolopi," *Studi Storici* XXX (1989): 379–399.

¹⁵ Favino, "Scienza ed erudizione," 331–370; Ead., "Ordini religiosi," 173–180; Ead., "Michellini, Famiano," *DBI* LXXIV (2010), 230–234; Ead., "Giovanni Alfonso Borelli's Last Will (1679, December 31st)," *Nuncius. Journal of the Material and Visual History of Science* XXXVII (2022): 144–173.

¹⁶ Stefania Montacutelli, "Da Galileo a Borelli e oltre: la filosofia naturale delle scuole pie," in *Conflicting duties: science, medicine and religion in Rome, 1550/1750*, eds. di Maria Pia Donato e Jill Kraye (London–Torino: Warburg institute–N. Aragno, 2009), 181–209; Ead., "Giovanni Alfonso Borelli," *Il Contributo italiano alla storia del Pensiero*, eds. di Antonio Clericuzio e Saverio Ricci (Roma: Istituto dell'Enciclopedia Italiana di scienze, lettere e arti, 2013).

sentata dagli scritti dei cronisti dell'ordine, come G. Giovannozzi, L. Picanyol, G. Sántha, I. Levo e O. Tosti.¹⁷ Questi autori individuano un legame profondo tra gli scolopi e la scienza galileiana, riconducendolo a Giuseppe Calasanzio, considerato il “primo scolopio galileiano.” Secondo la loro interpretazione, l'ordine avrebbe mostrato fin dalle origini una forte apertura verso le nuove scoperte scientifiche, grazie alla visione educativa del suo fondatore, che trasformò le scuole pie in un centro di formazione innovativo.

A sostegno della loro tesi, gli storici calasanziani richiamano il rapporto tra Calasanzio, Tommaso Campanella e Galileo, benché non esistano prove di un effettivo scambio epistolare con lo scienziato pisano.

Per Picanyol e gli altri cronisti scolopici, Calasanzio, Campanella e Galileo sarebbero accomunati da un destino simile: tutti e tre perseguitati per aver sfidato l'autorità culturale e teologica dominante, in particolare l'alleanza tra aristotelismo e potere gesuitico. Anche Calasanzio, infatti, subì la repressione del Sant'Uffizio, fu incarcerato e vide il suo ordine ridotto a semplice congregazione.

Malgrado ciò, l'idea di un ordine “galileiano” fin dalle origini, istituzionalmente e unanimemente schierato a favore dell'eliocentrismo, non è supportata dai dati storici. L'apertura alla scienza galileiana fu piuttosto il frutto di iniziative individuali, favorite da un contesto politico eccezionale.

Non esiste alcuna prova di corrispondenza tra Galileo e Calasanzio, e nelle migliaia di lettere del fondatore i riferimenti a questioni scientifiche o filosofiche sono rari, se non del tutto assenti, fatta eccezione per qualche sporadico accenno nel contesto fiorentino. Si trovano menzioni occasionali di studiosi come Cristoforo Clavio o Giovanni Camillo Gloriosi, ma nulla che indichi un coinvolgimento diretto nelle teorie galileiane.¹⁸

¹⁷ Giovanni Giovannozzi, *Scolopi galileiani: Lettura fatta alla società Colombaria il 25 Maggio 1915* (Firenze: Tipografia “L'arte della Stampa,” 1917); Leodogario Picanyol, “Le Scuole Pie e Galileo Galilei,” *Rassegna di storia e bibliografia scolopica XI–XII* (1942): 3–239; György Sántha, “L'opera delle Scuole Pie e le cause della loro riduzione sotto Innocenzo X,” *Archivium scholarum piarum* (da ora ArSP) XIII, 25 (1989): 1–134; Italo Levo, “Galileo e gli Scolopi e l'indirizzo scientifico nelle Scuole Pie dopo Galileo,” *Ricerche. Bollettino quadrimestrale degli Scolopi Italiani* (da ora: RiBQ) VIII, 2 (1988): 129–142; Osvaldo Tosti, “Il P. Francesco Michelini,” *RiBQ* IX, 3 (1989): 205–308; Id., “Il Padre Clemente Settimi,” *RiBQ* XIV, 2 (1994): 99–177.

¹⁸ Oltre alle centinaia di lettere ancora inedite conservate negli archivi scolopici, sono stati utilizzati come riferimento i seguenti epistolari pubblicati: Leodogario Picanyol, *Epistolario di San Giuseppe Calasanzio* (d'ora in poi: *Epist.*), voll. I–IX (Roma: Ed. Calasancianae, 1950–1956); György Sántha, ed., *Epistulae ad S. Iosephum Calasancium ex Europa Centrali (1625–1648)* (d'ora in poi: *EEC*) (Roma: Ed. Calasancianae, 1969); Id., *Epistulae ad S. Iosephum Calasancium ex Hispania et Italia (1616–1648)* (d'ora in poi: *EHI*), voll. I–III (Roma: Ed. Calasancianae, 1972); Id., *Epistolarium coetaneorum S. Iosephi Calasancii (1600–1648)* (d'ora in poi: *ECC*), voll. I–VII (Roma: Ed. Calasancianae, 1977–1982); Claudio Vilá Palá, *Epistolario di San Giuseppe Calasanzio* (d'ora in poi: *Epist.* 2), vol. X (Roma: Ed. Calasancianae, 1988).

Più concreta è invece la documentazione relativa a Campanella. Calasanzio lo cita in otto lettere scritte tra il 1631 e il 1632, tutte indirizzate a Giacomo Bandoni, rettore della Casa di Frascati, dove il filosofo soggiornò per un breve periodo.¹⁹ Tuttavia, anche in questo caso, il contenuto è di natura amministrativa e personale, privo di riferimenti scientifici.

Eppure, se un rapporto diretto tra Galileo e Calasanzio appare improbabile, è molto più verosimile l'esistenza di un dialogo con Campanella. Un indizio importante si trova in una lettera in cui Calasanzio, scrivendo a Bandoni, si dichiara disponibile a rispondere a Campanella "di presenza."²⁰

A rafforzare tale ipotesi è il permesso concesso agli scolopi di assistere alle lezioni che il filosofo calabrese teneva a Frascati: una scelta che dimostra l'attenzione alla formazione dei docenti e la disponibilità a confrontarsi con idee nuove, anche non ortodosse.

Il legame tra Campanella e gli scolopi, seppur limitato nel tempo, costituisce un'esperienza rilevante, soprattutto considerando il ruolo centrale del filosofo calabrese nella cultura del primo Seicento. Torturato, processato e incarcerato per oltre vent'anni, egli incarnava il volto più radicale del dissenso intellettuale contro le imposizioni dogmatiche della Chiesa posttridentina. Colpisce non solo il suo interesse per il progetto educativo calasanziano – volto all'istruzione dei ceti popolari – ma anche la sua capacità di coglierne l'impatto sociale. Figlio di un calzolaio, conosceva in prima persona le difficoltà di accesso al sapere e apprezzava la missione emancipatrice degli scolopi, tanto da dedicare un trattato in difesa dell'ordine.²¹

Non mancano figure che avrebbero potuto favorire un punto di contatto tra Campanella e Calasanzio, come Kaspar Schoppe, corrispondente del filosofo e sostenitore dell'insediamento scolopico a Firenze, o lo stesso Galileo, con cui Campanella intrattenne uno scambio epistolare.²² Tutto ciò dimostra quanto gli scolopi fossero inseriti nel vivace contesto scientifico e filosofico dell'epoca, anche se non chiarisce del tutto perché Cala-

¹⁹ *Epist.* IV, lett. 1694, 422; 1695, 423; 1697, 424; 1699, 425; *Epist.* V: lett. 1878, 112; 1886, 117; 1889, 119; 1894, 121.

²⁰ *Epist.* V, lett. 1894, 121.

²¹ Il testo è stato redatto da Tommaso Campanella nel 1631–32 ma rinvenuto solo nel 1932 da Picanyol, *Liber apologeticus*, 170–177; 217–223; 258–263; poi riedito in inglese nel 1984 da Kristian Jensen e A. Karen Liebreich, "Liber apologeticus contra impugnantes Institutum Scholarum Piarum," *ArSP* VIII, 15 (1984): 29–76; e in italiano nel 2015 con Maurizio Erto, ed., "Libro apologetico contro gli avversari dell'Istituto delle Scuole Pie," *Bruniana & Campanelliana* XXI, 1 (2015): 253–318.

²² Su Campanella: Luigi Amabile, *Fra Tommaso Campanella, la sua congiura, i suoi processi e la sua pazzia*, voll. I–III, (Napoli: 1882); Luigi Firpo, "Campanella, Tommaso," *DBI* XVII (1974): 372–401; Germana Ernst, *Religione, ragione e natura. Ricerche su Tommaso Campanella e il tardo Rinascimento* (Milano: Franco Angeli, 1991). Vedi anche la rivista *Bruniana & Campanelliana. Ricerche filosofiche e materiali storico-testuali* (Pisa–Roma: 1995–).

sanzio scelse di accordare fiducia a un personaggio così controverso e sorvegliato come Campanella. Ancora più sorprendente è la sua tolleranza verso Galileo, anch'egli sotto osservazione inquisitoriale.

Emblematico, in questo senso, è il permesso accordato agli scolopi fiorentini di frequentare la casa di Galileo durante il suo confino. Secondo Massimo Bucciantini, questa scelta fu parzialmente influenzata dalle pressioni del granduca di Toscana.²³ Tuttavia, ridurre la decisione a una semplice risposta politica sarebbe limitante. Più probabilmente, essa riflette la flessibilità intellettuale di Calasanzio e la sua apertura a sostenere percorsi educativi innovativi, anche quando non perfettamente allineati alla dottrina ufficiale.

Pur non essendo stato il promotore diretto della scuola di matematica fiorentina, Calasanzio non la ostacolò, ma anzi la sostenne. In altri casi, come per la fondazione di una scuola riservata ai patrizi a Firenze – scelta in apparente contrasto con la missione iniziale rivolta ai poveri – aveva accettato compromessi per garantire la sopravvivenza dell'ordine e mantenere buoni rapporti con il potere politico. Ma nel caso della scuola galileiana, la posta in gioco era più alta: il rischio di suscitare l'ostilità di Roma era concreto e si sarebbe in effetti realizzato. La mancata opposizione del fondatore può essere letta come una forma di tolleranza pragmatica, dettata tanto dalla necessità di mantenere buoni rapporti con i Medici quanto da una certa apertura personale verso le nuove scienze, senza che ciò implicasse necessariamente un'adesione piena e consapevole al copernicanesimo.

La differenza con altri ordini religiosi è evidente: nel 1618 il generale barnabita Ambrogio Mazenta costrinse Redento Baranzano a ritrattare le sue tesi copernicane espresse nell'*Uranoscopia seu de coelo*;²⁴ anche i gesuiti punirono severamente le deviazioni dottrinali di alcuni confratelli. Calasanzio, invece, non solo non ostacolò simili posizioni, ma arrivò persino a sostenerle. È quindi plausibile che, pur non partecipando direttamente alla ricerca scientifica, fosse interessato alle nuove idee matematiche e disposto ad accoglierle, almeno in parte.

Resta comunque chiaro che la sua priorità era un'altra: garantire un'istruzione di base utile e accessibile, capace di offrire ai giovani strumenti pratici per l'inserimento nel mondo del lavoro. In conclusione, se da un lato appare eccessivo il ruolo che alcuni storici scolopici attribuiscono a Calasanzio come promotore della rivoluzione scientifica, dall'altro va riconosciuta una certa sensibilità verso le scienze sperimentali, pur senza un coinvolgimento diretto. La sua posizione appare più vicina a una prudente tolleranza che a un'adesione esplicita.

L'interesse per la scienza galileiana non derivò dunque da una scelta personale di Ca-

²³ Bucciantini, "Eredità galileiana," 387–389.

²⁴ Cfr. Mario Tronti, "Baranzano Redento," *DBI* V (1963): 776–778; Michela Malpangotto, "Discussions coperniciennes au début du XVIII siècle: Le système du monde du P. Redento Baranzano enseignant en Savoie," *Archives internationales d'Histoire des sciences* 60, 2 (2010): 369–422.

lasanzio, ma si sviluppò concretamente a Firenze, grazie all'iniziativa di alcuni maestri e al decisivo sostegno della corte medicea. L'eccezionalità di questa collaborazione risalta ancor più se considerata nel contesto culturale e politico dell'epoca.

Con il ritorno di Galileo a Firenze nel 1610 – chiamato alla corte dei Medici come filosofo e matematico personale del granduca – si formò attorno a lui un fronte di opposizione determinato a contrastarne le idee astronomiche espresse nel *Sidereus Nuncius*. Tra i principali avversari vi erano Giovanni de' Medici, figlio naturale di Cosimo I, e l'arcivescovo Alessandro Marzi Medici, in carica dal 1605 al 1630. A sostenerli c'erano predicatori influenti come il domenicano Tommaso Caccini, protagonista di una violenta campagna contro Galileo che culminò con la denuncia del 20 marzo 1615 e con la condanna ufficiale del copernicanesimo nel 1616. Già due anni prima, un altro domenicano, Niccolò Lorini, aveva anticipato l'attacco inviando al Sant'Uffizio una copia della celebre lettera di Galileo a Benedetto Castelli.

Eppure, dopo la condanna del 1616, le autorità ecclesiastiche fiorentine evitarono lo scontro diretto. Le prediche ostili cessarono e, tra il 1616 e il 1630, gli atti dell'Inquisizione locale registrano solo due casi, entrambi legati a libri proibiti, senza menzione diretta di Galileo. Protetto dalla corte – prima da Cosimo II, poi da Ferdinando II – lo scienziato poté continuare le sue ricerche e mantenere una vasta rete di corrispondenza.²⁵

Se da un lato Galileo riuscì a sottrarsi al controllo diretto dell'arcivescovo, dall'altro la Chiesa non rinunciò a vigilare sul dibattito scientifico, soprattutto riguardo al moto della Terra e all'eliocentrismo. Marzi Medici attivò una rete di predicatori itineranti, come il francescano Angelo Celestino, incaricati di contrastare le idee galileiane nel contesto intellettuale cittadino. Tra gli oppositori più attivi vi fu anche Raffaello Delle Colombe, priore di Santa Maria Novella, figura di riferimento per il fronte antigalileiano negli anni Venti del Seicento. Con il sostegno dell'arcivescovo e di altri predicatori, Delle Colombe contribuì a consolidare un blocco culturale ostile alle nuove teorie scientifiche, che si estendeva anche all'università di Pisa.

Dopo la pubblicazione del *Dialogo sopra i due massimi sistemi del mondo* (1632) e la condanna di Galileo l'anno successivo, il clima si irrigidì ulteriormente. La pressione della Chiesa si fece più capillare e soffocante, trasformando Firenze in uno degli epicentri della repressione culturale post-galileiana. Proprio in questo contesto, l'apertura degli scolopi alla scienza moderna, e in particolare alle teorie galileiane, appare come un'anomalia significativa e degna di attenzione.²⁶

²⁵ Michele Cioni, *I documenti Galileiani del S. Uffizio di Firenze* (Firenze: Libreria editrice fiorentina, 1908), 20–21.

²⁶ Cfr. Bucciantini, *Contro Galileo*; Pagano, *I documenti del processo*, CII–CCVI; Annibale Fantoli, "Il processo del 1633," in *Il Caso Galileo*, eds. di Bucciantini, Camerota e Giudice, 187–212; Luigi Guerrini, *Galileo e la polemica anticopernicana a Firenze* (Firenze: Polistampa, 2009); Id., "The Archbishop and Astronomy. Alessandro Marzi-medici and the 1604 Supernova," in Co-

Paradossalmente, fu proprio grazie a Marzi Medici che gli scolopi ottennero il permesso di stabilirsi a Firenze. Durante il suo episcopato, l'arcivescovo diede un impulso decisivo al sistema educativo cittadino: inizialmente sostenendo Ippolito Galantini e la sua Congregazione di San Francesco della Dottrina Cristiana, poi i barnabiti, e infine proprio gli scolopi.

I rapporti cordiali tra Francesco Castelli e Marzi Medici suggeriscono che né Calasanzio né i primi scolopi mostrarono, almeno in un primo momento, particolare interesse per l'insegnamento delle scienze sperimentali.²⁷ Considerando l'atteggiamento dell'arcivescovo – poco incline verso questo tipo di sapere – è plausibile ritenere che anche un semplice sospetto in tal senso avrebbe potuto compromettere l'appoggio ecclesiastico e ostacolare l'insediamento dell'ordine in città.

Un documento del novembre 1629 conferma il clima favorevole: padre Arcangelo Galletti, collaboratore essenziale di Castelli nella fondazione della Casa scolopica fiorentina, riportava il favore manifestato dall'arcivescovo nei confronti di Francesco Michellini – futuro fondatore e indiscusso protagonista della scuola di matematica – lodato per le sue qualità di scrittore e matematico. L'incontro diretto tra Marzi Medici e Michellini si svolse dunque in un contesto sereno, senza attriti o segnali di diffidenza.²⁸

Fino al 1634, inoltre, non si registrano tensioni tra gli scolopi e le posizioni più conservatrici di Roma: l'ordine fiorentino sembrava infatti pienamente allineato con la condanna dell'eliocentrismo.

Il catalogo librario della scuola di quell'anno,²⁹ se da un lato testimonia un interesse crescente per la matematica – con quattro testi dedicati alla geometria euclidea – dall'altro include due opere chiaramente ostili alle idee galileiane: *Risposte piacevoli e curiose di Ludovico Delle Colombe e le Dubitationes di Claude Guillermet Beauregard (Berigardo)*.³⁰

La prima, pubblicata nel 1608, fu una risposta al *Discorso sulla stella nuova* firmato con lo pseudonimo Alimberto Mauri, dietro cui si celava Galileo stesso. In questo testo,

pernicus Banned. The Entangled Matter of the anti-Copernican Decree of 1616, eds. di Natacha Fabbri e Federica Favino (Firenze: Olschki, 2018), 101–126.

²⁷ APF, Reg. prov. 61–2, Documenti vari: 1628–1645, f. 1; EHI, 574.

²⁸ EHI, 1204.

²⁹ APF, Reg. prov. 61–5, Inventari diversi dell'anno 1634, 1640 e 1642, s. f.

³⁰ Ludovico Delle Colombe, *Risposte piacevoli e curiose alle considerazioni di Alimberto Mauri, fatte sopra alcuni luoghi del discorso del medesimo Ludovico d'intorno alla stella apparita l'anno 1604. Nelle quali risposte si trattano controversie d'Astrologia, Perspettiva, Filosofia, Teologia, e altre materie, non meno utili, che dilettevoli. Con tre tavole copiose; la prima delle quali contien le quistioni, la seconda le materie, la terza tutte le sentenze, motti, arguzie, similitudini istorie, e favole, che sono in questa opera* (Firenze: Caneò, 1608); Claudio Berigardo, *Dubitationes in Diaiogum Galilaei Galilaei Lyncei, in Gymnasio Pisano Mathematici Supraordinarii, auctore Claudio Berigardo, in eadem Academia philosophiam profitente* (Florentiae: ex typogr. Petri Nestri, sub signo Solis, MDCXXXII).

Delle Colombe attaccava apertamente sia Copernico sia Galileo, sostenendo una netta separazione tra filosofia e matematica e accusando i matematici di essere visionari privi di fondamento empirico. In una lettera del 1616 a Giovanni de' Medici, lo stesso autore si rallegrava della condanna dell'eliocentrismo. Ludovico, fratello maggiore di Raffaello Delle Colombe, fu in stretto contatto anche con il gesuita Clavio, al quale scrisse per congratularsi del suo rifiuto delle teorie galileiane.³¹

Siamo quindi di fronte a un autore, e a un'opera, lontani dallo spirito scientifico che alcuni storici calasanziani attribuiscono all'ordine fin dalle origini.

Il secondo testo, le *Dubitaciones* (1632), rispondeva al *Dialogo sopra i due massimi sistemi del mondo* di Galileo. Berigardo, arrivato a Firenze nel 1626 come segretario della granduchessa Cristina di Lorena, divenne l'anno seguente docente di filosofia allo Studio di Pisa. La sua breve operetta esprimeva alcune riserve sull'eliocentrismo, pur con toni meno polemici rispetto a quelli di altri critici contemporanei come Scipione Chiaramonti, Antonio Rocco o il gesuita Melchior Inchofer.³²

La presenza di questi due testi nella biblioteca calasanziana apre diverse ipotesi. È possibile che non tutti gli scolopi fossero inizialmente favorevoli al pensiero galileiano. Una seconda ipotesi, più speculativa vista la scarsità di risorse economiche dell'ordine, è che l'acquisto di questi volumi servisse a rassicurare l'Inquisizione o lo stesso arcivescovo. In alternativa, potrebbero essere stati usati dagli studenti come testi da confutare, in mancanza di opere dichiaratamente galileiane, allora proibite. Del resto, Galileo stesso possedeva molte opere dei suoi oppositori – da Chiaramonti a Rocco, da Inchofer a Delle Colombe – e ne annotava ampiamente i margini, a riprova di un uso critico e polemico.

Un'ulteriore possibilità è che quei volumi siano entrati nella biblioteca per via dei contatti tra Michelini e Delle Colombe, documentati da alcune lettere,³³ o forse anche con Berigardo, che insegnava a Pisa, città che Michelini frequentava abitualmente. Anche se la conoscenza diretta tra Michelini e Berigardo resta un'ipotesi, è plausibile che contatti di questo tipo abbiano favorito l'accesso a tali testi.

Michelini era noto, inoltre, per la sua capacità di procurarsi opere rare o costose, come dimostra l'acquisto del terzo volume del *De centro gravitas* del gesuita Paolo Guldino, reso

³¹ Cfr. Maria Muccillo, "Delle Colombe, Ludovico," *DBI* XXXVIII (1990): 29–30; Guerrini, *Galileo*.

³² Sul Berigardo: Antonio Favaro, "Oppositori di Galileo, IV. Claudio Berigardo," *Atti del. R. Istituto veneto di scienze, lettere ed arti* IXXIX (1919–1920): 39–92; Maria Bellucci, "La filosofia naturale di Claudio Berigardo," *Rivista critica di storia della filosofia* XXVI (1971): 363–411; Giorgio Stabile, "Il primo oppositore del "Dialogo": Claude Bérigard," in *Novità celesti e crisi del sapere. Atti del convegno internazionale di studi galileiani*, ed. di Paolo Galluzzi (Firenze: Giunti, 1984), 277–282.

³³ *EHI*, 1212.

possibile nel 1643 grazie all'intermediazione di Evangelista Torricelli.³⁴ A differenza dei due trattati precedenti, quest'opera non è riconducibile all'antigalileismo: pur criticando le teorie di Bonaventura Cavalieri sugli infinitesimi, accoglie implicitamente l'idea del movimento terrestre.³⁵

In definitiva, il quadro che emerge è complesso: lontano tanto da una piena adesione alle teorie galileiane quanto da un rifiuto sistematico. La presenza di testi antigalileiani non contraddice necessariamente l'apertura progressiva degli scolopi verso le scienze, ma testimonia piuttosto la loro collocazione in un ambiente culturale ancora instabile e profondamente segnato dalle pressioni ecclesiastiche.

Un elemento chiave che rese possibili gli sviluppi successivi fu la morte, nel 1630, dell'intransigente arcivescovo Marzi Medici. La sua scomparsa segnò la fine di una fase di forte controllo ecclesiastico sulla vita culturale fiorentina. Dopo la breve parentesi di Cosimo Bardi, conclusasi già nel 1631, la guida dell'arcidiocesi passò a Pietro Niccolini, che restò in carica fino al 1651.

Pur non avendo avuto un ruolo centrale nelle vicende legate a Galileo, Niccolini fu comunque oggetto di critiche da parte della Curia romana. Nel 1630, quando ricopriva l'incarico di vicario arcivescovile, aveva infatti concesso il *vidit* per la stampa del *Dialogo sopra i due massimi sistemi del mondo*, una decisione che a Roma fu interpretata come segno di eccessiva tolleranza verso l'eterodossia galileiana.

Lo stesso atteggiamento cauto e defilato si riscontrò nel processo contro Pandolfo Ricasoli, stimato teologo e direttore spirituale, vicino tanto agli scolopi quanto alla famiglia Medici. Ricasoli fu coinvolto in uno scandalo a sfondo sessuale, aggravato da sospetti di eresia quietista, legati al suo rapporto con la ricca vedova Faustina Mainardi. In quella circostanza, Niccolini si limitò a un ruolo secondario di supporto all'Inquisizione, evitando di esporsi pubblicamente nonostante la gravità delle accuse.

A Roma, gli si rimproverava anche una certa accondiscendenza verso i Medici, la cui politica culturale – promossa da Ferdinando II e dal principe Leopoldo – sosteneva apertamente Galileo e la scuola matematica calasanziana. In linea con le direttive granducali, Niccolini non solo evitò qualsiasi misura contro l'ordine calasanziano, ma nel 1636 ne approvò formalmente la permanenza a Firenze per un ulteriore quinquennio, confermandone poi il sostegno anche negli anni successivi, malgrado l'orientamento sempre più esplicito delle loro scuole verso l'insegnamento delle scienze e delle teorie ispirate al pensiero galileiano.³⁶

La scuola di matematica degli scolopi fu istituita ufficialmente nel 1633, con sede auto-

³⁴ Tosti, "Il P. Francesco," 323.

³⁵ Paolo Guldin, *De centro gravitatis, Liber tertius, De fructu ex usu centri gravitatis binarium specierum quantitatis continuæ, collecto; qui est geometria rotundi* (Viennæ: Cosmerovius, 1641).

³⁶ Cfr. Pagano. *I documenti*; Guerrini, *Galileo*; Andrea Zagli, "Niccolini, Francesco," *DBI LXXVIII* (2013); APF, Reg. prov. 61–1, Varia Florentina 1619–1696.

noma in via dello Studio, nello stesso anno in cui Galileo venne definitivamente confinato nella villa di Arcetri. Il primo docente fu Francesco di San Giuseppe, al secolo Famiano Michelini. La sua presenza a Firenze è attestata già nel 1629, durante le trattative con la corte medicea per l'insediamento dell'ordine.³⁷ Inviato da Calasanzio per dirigere la scuola di abaco, Michelini si ritrovò presto coinvolto anche in lezioni private di algebra richieste da membri dell'élite cittadina.

Lettere di padre Galletti a Calasanzio rivelano che, fin dai primi mesi del suo soggiorno, Michelini entrò in contatto con esponenti di spicco della comunità scientifica fiorentina: Galileo, Mario Guiducci e persino Ludovico Delle Colombe. Mentre insegnava nella scuola di abaco – frequentata da giovani artigiani e mercanti – offriva anche un insegnamento più avanzato in ambienti esclusivi, rivolto a patrizi interessati alla matematica.³⁸ Questa duplice attività mostra chiaramente che la scuola di matematica non era parte del progetto originario calasanziano, ma nacque come risposta a una domanda formativa emergente da parte del patriziato locale, sostenuta attivamente dalla corte medicea, che riconobbe negli scolopi – e nella loro relativa autonomia rispetto ai gesuiti – un'opportunità per promuovere una formazione scientifica di alto livello in linea con le proprie strategie culturali.

Per comprendere l'orientamento della scuola, è utile considerare il profilo di Michelini: allievo del matematico Giovanni Camillo Gloriosi, frequentatore delle lezioni del somasco Antonio Santini, e in contatto con l'astronomo Giovanni Battista Baliani. Questi legami inserirono il padre Francesco nel circuito della scienza galileiana. Proprio Baliani, in una lettera del 7 settembre 1629, lo presentò a Galileo lodandone il talento matematico.³⁹

Michelini frequentò con regolarità la villa di Arcetri, "il Gioiello," dove entrò stabilmente nel circolo galileiano. Tra le sue frequentazioni figurava anche Mario Guiducci, con cui era in contatto già dal 1629.⁴⁰ Guiducci, noto galileiano, era impegnato nella gestione di un collegio per l'istruzione primaria a Firenze, circostanza che ha fatto ipotizzare possibili legami con le scuole pie, sebbene non vi siano menzioni dirette nelle carte calasanziane.⁴¹

Il rapporto tra Michelini e Galileo è invece ben documentato: ci sono pervenute dodici lettere, scritte tra l'8 aprile 1634 e il 7 dicembre 1639.⁴² Le prime due furono inviate durante i soggiorni romani del padre Francesco, quando entrò in contatto con il vivace ambiente

³⁷ APF: Reg. prov. 61–2, n. 1; Reg. prov. 61–3, Venuta e mutazione de' superiori, s. f.; *Epist.* 580.

³⁸ *EHI*, 1212.

³⁹ Antonio Favaro, *Le Opere di Galileo Galilei*, vol. XVI (Firenze: Tip. di G. Barbèra, 1904), lett. 1959, 46–47.

⁴⁰ *EHI*, 1212.

⁴¹ Cfr. Paolo Galluzzi, *Amici e corrispondenti di Galilei*, vol. II (Firenze: Salimbeni, 1985), 1413–1474; Bucciantini, *Contro Galileo*; Baldini, *Legem*; Federica Favino, "Guiducci, Mario," *DBI LXI* (2004).

⁴² Antonio Favaro, *Le Opere di Galileo Galilei*, voll. I–XX (Firenze: Tip. di G. Barbèra, 1890–1909).

scientifico raccolto attorno a Benedetto Castelli nel monastero di San Callisto.⁴³ Quel contesto ospitava alcuni dei protagonisti più significativi della nuova scienza: Giovanni Alfonso Borelli, Evangelista Torricelli, Bonaventura Cavalieri, Raffaello Magiotti, Tommaso Rinuccini e Vincenzo Renieri. Michelini si integrò rapidamente, guadagnandosi la stima di Castelli, che ne lodava l'intelligenza, la cultura e lo spirito vivace.⁴⁴

La corrispondenza tra Castelli e Galileo conferma la considerazione riservata a Michelini all'interno del circolo. Anche dopo il suo ritorno a Firenze, i legami con Castelli e gli altri membri restarono forti: nel 1635, l'abate esprime entusiasmo nell'apprendere che lo scolopio era a fianco di Galileo.⁴⁵ I contatti con Torricelli sono documentati da una lettera del 3 febbraio 1643, in cui i due discutono di geometria con l'ausilio di schemi illustrativi.⁴⁶ Con Raffaello Magiotti sopravvivono due lettere, datate 25 gennaio 1636 e 25 aprile 1637, nelle quali i due si confrontano su teoremi e questioni scientifiche.⁴⁷ Magiotti menziona Michelini in tre lettere indirizzate a Galileo tra il 1634 e il 1636, segno di un dialogo scientifico continuo.⁴⁸

Durante questo periodo, il padre Francesco mantenne rapporti anche con Tommaso Rinuccini, tornato a Firenze nel 1635 come gentiluomo di camera di Ferdinando II e copiere della granduchessa Vittoria della Rovere,⁴⁹ e con Vincenzo Renieri, che nel 1648 gli sarebbe subentrato nella cattedra di matematica all'Università di Pisa.⁵⁰

Nel corso del suo secondo soggiorno romano, Michelini si impegnò per ottenere l'invio di nuovi insegnanti alla scuola di matematica di Firenze. Dovette, però, scontrarsi con la limitata disponibilità dell'ordine, che riuscì a destinare solo due giovani religiosi, ostacolato dalla carenza di risorse e dagli spazi ristretti nella sede fiorentina.⁵¹

Nonostante le difficoltà logistiche, il padre Francesco trovò conforto nel sapere che i suoi confratelli avevano continuato a frequentare la villa di Arcetri anche in sua assenza. Questo suggerisce che il legame tra gli scolopi e Galileo non fosse limitato alla figura di Michelini, ma coinvolgesse anche altri membri della comunità, come Angelo Morelli, Salvatore Grise e Domenico Rosa.

Morelli, toscano e discepolo di Campanella, arrivò a Firenze nel 1632 per dirigere la

⁴³ Favaro, *Le Opere XVI*, lett. 2917, 76; lett. 2994, 139–140.

⁴⁴ Ibid., lett. 2916, 75.

⁴⁵ Ibid., lett. 3077, 210–211.

⁴⁶ Tosti, "Il P. Francesco," 323.

⁴⁷ Favaro, *Le Opere XVI*, lett. 3251, 384; *XVII*, lett. 3467, 64–66.

⁴⁸ Ibid., lett. 3008, 152–153; lett. 3196, 323–324; lett. 3396, 521–522.

⁴⁹ Cfr. Salvino Salvini, *Fasti consolari dell'Accademia fiorentina* (Firenze: 1717), 479–482; Stefano Tomassetti, "Rinuccini, Tommaso," *DBI LXXXVII* (2016).

⁵⁰ Antonio Favaro, "Amici e Corrispondenti di Galileo Galilei XII. Vincenzo Renieri," *Atti del Reale Istituto Veneto di Scienze, Lettere ed Arti LXIV*, 5 (1904): 111–195.

⁵¹ Favaro, *Le Opere XVI*, lett. 2994, 139–140.

classe di abaco e fu probabilmente il secondo scolopio a incontrare Galileo.⁵² Grise, fratello operaio e allievo di Benedetto Castelli, autore di un trattato di algebra pubblicato nel 1631, fu nominato supplente di Michelini nel 1633 e viene menzionato da Raffaello Magiotti in una lettera del 1636 come figura di riferimento in ambito matematico.⁵³ Domenico Rosa, anch'egli chierico operaio, giunse a Firenze nello stesso periodo e vi insegnò per cinque anni.⁵⁴ A loro si aggiunsero i giovani Clemente Settimi e Giovanni Battista De Ferraris, quest'ultimo trasferito a Genova al termine dell'anno scolastico.⁵⁵

Grazie a questo rafforzamento dell'organico, Michelini poté assumere nuovi incarichi presso la corte medicea. Dal 1635 divenne maestro dei paggi e, dal 1638, insegnante d'algebra ai fratelli del granduca Ferdinando II, Giovanni Carlo e Leopoldo. In una lettera del 29 marzo 1638 indirizzata a Galileo, lo scolopio lo ringraziava per l'intercessione presso la corte, lasciando intendere che l'incarico gli fosse stato affidato grazie al prestigio e al sostegno dello scienziato.⁵⁶

L'analisi dei primi insegnanti della scuola di matematica mostra che molti provenivano dal gruppo dei fratelli conversi, successivamente elevati a chierici operai, una scelta atipica rispetto alla tradizione che riservava le cattedre di grammatica ai sacerdoti. Eppure, fu proprio da questo ambito che emersero personalità di spicco come Michelini, Grise, Romani e Ambrogio Ambrogi. L'apertura dell'insegnamento a un pubblico più ampio rese possibile l'ascesa di nuovi talenti, spesso in diretto contatto con Galileo, la cui influenza fu decisiva per la loro formazione.

Malgrado ciò, l'11 novembre 1636, Michelini e Ambrogi furono ordinati sacerdoti per meriti straordinari, una deroga concessa per volontà papale, che suscitò tensioni interne. Secondo Picanyol, la richiesta potrebbe essere partita dai Medici.⁵⁷

Ancora una volta, il tacito assenso di Calasanzio non può essere ignorato. L'ordinazio-

⁵² Cfr. APF, Reg. prov. 61–3; Reg. prov. 61–4, Memorie de Pri e fratelli stati di famiglia in quella casa delle Scuole Pie di Firenze e degli Ordinari in essa da marzo 1646 a ottobre 1661; *EHI*, 1490–1492; Giovanni Giovannozzi, *Lettere inedite di G. A. Borelli al P. Angelo di S. Domenico, sulla versione di Apollonio* (Firenze: 1916); Picanyol, “Le Scuole Pie e Galileo,” 100–106; Valeria Vagnoli, “Morelli, Angelo,” *DBI* LXXXVI (2012): 580–583.

⁵³ Cfr. APF, Reg. prov. 61–3; Reg. prov. 61–4; *Epist.* IV, lett. 1925, 140; lett. 2225, 396; lett. 2302, 439; *Epist.* VI, lett. 2358, 15; Picanyol, “Le Scuole Pie e Galileo,” 111–115; Favaro, *Le Opere XVI*, lett. 3251, 384.

⁵⁴ Cfr. APF, Reg. prov. 61–3; Reg. prov. 61–4; Picanyol, “Le Scuole Pie e Galileo,” 121–122; Giovanni Ausenda, “Rosa, Domenico,” in *Diccionario Enciclopédico Escolapio*, vol. II. (Salamanca: Ed. Calasancias, 1983; da ora *DEE*), 475.

⁵⁵ Cfr. APF, Reg. prov. 61–3; Reg. prov. 61–4; Picanyol, “Le Scuole Pie e Galileo,” 99–100; 118–120; Giovanni Ausenda, “Settimi, Clemente,” in *DEE*, 510–511; Tosti, “Il Padre Clemente,” 99–177; Stefania Montacutelli, “Settimi, Clemente,” *DBI* XCII (2018): 340–342.

⁵⁶ Favaro, *Le Opere XVII*, lett. n. 3715, 321–322.

⁵⁷ Picanyol, “Le Scuole Pie e Galileo,” 83–91.

ne serviva probabilmente a rafforzare il profilo istituzionale della scuola, ormai affermata, affidandola a maestri formalmente sacerdotali. Il fondatore, del resto, non poteva permettersi di perdere Michelini, ormai centrale nelle relazioni con il granduca e il suo entourage.

L'impegno a corte assorbì progressivamente il padre Francesco, ma non interruppe il suo legame con Galileo. Le lettere testimoniano un rapporto affettuoso e ricco di scambi scientifici: Michelini inviava piccoli doni al maestro, gli chiedeva chiarimenti su teoremi da insegnare ai principi⁵⁸ e, nella lettera del 18 marzo 1638, riferiva di aver seguito lezioni del servita Poggi e del gesuita Niccolò Zucchi, promettendo di condividerne i contenuti con Galileo.⁵⁹ Questo episodio mostra che lo scolopio non era solo un allievo, ma un interlocutore attivo nel dibattito scientifico, e che anche i gesuiti, nonostante l'opposizione istituzionale, continuavano a contribuire al dialogo scientifico, coinvolgendo Galileo stesso.

Con il tempo, gli impegni di Michelini a corte si intensificarono, rendendolo sempre meno disponibile. Le sue risposte si fecero più rare, come lamentava Galileo in una lettera a Castelli del luglio 1638, dove si diceva frustrato per l'irreperibilità dell'amico, ormai pienamente assorbito dall'insegnamento dell'algebra ai principi medicei.⁶⁰

L'ultima lettera nota tra Michelini e Galileo è datata 7 dicembre 1639. In essa, lo scolopio presenta Ambrogio Ambrogi come suo naturale successore e lo raccomanda a Galileo affinché lo accolga tra i suoi discepoli.⁶¹ Ambrogi, già vicino a Benedetto Castelli, entrò ufficialmente nel circolo galileiano pochi giorni dopo, come conferma una lettera in cui Galileo lo descrive a Castelli come "un soggetto molto laudabile [...] adornato di ogni scienza, colmo di virtù, religione e santità."⁶²

Con il nuovo ingresso, il gruppo degli scolopi fiorentini legati allo scienziato pisano si ampliò ulteriormente. Sebbene nessuno di loro abbia lasciato opere di rilievo, le lettere dimostrano quanto fossero stimati dai contemporanei e come, pur nella discrezione, abbiano contribuito attivamente alla diffusione del pensiero scientifico galileiano all'interno delle scuole pie.

Malgrado l'intervento di Ambrogi e il suo incontro personale con Galileo, fu Clemente Settimi a raccogliere pienamente l'eredità di Michelini. Allievo diretto del maestro, Settimi ne assunse gradualmente il ruolo, diventando il nuovo punto di riferimento sia per la direzione della scuola di matematica che per le visite settimanali alla villa di Arcetri.

Conosciuto nell'ordine come Clemente di San Carlo, Settimi era arrivato a Firenze nel giugno 1635 insieme a Giovanni Battista De Ferraris, con l'incarico di affiancare Michelini nell'insegnamento. Appena un anno dopo, nel dicembre 1636, fu ordinato sacerdote,

⁵⁸ Vedi: Favaro, *Le Opere XVII*, lett. 3809, 399–400; lett. 3816, 407; lett. 3821, 411–412; *XVIII*, lett. 3842, 24–25.

⁵⁹ Ibid., lett. 3708, 316.

⁶⁰ Ibid., lett. 3765, 359.

⁶¹ Ibid., lett. 3948, 128.

⁶² Ibid., lett. 3950, 129–130.

segno della fiducia riposta in lui;⁶³ nel catalogo scolastico del 1638 è già indicato come “matematico ed economo,” a conferma del ruolo acquisito.⁶⁴

Con l’aggravarsi della cecità di Galileo, le visite periodiche di Morelli, Ambrogio e altri confratelli non bastavano più. Fu allora che la presenza di padre Clemente divenne indispensabile. Sempre più vicino allo scienziato, egli assunse il compito di lettore personale e assistente quotidiano, aiutandolo nella consultazione dei testi e nella discussione dei contenuti scientifici. In una lettera dettata a Baliani nel gennaio 1639, Galileo riconosce l’aiuto essenziale del giovane scolio, che trascorreva intere giornate con lui per leggere, commentare e ragionare insieme su dimostrazioni e teorie.⁶⁵

Il 16 aprile 1639, su intercessione della corte medicea – come risulta dalla corrispondenza tra l’ambasciatore Francesco Niccolini e il segretario Andrea Cioli – Calasanzio concesse formalmente a Settimi il permesso di pernottare ad Arcetri per assistere Galileo nelle ore notturne. Le parole rivolte al ministro di Firenze Domenico Romani rivelano un consenso pieno e consapevole:

[...] Et se per caso il Sig. Galileo dimandasse, che qualche notte restasse là il P. Clemente, V. R. glielo permetta, e Dio voglia che ne sappia cavare il profetti che doveria.⁶⁶

Questa comunicazione chiarisce che il fondatore non solo era perfettamente informato, ma approvava la vicinanza del suo religioso a Galileo. Le sue iniziali perplessità non riguardavano tanto la figura dello scienziato condannato, quanto l’eventuale scandalo interno all’ordine per un padre che trascorreva notti fuori dalla comunità.⁶⁷ Come osserva M. Bucciantini, la pressione della Corte fu determinante per superare ogni resistenza.⁶⁸ Dalle lettere emerge una decisione ponderata, volta a bilanciare le aspettative medicee con le preoccupazioni interne, in un contesto in cui alcuni maestri avevano ormai maturato un genuino interesse per la scienza galileiana.

Le conseguenze di questa vicinanza furono rilevanti. La frequentazione quotidiana di Galileo offrì a Settimi l’occasione di approfondire la propria formazione scientifica e di entrare in contatto con alcuni tra i più noti matematici del tempo: Vincenzo Renieri, Tommaso Rinuccini, Antonio Nardi, Benedetto Castelli, Raffaello Magiotti ed Evangelista

⁶³ Cfr. APF: Reg. prov. 61–3; Reg. prov. 61–4; Picanyol, “Le Scuole Pie e Galileo,” 99–100; 118–120; Tosti, “Il Padre Clemente,” 99–177; Montacutelli, “Settimi, Clemente.”

⁶⁴ APF, Reg. Prov. 61–6, Memorie de’ Padri, che con la bontà della vita, o eccellenza del sapere hanno illustrato questa Casa delle Scuole pie in Fiorenza, s. f.

⁶⁵ Favaro, *Le Opere XVIII*, lett. 3829, 11.

⁶⁶ *Epist. VII*, lett. 3074, 65; Favaro, *Le Opere XVIII*, lett. 3865, 41.

⁶⁷ Favaro, *Le Opere XVIII*, lett. 3866, 42.

⁶⁸ Bucciantini, “Eredità galileiana,” 379–399.

Torricelli.⁶⁹ La prossimità fisica con Galileo ridusse però le necessità di comunicazione scritta: delle lettere tra i due sono sopravvissute solo tre. La prima, datata 13 maggio 1640, fu inviata da Siena, dove Settimi agiva da intermediario tra il principe Leopoldo e Michelini;⁷⁰ le altre due risalgono ai suoi spostamenti a Roma nel 1640–41, durante i quali aggiornava Galileo sui propri viaggi e trasmetteva saluti da altri studiosi.⁷¹

Sotto la direzione di padre Clemente, la scuola di matematica visse una svolta decisiva. Da semplice spazio per lezioni private, si trasformò in una vera scuola superiore, con un'attività regolare avviata nell'anno scolastico 1638–1639. Nonostante i progressi, i locali di via dello Studio si rivelarono presto inadeguati. Già nel 1640, Settimi iniziò a sollecitare un trasferimento in una sede più adatta.⁷² Si valutò l'acquisto di una casa appartenente alla famiglia Minucci, ma il progetto naufragò per contrasti interni e, soprattutto, per le denunce mosse da Mario Sozzi. Le tensioni culminarono nei processi romani e portarono alla chiusura definitiva della scuola.⁷³

Sozzi, arrivato a Firenze nel novembre 1639 per incarichi liturgici e pastorali, era entrato in conflitto con i confratelli locali, probabilmente per effetto del clima teso legato al caso Ricasoli, di cui era stato il principale accusatore. Nel settembre 1640 Calasanzio tentò di trasferirlo a Narni, ma Sozzi, forte dei suoi contatti con l'Inquisizione fiorentina e romana, riuscì a restare, aggravando le frizioni. Lo scontro esplose il 7 febbraio 1641 con un alterco fisico tra Sozzi e Settimi: ferito, Sozzi si rifugiò presso l'inquisitore Giovanni Muzzarelli, cui presentò un dettagliato resoconto per il Sant'Uffizio. Si descrisse come vittima di violenza e soprusi, costretto a letto per otto giorni. Muzzarelli avviò un procedimento per indisciplina e trasmise gli atti al cardinale Francesco Barberini, che convocò gli imputati a Roma.⁷⁴

Oltre ad Ambrogio, Settimi e Morelli, tra i religiosi colpiti dalle accuse di Sozzi vi era anche Carlo Conti, noto come padre Carlo di S. Gasparo. Formatosi a Frascati nel 1631 sotto la guida di Tommaso Campanella, non fu subito destinato a Firenze, ma inviato in Moravia, a Nikolsburg (oggi Mikulov), dove insegnò umanità. È però significativa una sua lettera indirizzata a Galileo, del 18 agosto 1638, poco dopo il suo arrivo a Firenze, in cui

⁶⁹ Favaro, *Le Opere XVIII*, lett. 3841, 23–24; lett. 3879, 51–52; lett. 3884, 56–59; lett. 3891, 71; lett. 3900, 81–82; lett. 3925, 108–110; lett. 3934, 116; lett. 4144, 331–332.

⁷⁰ Favaro, *Le Opere XVIII*, lett. 4005, 189–190.

⁷¹ Ibid., lett. 4127, 317–318; lett. 4134, 323–324.

⁷² APF: Reg. prov. 61–6, s. f.; Reg. prov. 61–9, f. 34, lett. XVI, XIX.

⁷³ Ibid.; Ivi, lett. XIV; XV; XVI; XXVI.

⁷⁴ Cfr. Picanyol, "Le Scuole Pie e Galileo," 120–125; Bucciantini, "Eredità galileiana," 379–399; Tosti, "Il P. Clemente," 171–177; Claudio Vilá Palá, "El P. Mario Sozzi. I. Datos anteriores a su ingreso en la Escuela Pía," *ArSP* III, 5 (1979): 57–86; Id., "El P. Mario Sozzi. II. Su actuación en los años 1631–1639," *ArSP* IV, 7–8 (1980): 87–140; Id., "Mario Sozzi: ultimo cuatrienio de su vida (1639–1643)," *ArSP* XV, 29 (1981): 83–182; Alberto Tanturri, "Sozzi, Mario," *DBI* XCIII (2018): 406–408.

ricorda un precedente incontro con lo scienziato, durante il quale gli aveva recitato alcuni suoi “versacci,” segno di un rapporto personale già avviato.⁷⁵

Sebbene lo storico scolopico G. Giovannozzi lo liquidi con toni sprezzanti, giudicandolo privo di meriti e storicamente irrilevante – un’opinione probabilmente viziata dall’abbandono dell’ordine da parte di Conti dopo la “riduzione innocenziana” – il suo coinvolgimento nel mondo galileiano conferma quanto ormai radicata fosse l’influenza dello scienziato pisano tra gli scolopi fiorentini. La scelta di destinarlo a Firenze, piuttosto che inviare figure più ortodosse, suggerisce un’intenzione precisa da parte di Calasanzio, che pur mantenendo un profilo prudente, sostenne di fatto la prosecuzione di un’esperienza scolastica ispirata alla scienza galileiana.

Il primo tentativo concreto di colpire la scuola di matematica da parte delle autorità interne all’ordine si verificò solo durante il Capitolo generale dell’aprile 1641, a Roma. In quella sede, l’istituzione fiorentina fu duramente criticata e la sua esistenza messa in discussione. Il timore di un intervento del Sant’Uffizio, ancora impegnato a esaminare le denunce di Sozzi, rafforzò le posizioni degli oppositori del galileismo. Alla fine, però, non fu approvata alcuna misura restrittiva contro l’insegnamento delle matematiche. Secondo L. Picanyol, fu l’abilità diplomatica di Clemente Settimi a scongiurare il provvedimento;⁷⁶ per M. Bucciantini, il merito va invece al granduca Ferdinando II, intervenuto tramite l’ambasciatore Francesco Niccolini.⁷⁷ Una sua lettera del 20 aprile 1641 al Bali Giovanni Battista Gondi testimonia l’interesse diretto del granduca nel promuovere l’insegnamento delle scienze, anziché reprimerlo.⁷⁸

Poche settimane dopo, arrivò un altro segnale positivo. In una lettera dell’11 maggio 1641 indirizzata al principe Leopoldo, Settimi annunciava l’assoluzione degli scolopi coinvolti e il loro prossimo rientro a Firenze, anche se non tutti fecero effettivamente ritorno.⁷⁹

Ma il clima mutò rapidamente. Nell’ottobre dello stesso anno, Mario Sozzi presentò una nuova denuncia al Sant’Uffizio, accusando Michelini, Settimi, Ambrogio, Conti e Morrelli di essere “seguaci di Galileo” e di diffondere attivamente le sue teorie nella scuola. Secondo Sozzi, quei religiosi non solo consideravano la matematica la più certa delle scienze, ma identificavano nel metodo galileiano l’unico modo autentico di filosofare, attribuendogli persino un valore spirituale. Alcuni di loro, affermava, lo consideravano lo strumento più vero per conoscere Dio.⁸⁰

In un terzo documento inviato all’Inquisizione, Sozzi articolò con maggiore precisione le sue accuse, attribuendo agli scolopi dottrine ritenute eretiche. Michelini veniva indicato

⁷⁵ APF, Reg. prov. 613, s. f.; Reg. prov. 61–4, s. f.; Favaro, *Le Opere XVII*, lett. 3777, 368.

⁷⁶ Picanyol, “Le Scuole Pie e Galileo,” 98–99.

⁷⁷ Bucciantini, “Eredità galileiane,” 391–392.

⁷⁸ ECC, VI, 2938.

⁷⁹ Archivio di Stato di Firenze (da ora ASF), Mediceo Del Principato, f. 5561, 673.

⁸⁰ ECC, V, 2524–2526.

come il capo del gruppo e accusato di insegnare pubblicamente idee atomistiche e teorie eliocentriche, in aperto contrasto con la filosofia aristotelica che, secondo la denuncia egli disprezzava. Galileo veniva invece celebrato da questi religiosi come guida indiscussa, le cui affermazioni dovevano ritenersi incontestabili. Anche Settimi e Ambrogio furono coinvolti: il primo per aver negato la creazione divina, sostenendo un universo regolato solo dai cieli; il secondo per aver professato in modo radicale l'atomismo, arrivando a mettere in dubbio perfino l'esistenza dei colori, in palese contraddizione con la dottrina della Chiesa.⁸¹

Sebbene eccessive, queste accuse riflettono il diffuso timore che il galileismo potesse minare l'ordine teologico e culturale. In un'epoca in cui l'atomismo e la matematica sperimentale erano considerate discipline pericolose, insegnarle all'interno di un contesto religioso appariva di per sé un atto sovversivo.

Gli storici scolopici tendono a leggere l'iniziativa di Sozzi come il frutto di rancori personali e di rivalità interne. Bucciantini propone un'interpretazione più ampia: gli attacchi a Michelini – che insegnava matematica ai principi Giovan Carlo e Leopoldo – erano anche una forma indiretta di critica alla politica culturale dei Medici. Nelle sue denunce, Sozzi contestava non solo l'insegnamento della “nuova filosofia,” ma anche il coinvolgimento diretto del granduca e di Leopoldo nel promuoverla. Accusava Michelini di educare i giovani principi secondo le teorie di Galileo, di far circolare copie dei suoi scritti tra gli studenti e di ricevere fondi dalla corte per sostenere allievi promettenti come Candido Del Buono, il cui fratello percepiva uno stipendio granducale.⁸²

In quest'ottica, la denuncia assumeva anche il tono di un avvertimento politico: un messaggio rivolto alla dinastia medicea affinché interrompesse ogni sostegno a quelle scuole e a quei religiosi che si facevano portatori del pensiero galileiano.⁸³

Alla fine, tra tutti gli accusati, fu Settimi l'unico a dover comparire davanti al Sant'Uffizio. Michelini, protetto dalla corte, non venne coinvolto formalmente, mentre gli altri scolopi avevano già lasciato Firenze. Morelli, citando “gravi difficoltà” nel restare in città, ottenne l'autorizzazione al trasferimento a Roma il 30 novembre 1640.⁸⁴ Ambrogio, dopo vari spostamenti, si stabilì definitivamente a Napoli il 22 giugno 1641.⁸⁵ Conti, dopo un periodo a Pisa, rientrò brevemente a Firenze nel 1642, per poi essere assegnato a Genova.⁸⁶

Rimasto solo, Settimi fu costretto a recarsi a Roma per rispondere alle accuse. Numerose testimonianze documentano l'impegno della corte medicea nel difendere gli scolopi, con il coinvolgimento del cardinale Giulio Sacchetti e una fitta corrispondenza tra l'amba-

⁸¹ Ibid.

⁸² Ibid.

⁸³ Bucciantini, “Eredità galileiane,” 393–394.

⁸⁴ *Epist.*, VII, lett. 3537, 324.

⁸⁵ *ECC*, V, 2938.

⁸⁶ *Epist.*, VII, lett. 3870, 53.

sciatore Niccolini e il Bali Gondi.⁸⁷

Anche grazie all'appoggio dei Medici, lo scolopio fu assolto; tuttavia, fu obbligato a trasferirsi in Sicilia. Nel frattempo, la scuola di matematica venne chiusa, il gruppo dei galileiani disperso, e Sozzi fu ricompensato con la nomina a provinciale dell'ordine in Toscana, per decreto del Sant'Uffizio, il 4 dicembre 1641.

La protezione medicea non fu sufficiente neanche a impedire la cosiddetta "riduzione" imposta da papa Innocenzo X nel 1646, che declassò l'ordine degli scolopi a semplice congregazione priva di voti solenni, aprendo una lunga fase di declino. L'ordine recuperò il proprio status solo nel 1669, grazie a papa Clemente IX – ancora una volta, anche per merito dell'intercessione medicea.

Eppure, nonostante le repressioni, la scuola ebbe il tempo per lasciare un'impronta profonda: formò un'intera generazione di giovani patrizi fiorentini e contribuì in modo decisivo alla nascita di nuove esperienze culturali, prima fra tutte l'Accademia del Cimento. Fondata da Leopoldo de' Medici nel 1657, fu la prima istituzione scientifica europea basata sul metodo sperimentale galileiano, e annoverava tra i suoi membri molti ex allievi delle scuole pie.⁸⁸

Il più celebre tra questi fu Vincenzo Viviani (1622–1703). Appartenente a una famiglia patrizia, fu allievo di padre Clemente Settimi e, dal 1639, segretario personale di Galileo. Si trasferì ad Arcetri per assisterlo nella redazione dei suoi ultimi scritti e, dopo la sua morte, proseguì gli studi sotto la guida di Evangelista Torricelli. Nel 1647 fu nominato matematico di corte e ingegnere della Magistratura della Parte Guelfa.

Viviani contribuì a diversi ambiti della scienza. Nel 1692 formulò un celebre problema geometrico risolto attraverso la curva oggi nota come "finestra di Viviani." In fisica, riprese gli studi galileiani sul moto, osservò la rotazione del piano di oscillazione del pendolo, collaborò con Torricelli agli esperimenti barometrici, e progettò strumenti per misurare la

⁸⁷ Il rapporto tra i Medici e gli scolopi, così come i tentativi di intermediazione a favore dell'ordine, sono ampiamente documentati nella mia tesi di dottorato "Gli Scolopi nel Granducato mediceo (1630–1737). Storia del rapporto culturale, politico ed economico tra le Scuole Pie, la corte e la società toscana" (PhD diss., Università degli studi di Firenze e Università di Siena, 2025).

⁸⁸ Per l'Accademia del Cimento vedi: W.E. Knowles Middleton, *The Experimenters. A Study of the Accademia del Cimento* (Baltimore–London: Johns Hopkins University Press, 1971); Paolo Galluzzi, "L'Accademia del Cimento: «gusti» del Principe, filosofia e ideologia dell'esperimento," *Quaderni Storici* XVI, 48 (1981): 788–844; Id., *Scienziati a corte. L'arte della sperimentazione nell'Accademia galileiana del Cimento (1657-1667)* (Livorno: Sillabe, 2001); Mordechai Feingold, "The Accademia del Cimento and the Royal Society," in *The Accademia del Cimento and its European context*, eds. di Marco Beretta, Antonio Clericuzio e Lawrence M. Principe (Sagamore Beach: Science history publications, 2009), 229–242; Jean Boutier, Maria Pia Paoli e Claudia Tarallo, eds., *Dizionario storico delle Accademie toscane. Secoli XVI–XVIII, Firenze*, vol. I (Pisa: Pacini, 2024).

pressione atmosferica in funzione dell'altitudine. Con Giovanni Alfonso Borelli calcolò la velocità del suono in aria, applicando un metodo diretto suggerito da Galileo.

Egli fu anche un instancabile divulgatore. La sua edizione degli *Elementi* di Euclide, pubblicata nel 1690 e reinterpretata alla luce del pensiero galileiano, conobbe una fortuna duratura e fu riproposta come testo scolastico fino al XIX secolo. Fu autore anche di una biografia di Galileo, il *Racconto storico della vita di Galileo*, che però incontrò ostacoli da parte della censura ecclesiastica.

Figura centrale dell'Accademia del Cimento, Viviani fu accolto nella Royal Society di Londra nel 1696 e, tre anni dopo, nell'Académie des sciences di Parigi. La sua importanza storica risiede anche nel suo ruolo di ponte tra due epoche della scienza: da un lato i galileiani, di cui fu diretto erede; dall'altro i newtoniani e i leibniziani, rappresentati dal suo allievo e collaboratore Luigi Guido Grandi.

Al centro di tutto vi era la Corte medicea. Il suo sostegno alla scienza non era solo un gesto culturale, ma parte di una strategia politica ben precisa: rafforzare il prestigio intellettuale di Firenze e Pisa e migliorare le condizioni economiche del Granducato. Questo obiettivo si traduceva nello sviluppo di tecnologie applicate a ingegneria civile, meccanica, idraulica e architettura militare. La fama di Galileo e il favore dei Medici attirarono in Toscana studiosi da tutta Europa, interessati a un ambiente in cui la scienza era non solo coltivata, ma sostenuta e valorizzata come motore di progresso concreto.

Le scuole pie svolsero un ruolo importante nel processo di rinnovamento culturale del Granducato, soprattutto attraverso la scuola di matematica, creata fin dall'inizio per rispondere alle nuove inclinazioni scientifiche della Corte medicea, fortemente influenzate dal pensiero galileiano. In pochi anni, l'istituto calasanziano fu in grado di formare figure come Vincenzo Viviani, capaci di trasferire le conoscenze acquisite nel contesto scolopico in ambienti strategici come la corte granducale e le accademie scientifiche.

Anche i gesuiti contribuirono in modo rilevante al vivace panorama intellettuale della Toscana, come dimostrano non solo le attività di Niccolò Zucchi, ma anche il caso emblematico di Honoré Fabri. Gesuita francese, fu trasferito nel 1646 dal collegio di Lione a Roma per le sue simpatie copernicane e galileiane, viste con sospetto dai superiori. A Roma operò presso l'osservatorio del Collegio Romano, collaborando con scienziati come Francesco Eschinardi, Gilles François de Gottignies e Giuseppe Ferroni, e mantenendo rapporti diretti con il principe Leopoldo. La sua partecipazione ai lavori dell'Accademia del Cimento conferma il suo coinvolgimento nella vita scientifica italiana e il legame tra Compagnia di Gesù e cultura sperimentale.

Fabri mantenne anche una fitta corrispondenza con Giovan Battista Riccioli, figura di spicco dell'osservatorio gesuita di Santa Lucia a Bologna, noto per il suo approccio empirico e per i suoi studi sul moto dei corpi⁸⁹. L'intreccio di queste relazioni mostra come, all'in-

⁸⁹ Aricò, "Les yeux d'Argos," 285–303.

terno della Compagnia, non mancassero posizioni favorevoli alla scienza sperimentale, e come l'iniziativa dei singoli potesse innescare reti di collaborazione che andavano oltre i confini delle rigide gerarchie ecclesiastiche. In molti casi, questo contribuì al lento ma progressivo superamento della tradizione aristotelica, senza rinunciare all'identità religiosa.

Tutti questi sviluppi si inserirono nel clima culturale toscano a cavallo tra Seicento e Settecento, segnato da un confronto sempre più aperto tra scienza e religione. Il dibattito su come conciliare l'orizzonte cattolico con le nuove correnti filosofiche e scientifiche fu alimentato dalla politica culturale dei Medici: una strategia prudente ma lungimirante, che cercava di bilanciare le richieste della Santa Sede e del Sant'Uffizio con l'esigenza di promuovere il progresso scientifico, il prestigio delle istituzioni fiorentine e lo sviluppo tecnico del Granducato.⁹⁰

L'esperienza della scuola di matematica delle scuole pie, pur nella sua breve durata e nella sua natura controversa, lasciò tracce significative. Contribuì a diffondere, almeno in alcuni settori dell'ordine, una sensibilità verso la scienza sperimentale che, nei decenni successivi, avrebbe trovato nuove espressioni, sempre però condizionate dal contesto politico e dalle protezioni locali.

Questa apertura alla nuova scienza si diffuse anche grazie a figure come Angelo Morelli, che trasmise il proprio bagaglio intellettuale al noviziato di Chieti. Tra i suoi allievi spiccò Carlo Giovanni Pirroni, futuro generale dell'ordine dal 1677 al 1685, protagonista di un'importante riforma normativa e organizzativa. Dopo la difficile fase della "riduzione," lo studentato di Chieti, sotto la guida di Morelli, si affermò come centro nevralgico per la formazione della nuova élite scolastica, attirando studenti da tutta Italia.⁹¹

Tra i nomi più significativi formati in quel contesto vi furono Giuseppe Vassallo – che più tardi operò anche a Firenze – e i toscani Giovan Battista de Francisci, Giovanni Magagnini, Bonifazio Landi e Francesco Maria Balestri, futuro provinciale della Toscana. Un'altra figura centrale fu Domenico Rossi, docente di matematica e allievo di Borelli al collegio romano di S. Pantaleo.⁹²

Proprio Giovanni Alfonso Borelli (1608–1679), insieme a Morelli, fu uno dei principali artefici della diffusione del galileismo tra gli scolopi. Fin dagli anni Trenta del Sei-

⁹⁰ Vincenzo Ferrone, *Scienza Natura religione. Mondo newtoniano e cultura italiana nel primo Settecento* (Napoli: Jovene editore, 1982); Marcello Verga, "La cultura del Settecento. Dai Medici ai Lorena," in *Storia della civiltà toscana*, vol. IV: *I Lumi del Settecento*, ed. di Furio Diaz (Firenze: Cassa di Risparmio, 1999), 125–152.

⁹¹ Cfr.: Leodogario Picanyol, *Brevis conspectus historico-staticus ordinis Scholarum Piarum* (Roma: Apud Curiam Generalitiam, 1932); Id., "Le Scuole pie e Galileo," 184–186; Maurizio Sangalli, "Un Generale alle prese con la riorganizzazione delle Scuole Pie: Carlo Giovanni Pirroni e le sue prime quattro circolari 1677–1681," *ArSP XXXIV* (1996): 15–44; Alberto Tanturri, "Gli Scolopi nel Mezzogiorno d'Italia in età moderna," *ArSP XIV*, 50 (2001).

⁹² Picanyol, "Le Scuole Pie e Galileo," 177–194.

cento frequentò ambienti scientifici d'avanguardia, entrando in contatto con Michelini, Benedetto Castelli, Torricelli e Ricci. Tra il 1641 e il 1642 visse a Firenze, dove, pur non essendo certo un suo incontro diretto con Galileo, respirò quell'atmosfera intellettuale che ne plasmò definitivamente l'orientamento scientifico. Le sue prime opere riflettono chiaramente l'influsso galileiano, sia nei contenuti sia nel metodo.

Nel 1656 ottenne la cattedra di matematica a Pisa – già occupata da Michelini – e avviò una serie di progetti innovativi, tra cui una nuova edizione degli *Elementi* di Euclide (1658). Collaborò attivamente con il principe Leopoldo e l'Accademia del Cimento, conducendo esperimenti sulla propagazione del suono, la pressione atmosferica, la resistenza dei materiali e lo studio dei fenomeni idraulici.

Il suo contributo più significativo fu però in campo astronomico. Le sue osservazioni telescopiche, raccolte nel *Theoricae mediceorum planetarum*, rappresentano una chiara adesione al copernicanesimo e al galileismo, anticipando in alcuni punti le future teorie newtoniane.

Nel 1667, a seguito di contrasti con Viviani e l'ambiente accademico fiorentino, Borelli lasciò la Toscana. Dopo un periodo a Messina, dove fu coinvolto in una sommossa anti-spagnola, tornò a Roma nel 1673. Qui, grazie all'appoggio di Pirroni, ottenne una cattedra presso le scuole pie di S. Pantaleo, dove fondò un'accademia matematica centrata sull'insegnamento della geometria euclidea e della meccanica galileiana. Rimase in carica fino alla morte, lasciando un'eredità scientifica profonda.⁹³

Secondo Stefania Montacutelli, l'ingresso di Borelli a S. Pantaleo non fu casuale, ma parte di una strategia ben definita: la sua nomina, voluta da Pirroni, mirava a consolidare la trasmissione del pensiero galileiano all'interno dell'ordine, attraverso la formazione di una nuova generazione di religiosi destinati all'insegnamento nei collegi e nelle università.⁹⁴

Le ragioni alla base di questa scelta erano due: da un lato, molti membri dell'ordine, incluso Pirroni, avevano ricevuto una formazione ispirata al galileismo; dall'altro, la padronanza della nuova scienza rappresentava un fattore di prestigio per l'identità educativa delle scuole pie.

⁹³ Cfr. L. Picanyol, *Alfonso Borelli e il P. Carlo Giovanni Pirroni delle Scuole Pie* (Roma: Apud Curiam Generalitiam, 1933); Id., *Le scuole pie e Galileo*, 192–194; Ugo Baldini, "Borelli, Giovanni Alfonso," *DBI* XII (1970): 543–551; Id., "Giovanni Alfonso Borelli e la rivoluzione scientifica," *Physis*, XVI, 2 (1974): 97–128; Id., "Gli studi su Giovanni Alfonso Borelli," in *La scuola galileiana: prospettive di ricerca. Atti del Convegno di studio di Santa Margherita Ligure (26-28 ottobre 1978)*, eds. di Giovanni Arrighi, Ugo Baldini, Lanfranco Belloni, Sergio De Angeli, Paolo Galluzzi e Maurizio Torrini (Firenze: La Nuova Italia, 1979), 111–135; Luigi Guerrini, "Matematica ed erudizione. Giovanni Alfonso Borelli e l'edizione fiorentina dei libri V, VI e VII delle Coniche di Apollonio di Perga," *Nuncius. Journal of the Material and Visual History of Science* XIV, 2 (1999): 505–568; Montacutelli, "Da Galileo a Borelli," 181–209; Ead., "Giovanni Alfonso Borelli"; Favino, "Giovanni Alfonso Borelli's Last Will (1679, December 31st)," 144–173.

⁹⁴ Montacutelli, "Da Galileo a Borelli," 195–197.

I risultati furono tangibili. Dall'accademia di Borelli emerse un gruppo compatto di allievi che continuarono la sua opera in tutta la penisola. Tra loro, si distinsero figure come i napoletani Agostino di S. Tommaso d'Aquino e Domenico di S. Giuseppe, il fiorentino Antonio di S. Filippo, il genovese Giovanni Francesco di S. Romolo, il savoiano Carlo Emanuele del Beato Amedeo e il romano Filippo Maria Tosini di S. Michele.⁹⁵

Quest'ultimo ebbe un ruolo particolarmente rilevante. Dopo un periodo di formazione a S. Pantaleo (1677–1679), fu inviato a Città della Pieve, dove il generale Foci gli affidò la cattedra di matematica. Nel 1693 venne nominato lettore all'Università di Urbino; dopo la soppressione della cattedra nel 1697, divenne rettore della Casa di Poli, poi dello studentato di Narni, dove morì nel 1704. La sua opera *Trattato in difesa del sistema copernicano* (1696) testimonia l'impatto dell'accademia borelliana e l'impegno dell'ordine a favore della nuova scienza, al punto da sostenere apertamente la pubblicazione di un testo tanto innovativo.⁹⁶

Se dunque, in alcuni contesti – come il collegio romano di S. Pantaleo – l'apertura al metodo sperimentale galileiano trovò spazi di sviluppo, anche a Firenze, nonostante la chiusura della scuola autonoma, l'insegnamento delle scienze continuò nelle classi superiori di filosofia e teologia.⁹⁷ Questa continuità, però, rimase legata alle protezioni politiche locali e all'iniziativa di singoli maestri, più che a un indirizzo unitario dell'ordine. Il suo mantenimento non fu affatto scontato, neppure nel XVIII secolo, nonostante la protezione dei Medici e il ruolo dell'Accademia del Cimento nel promuovere la circolazione delle teorie scientifiche moderne nel Granducato.

Un primo campanello d'allarme arrivò nel 1669–1670, con la repressione e l'espulsione dall'ateneo pisano dei sostenitori di una corrente atomista e materialista di orientamento galileiano.⁹⁸ Un secondo segnale fu l'insuccesso dei tentativi di Viviani, del cardinale Leopoldo e di altri intellettuali di ottenere una piena riabilitazione dell'opera di Galileo. Per oltre cinquant'anni, infatti, la ristampa dei suoi scritti in Italia fu ostacolata dal rigido controllo esercitato dalle autorità ecclesiastiche. La prima edizione fiorentina vide la luce solo nel 1718, ma fu un insuccesso: priva delle opere fondamentali e priva di un ordine coerente, venne subito criticata per la sua inefficacia.⁹⁹

Questa vicenda è indicativa delle difficoltà incontrate dagli intellettuali italiani nel fronteggiare una censura ancora capace di imporsi attraverso l'apparato ecclesiastico. Sebbene la posizione ufficiale della Compagnia di Gesù nei confronti della scienza galileiana

⁹⁵ Picanyol, *Alfonso Borelli*, 1–35.

⁹⁶ Per le notizie su Tosini vedi: Montacutelli, "Da Galileo a Borelli," 181–209.

⁹⁷ Cfr. APF, Reg. Sch. 1: Prefettura 1695–1698; Reg. Sch. 2: Prefettura 1699–1702; Reg. Sch. 3: Prefettura 1702–1712; 1706–1708; 1709–1712; Reg. Sch. 4: Prefettura 1717–1718; 1718–1719; 1726–1727; 1727–1728; 1736–1737; 1736–1739.

⁹⁸ Ferrone, *Scienza*, 109–148; Verga, "La cultura," 125–152.

⁹⁹ Ferrone, *Scienza*, 109–148.

e newtoniana rimanesse ostile, nel corso del Settecento iniziarono a emergere segnali di frattura interna.

Un episodio emblematico fu la polemica sollevata negli anni Ottanta del Seicento da un breve scritto del gesuita lucchese Giovanni Francesco Vanni, che attaccava un teorema galileiano sulle leggi del moto. Il testo provocò reazioni accese e coinvolse figure di primo piano come Leibniz, Bernoulli, Viviani e Grandi. Come osserva Maurizio Torrini, l'intervento di Vanni non fu soltanto un attacco alla fisica galileiana, ma un chiaro avvertimento ai membri della Compagnia che iniziavano a sostenere Galileo apertamente, mettendo in discussione l'ortodossia ufficiale dell'ordine.¹⁰⁰

Un altro caso esemplare di trasmissione del pensiero galileiano all'interno della Compagnia di Gesù è quello di Giuseppe Ferroni, originario di Pistoia. Formatosi a Pisa sotto la guida di Borelli, fu proprio quest'ultimo a introdurlo al galileismo e a favorirne la collaborazione con Viviani. Grazie all'influenza di Borelli presso la Corte medicea, Ferroni ottenne nel 1686 la cattedra di matematica all'Università di Siena, che mantenne fino alla morte nel 1709. Parallelamente, insegnava presso il collegio gesuita Tolomei.¹⁰¹

Sebbene il numero degli studenti inizialmente fosse esiguo (tra sei e quindici nei primi cinque anni), l'impatto della sua attività fu tutt'altro che marginale. Per oltre vent'anni, un gesuita vicino al galileismo riuscì a insegnare simultaneamente in un'università prestigiosa e in una scuola della Compagnia. Le sue lettere a Viviani rivelano un crescente sconforto per la mancata pubblicazione dei suoi lavori, ostacolata, a suo dire, da censori ancora legati all'aristotelismo. Il suo percorso, tuttavia, non fu isolato: Ferroni intrattenne contatti anche con Honoré Fabri, contribuendo a consolidare una rete di gesuiti favorevoli alla nuova scienza all'interno di un ordine generalmente ostile.

Anche nel mondo scolastico, un secolo dopo l'esperienza della scuola di matematica fiorentina, emerse una figura che ne raccolse l'eredità: Odoardo Corsini (1702–1764). Per dieci anni insegnò nelle scuole pie di Firenze, mantenendo vivo l'interesse per le scienze sperimentali e cercando nuove connessioni tra filosofia e teologia. Questo spirito si riflette nelle sue *Institutiones*, un'opera in sei volumi che spazia dalla logica alla fisica, dall'astronomia alla cosmologia, dalla psicologia alla metafisica, fino alla geometria.¹⁰²

Nonostante fossero trascorsi oltre ottant'anni dalla chiusura della scuola galileiana, la storia sembrò ripetersi: nel 1725 Corsini fu accusato di eterodossia e sospeso dall'insegnamento per aver incoraggiato i suoi studenti alla lettura di autori considerati non ortodossi. Tuttavia, ottenne il sostegno della Casa Medici, decisa a difendere il primato culturale to-

¹⁰⁰ Maurizio Torrini, *Dopo Galileo. Una polemica scientifica (1684-1711)* (Firenze: Olschki, 1979), 36–77.

¹⁰¹ Verga, "La cultura," 125–152; Paul F. Grendler, *The Jesuits and Italian Universities 1548–1773* (Washington: The Catholic University of America Press, 2017), 384–392.

¹⁰² Edoardo Corsini, *Institutiones philosophicae ac mathematicae ad usum Scholarum Piarum*, voll. I–VI (Firenze: 1731–1734).

scano. Grazie a questo appoggio, fu nominato titolare della cattedra di matematica e logica all'Università, e nel 1754 fu eletto generale dell'ordine, a conferma del suo pieno reintegro e della rivalutazione del suo operato.¹⁰³

Tra i suoi allievi più brillanti vi fu Stanislao Canovai (1740–1812), matematico fiorentino autore delle *Lezioni elementari di matematiche*, un manuale ispirato al modello newtoniano-euleriano. Canovai, con il suo discepolo Del Ricco, assunse la direzione dell'osservatorio Ximeniano, fondato nel 1756 dal gesuita Leonardo Ximenes, e poi affidato a Giovanni Inghirami, noto per la realizzazione della carta topografica della Toscana.

La vicenda, che attraversa oltre un secolo – dalle prime lezioni di Michelini ad Arcetri fino alla cattedra universitaria di Corsini – mostra con chiarezza quanto complesso sia stato il rapporto tra gli scolopi e la scienza galileiana, sempre sospeso tra adesione intellettuale e condizionamenti politici.

Tale complessità impone di affrontare una questione interpretativa di fondo: stabilire se l'esperimento della scuola di matematica degli scolopi costituisca un autentico caso di galileismo istituzionalizzato all'interno di un ordine religioso, o piuttosto il risultato della tolleranza medicea – un fenomeno reso possibile da un contesto politico favorevole, ma non necessariamente fondato su una piena adesione epistemologica al metodo galileiano.

La risposta non può essere univoca. Da un lato, la presenza continuativa di maestri come Michelini, Settimi, Ambrogio e Morelli, la loro frequentazione diretta di Galileo, l'insegnamento di contenuti esplicitamente copernicani e l'uso dei suoi testi testimoniano un orientamento scientifico chiaro e consapevole. La formazione di figure come Viviani, Del Buono e altri membri dell'Accademia del Cimento conferma l'efficacia didattica e la coerenza metodologica della scuola.

Dall'altro, è innegabile che tale esperienza fu resa possibile soprattutto dalla protezione politica della corte granducale. Senza l'appoggio di Ferdinando II e del principe Leopoldo, difficilmente gli scolopi avrebbero potuto resistere alle pressioni del Sant'Uffizio e alle denunce interne. La chiusura della scuola nel 1641, seguita dalla dispersione dei maestri, dimostra quanto fragile fosse quell'equilibrio una volta venuto meno il sostegno mediceo. Anche la successiva ripresa settecentesca dell'insegnamento scientifico – con Corsini e Canovai – si inserì sempre all'interno di una strategia culturale sostenuta dai granduchi, prima i Medici, poi i Lorena.

Inoltre, non tutti gli scolopi aderirono convintamente al galileismo. Le tensioni interne, le accuse di Sozzi, la presenza di testi antigalileiani nella biblioteca della scuola, e la stessa prudenza di Calasanzio suggeriscono che la scelta di sostenere le scienze sperimentali non fu unanime né priva di ambiguità. L'ordine, nato per educare i poveri, si trovò coinvolto in

¹⁰³ Cfr. Picanyol, *Brevis conspectus*, 25–26; Ugo Baldini, "Corsini, Odoardo," *DBI* XXIX (1983), 620–625; Favino, "Ordini religiosi."

un progetto elitario e controverso, rispondente più alle esigenze del patriziato fiorentino che alla sua missione originaria.

In conclusione, definire la scuola di matematica scolopica semplicemente “galileiana” sarebbe forse eccessivo: essa fu piuttosto “medicea” nelle sue origini e nella sua sopravvivenza, espressione di una politica culturale illuminata ma anche strumento di prestigio dinastico. Ciò non toglie che, al suo interno, si formò una generazione di maestri genuinamente convinti della validità del metodo sperimentale, capaci di trasmetterlo con rigore e passione. L'esperimento scolopico rappresenta dunque un caso ibrido: non un'istituzionalizzazione autonoma del galileismo, ma neppure una mera concessione esteriore. Fu piuttosto un incontro, fragile ma fecondo, tra scienza, politica e religione, in cui ciascuna componente influenzò profondamente le altre, lasciando un'impronta duratura nella storia educativa e scientifica italiana.

Riferimenti

Abbreviazioni

APF=Archivio della Provincia Italiana dei P. P. Scolopi a Firenze

ASF=Archivio di Stato di Firenze

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The solar eclipse of 1654 in the Grand Duchy of Lithuania: astrological prognosis, discussion, and the war

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Abstract

The French historian Élisabeth Labrousse dedicated a monograph to the solar eclipse of 1654, an event that sparked widespread debate and panic across Europe, including in Poland and the Grand Duchy of Lithuania. This study examines predictions related to the eclipse in: the work of Andrea Argoli, the European almanacs and calendars, the prognostication by the German astrologer Stefan Furman as well as the contemporary discussions in Poland and the Grand Duchy of Lithuania. Special attention is given to the role of astrology in elite society, particularly its connection to war prophecies. The eclipse became a focal point for clergy, nobility, and astrologers, whose interpretations varied dramatically. Forecasts of imminent wars and epidemics exacerbated apocalyptic fears, with many perceiving war as a literal apocalypse.

Keywords

the solar eclipse of 1654, astrology, European almanacs, Grand Duchy of Lithuania

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The history of astrology encompasses many topics and branches. Astrological forecasts and calendars also attract significant attention in historiography. Scholars have analyzed the solar eclipse of 12 August 1654, because of the great panic and much discussion that this event provoked in European countries. French historian Elisabeth Labrousse devoted a special book to this event. She investigated European astrological literature related to the solar eclipse.¹ The historiography of eclipse interpretation has predominantly centered on Western Europe. Even within this established framework, however, the topic requires further scholarly development.² Poland and the Grand Duchy of Lithuania (GDL) were not included in this analysis. Incorporating these cases would not only broaden the general understanding of the phenomenon, but also highlight the specific dynamics of 17th-century intellectual exchange in this region.

The work analyzes the prediction of the eclipse of 1654 in Andrea Argoli's work, European calendars, the prognostication of German astrologer Stefan Furman, and the discussion of the prognosis in Poland and the Grand Duchy of Lithuania, as well as its implementation. Additionally, attention is drawn to the role of astrology in politics and international relations, its use for decision-making, and its impact on society. Emphasis is also placed on the significance of astrology in emergency situations such as war. All these questions are traced through the analysis of various types of sources such as astrological literature, religious literature, correspondence, diaries, memoirs, etc.

This study has a threefold purpose. First, it aims to reconstruct the specific astrological forecast for the 1654 eclipse by tracing its transmission from Andrea Argoli's work through European calendars and local prognostications. Second, it seeks to investigate the political and social functions of astrology by demonstrating its concrete role in statecraft, international relations, and popular belief. Finally, it endeavors to assess the practical influence of this forecast on decision-making and public sentiment during the pre-war period.

¹ Lynn Thorndike, *A History of Magic and Experimental Science. The Seventeenth Century*, vol. 7 (New York; London: Columbia University Press, 1959), 309; Elisabeth Labrousse, *L'Entrée de Saturne au Lion. L'éclipse du soleil du 12 août 1654* (La Haye: M. Nijhoff, 1974); William Burns, "The Terriblest Eclipse That Hath Been Seen in Our Days': Black Monday and the Debate on Astrology during the Interregnum William," in *Rethinking the Scientific Revolution*, ed. by Margaret Osier (Cambridge, New York: Cambridge University Press, 2000), 137–152; Jacques Halbronn, "Questions autour du texte sur l'éclipse de 1654 attribué à Gassend," in *Gassendi et la modernité*, ed. by Sylvie Taussig (Turnhout: Brepols Publishers, 2008), 311–346.

² Michel de Certeau, "Élisabeth Labrousse, *L'Entrée de Saturne au Lion. L'Éclipse de soleil du 12 août 1654*," *Annales. Economies, sociétés, civilisations* 5 (1975): 1138–1141.

Italian astrologer Andrea Argoli and his work devoted to the solar eclipse of 1654

In Western astrology, apocalyptic predictions were very common between the fourteenth and the seventeenth centuries.³ The solar eclipses of 1300, 1478, and 1654 also provoked a number of prophecies.⁴ Eclipses occurring in the fire signs of the zodiac (Aries, Leo, Sagittarius) received particular scrutiny due to their perceived connection to the outbreak of wars.

It was common practice in astrological calendars to include forecasts of eclipses, which were often associated with severe consequences.⁵ In astrological literature solar eclipses were regarded to have a negative social impact. Particular scrutiny was given to eclipses that aligned with the major conjunction cycles – specifically those of Saturn-Jupiter and Saturn-Mars. Both astrological and historical literature frequently associated misfortunes with these celestial events.⁶

The beginning of the discussion and panic of 1654 was started by the Italian mathematician, astrologer, astronomer and physician Andrea Argoli (1570–1657). He was professor of mathematics at the Sapienza University of Rome, then at the University of Padua. Argoli's publications spanned several fields, from mathematics to medicine, astrology, and astrological medicine. His ephemerides were highly popular and went through multiple reprints. The book *De Diebus Criticis* was particularly valuable to medical astrology, as it presented individuals' natal horoscopes as well as the horoscope of the illness that caused the death. Argoli defended the astrological interpretation of critical days. He regarded the astrological positions of heavenly bodies as directly influencing a disease's course and duration.⁷ Owing to his prominent standing in the scientific community, his publications

³ Bernard Capp, *Astrology and the Popular Press: English Almanacs 1500–1800* (New York: Cornell University Press, 1979), 164–179.

⁴ Giangiacomo Gandolfi, “L'Eclissi e l'Orbe Magno del Leone: l'affresco astrologico del Duomo di Montagnana come profezia anti-turca,” in *Il Cielo in Terra ovvero della giusta distanza*, cura di Valentina Girotto e Guido Rosada (Padova: Padova University Press, 2015), 213.

⁵ Kelly Marie Smith, “The Science of Astrology: *Schreibkalender*, Natural Philosophy, and Everyday Life in the Seventeenth-Century German Lands” (PhD diss. University of Cincinnati, 2003), 167–182.

⁶ For example, the Kazimierz councilman Goliński Marcin connected eclipses and misfortunes. Мартин Голінський [Marcin Goliński], *Silva rerum: (1648–1665)*. Ч. II: (1650–1653), упоряд. Ярослав Федорук; латин. Маркіян Домбровський, Орест Заяць (Київ: Лну ім. Івана Франка, ІУАД ім. М. С. Грушевського, 2020), 81–91.

⁷ Andrea Argoli, *De Diebus Criticis et agreorum decubitu libri duo* (Patavii: Apud Paulum Frambottum, 1639); Andrea Argoli, *Ephemerides annorum L iuxta Tychonis hypotheses, et accurate e coelo deductas observationes ab anno 1630 ad annum 1680* (Venetijs: Apud Pavlvm Frambottum, 1638); Andrea Argoli, *Ptolemaeus parvus* (Lyon: Joseph and Pierre Vilort 1652); Mario Gliozzi, “Andrea Argoli,” in *Dizionario Biografico degli Italiani*, vol. 4 (Roma, Istituto dell'Enciclopedia Italiana, 1962), 132–134; Thorndike, *A history of magic and experimental science*, vol. 7, 122–124; Walter Pagel and F. N. L. Poynter, “Harvey's doctrine in Italy: Argoli (1644) and

naturally attracted considerable attention from diverse audiences.⁸

In 1652 Andrea Argoli published his work devoted to the solar eclipses of 1652 and 1654. The solar eclipse was at 11:22 a.m. on 12 August 1654 in Rome. The author gave a complete astrological description of the event. The positions of the Sun ($19^{\circ}32'$), the Moon ($19^{\circ}32'$), Saturn ($28^{\circ}50'$), Mars ($12^{\circ}24'$), the star Regulus⁹ (28°), and Dragon Tail ($25^{\circ}2'$) were in the fiery sign Leo (Fig. 1). The astrologer considered that at the time of the eclipse Saturn and Mars would be practically in conjunction. This was the reason for prognosticating war and violence for the next two years. The celestial phenomenon was supposed to affect the countries, regions and cities that were considered to be under the rulership of the sign Leo (Italy, Cremona, the Alps, Poland, etc.) and Sagittarius (Spain, Dalmatia, etc.). In the horoscope of the event, the eclipse was situated in the tenth house, which was associated with power. This gave grounds to predict an untoward influence on monarchs and the representatives of high society. The effect was expected on 12 December 1654 and 9 April 1655. Argoli connected this solar eclipse with another solar eclipse on 8 April 1652 which took place in the fiery sign of Aries. There was a quadrature between Saturn and the luminaries. In both cases, the events took place in a fiery sign and Saturn was related to them. Additionally, the eclipses were connected with the cycle of the conjunction of Saturn and Jupiter¹⁰ in the fiery signs, which began

Bonaccorsi (1647) on the Circulation of the Blood,” *Bulletin of the History of Medicine* 34, 5 (1960): 419–429; Darrel H. Rutkin, “Various Uses of Horoscopes: Astrological Practices in Early Modern Europe,” in *Horoscopes and Public Spheres: Essays on the History of Astrology*, ed. by Guenther Oestmann, H. Darrel Rutkin, Kocku von Stuckrad (Berlin: De Gruyter, 2005), 179–180; Broecke, Vanden Steven. “Evidence and Conjecture in Cardano’s Horoscope Collections,” in *Horoscopes and Public Spheres: Essays on the History of Astrology*, ed. by Guenther Oestmann, H. Darrel Rutkin, Kocku von Stuckrad (Berlin: De Gruyter, 2005), 217–218; Martin Craig, “Medicine and the heavens in Padua’s Faculty of Arts, 1570–1630,” *The British Journal for the History of Science* 57 (2024): 545–559.

⁸ For instance, the Carniolan provincial governor Volf Engelbert Turjaški (1610–1673) owned six books by Argoli in his library, among them the *Dissertatio in Eclipsim Solis 12. Augusti 1654*. Stanislav Južnič, “Kopernik v Turjaški in Licejski Knjižnici,” *Knjižnica: Revija Za področje Bibliotekarstva in Informacijske Znanosti* 50, 1–2 (2006): 7–33.

⁹ The star Regulus is situated in the constellation Leo. It is regarded as a royal star. According to Ptolimiy it affects as Mars and Jupiter and brings violence, success, glory, and ruin. Илья Касавин, *Знание за пределами науки. Мистицизм, герметизм, астрология, магия в интеллектуальных традициях I–XIV веков* (Москва: Республика, 1996), 101.

¹⁰ The Jupiter-Saturn conjunction occurs every 20 years. It is very important cycle for a forecast of social events. Argoli paid special attention to this topic at the beginning of his work. Andrea Argoli, *Dissertatio in Eclipsim Solis 12. Augusti 1654. Et aliqua in Eclipsim Solis 1652. 8. Aprilis*, ([S.l.], 1652), 3–6. Abū Mašar analyzed all types of conjunctions. Later, this topic was addressed by Messahalalah, Alcabitus, and etc. It was further developed in the 16th–17th centuries by Cyprian Leowitz, Lorenz Eichstaedt, William Lilly, and etc. Charles Burnett and

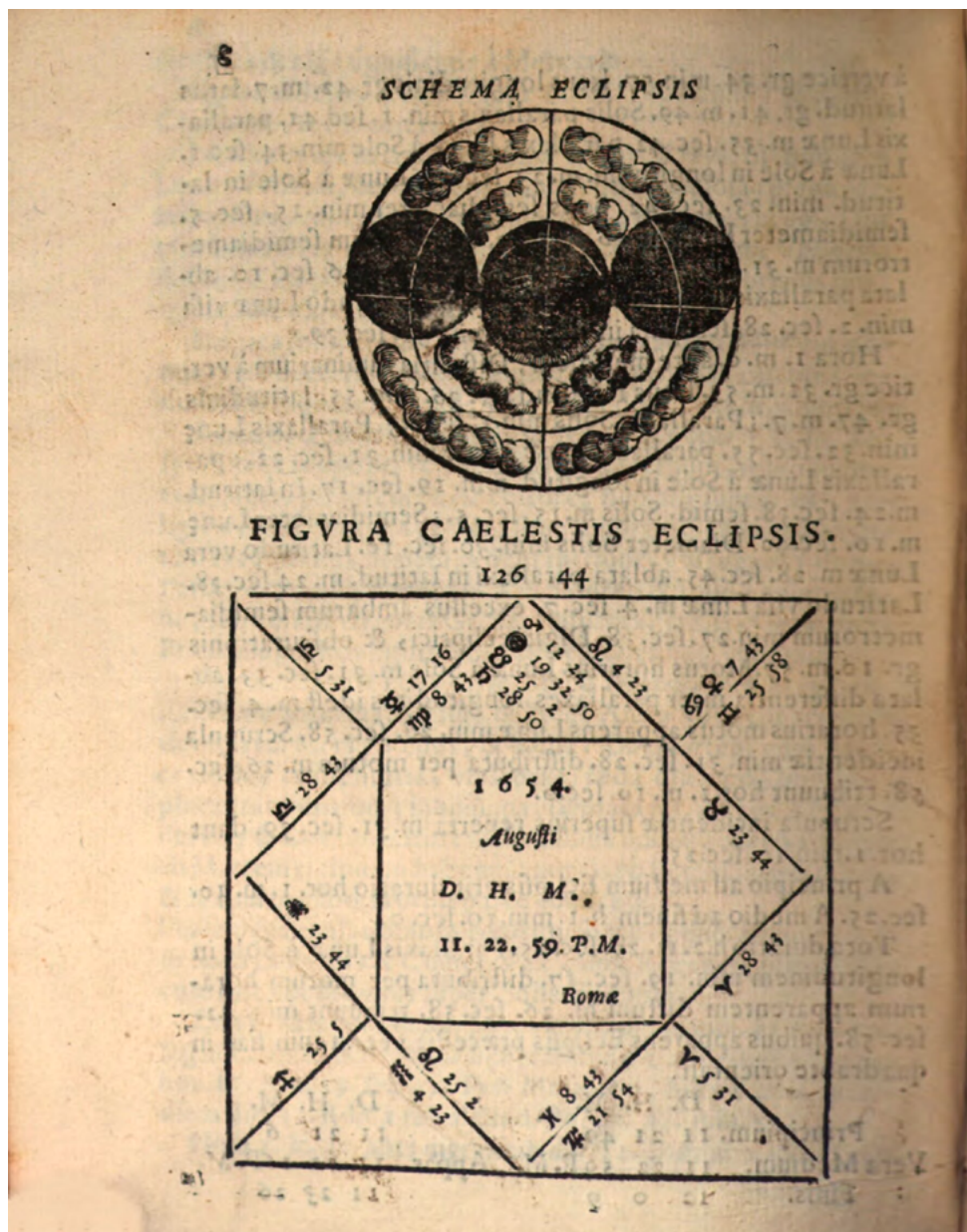


Fig. 1 – Argoli, Andrea, *Dissertatio in Eclipsim Solis 12. Augusti 1654. Et aliqua in Eclipsim Solis 1652. 8. Aprilis*, [S.l.], 1652. The horoscope of the eclipse of 12 August 1654.

in 1623 and continued in 1663.¹¹ Argoli considered eclipses in connection with other events, a technique that was prevalent in the astrology of that time.

The calendars of Polish astrologers also mentioned the eclipse of 1652, which was characterized as dangerous. Occurring under the influence of Mars, the eclipse was prognosticated to portend fire, plague, sudden death, and dangers to the upper classes. These malefic effects were forecast to persist until 1654.¹²

Argoli's prognostication was based on Ptolemy's *Tetrabiblos*, chording to which the forecast consists of four points: 1) the locality: the countries or cities that will be affected, which the position of Saturn, Jupiter and Mars influences greatly; 2) chronology: the time of the forecast and its duration; 3) classification: what kind of events will happen; 4) a special section determining the quality of the event.¹³

Why did Argoli draw such conclusions? The eclipse occurred in the sign of Leo, with Saturn and Mars, two malefic planets,¹⁴ positioned near the luminaries.¹⁵ At that moment, they were on the path of conjunction, which occurred on September 12, 1654. The cycle of the Saturn-Mars conjunction lasts for two years. The zodiac sign influences the nature of the prognosis. Some astrologers correctly stated that a conjunction of Saturn and Mars could not occur on August 12. But these trends were taken into account in the prognostication. Thus, the conjunction cycle of Saturn and Mars in 1654 was considered ominous, particularly as it coincided with the eclipse. Moreover, the eclipse occurring in the area of the Dragon's Tail further exacerbated the unfavourable conditions. While it was regarded that the Dragon's Head had a positive influence, the Tail was considered negative. It is important to note that astrological cycles do not directly cause events on the day of the conjunction, but rather set trends for a certain period.

Keiji Yamamoto, *The Great Introduction to Astrology by Abū Mašar*, vol. 1, (Leiden, Boston: Brill, 2019), 749–761; Edward Stewart Kennedy and David Pingree, *The Astrological History of Masha'allah* (Cambridge, MA: Harvard University Press, 1971), 39–88; Lorenz, Eichstaedt. *Prognosticon de conjunctione magna Saturni et Iovis in trigono igneo Leonis circa annum [...] 1623* (Alten Stettin: Schultz, 1622); William Lilly, *The world's catastrophe, or Europes many mutations untill, 1666. The fate of Englands monarchy untill its subversion. Government of the vworld under God by the seven planetary angels; their names, times of government. An exact type of the three suns seen in Cheshire and Shropshire, 3 April 1647* (London: Printed for John Partridge and Humphrey Blunden, 1647), 7–16.

¹¹ Argoli, *Dissertatio in Eclipsim Solis 12. Augusti 1654*, 4–20.

¹² Goliński, *Silva rerum: (1648–1665)*, 262–264, 317.

¹³ Klaudiusz Ptolemeusz, *Czworoksiąg = Tetrábiblos*, przeł. i przypisami opatrzył Grzegorz Muszyński (Wrocław: Wydawnictwo Uniwersytetu Wrocławskiego, 2012), 121–132.

¹⁴ Saturn and Mars are both classically considered malefic planets: Saturn due to its excessively cold nature, and Mars because of its excessively dry nature.

¹⁵ Traditional astrologers designated the Sun and Moon as the “luminaries,” the two brightest and most significant heavenly bodies. The name originates from their role as sources of light.

Argoli examined the eclipses in the connection with other events. This technique was used in that period in astrology. In forecasts astrologers took into account links between eclipses, ingressions of the Sun, the cycles of the Saturn-Mars and Saturn-Jupiter conjunctions.¹⁶ The astrologer described the eclipse as hazardous, though not world-ending.

The astrological discussion of Argoli's work

Argoli's work generated considerable public excitement, leading to the appearance of Pseudo-Argoli publications in France. For example, a brochure from 1654 in French contained some information from Argoli's work, but presented its consequences in an apocalyptic manner, predicting the end of the world in 1656. The next book with the same context was published in Danish, listing Angulus Andreus as an author.¹⁷ Notably, Argoli did not include this prediction in his work. Such publications caused widespread panic, as they circulated in France, Italy, Spain, and Germany. Some even recommended that people stay indoors on the day of the solar eclipse.¹⁸

The information about this eclipse spread across other countries. Two special leaflets were printed in Holland. The same text was accompanied by different images. It contained astronomical and astrological information of the event based on Argoli's works (Fig. 2).¹⁹ Albert Linemann (1603–1653), a mathematician, astronomer and astrologer from Königsberg, analyzed the solar eclipse in his prognostication for 1654.²⁰ A collection of some astrologers' works devoted to the eclipse was published in Nurnberg in 1654. Stefan

¹⁶ An unknown Portuguese author employed this technique to analyze and predict the consequences of the 1652 and 1654 eclipses, reaching conclusions similar to Argoli's. The manuscript was written for Prince Don Theodosio. Luís Campos Ribeiro, *Jesuit Astrology. Prognostication and Science in Early Modern Culture* (Leiden; Boston: Brill, 2023), 509–517.

¹⁷ *Prédiction merveilleuse du sieur Andreas, astrologue & mathématicien de Padoue, sur l'éclipse de soleil qui se fera le douziesme jour d'aout 1654* (Paris: Par Jacques Beslay, 1654); Andreus Angulus, *Prophecie ofte voorsegginghe van de toekomende ionghsten dach des Heeren* (Padua, 1653), 4–5, 20–23; Halbronn, "Questions autour du texte sur l'éclipse de 1654 attribué à Gassend," 319–343.

¹⁸ Lynn Thorndike, *A history of magic and experimental science. The seventeenth century*, vol. 8 (New York; London: Columbia University Press, 1959), 309.

¹⁹ There are two pamphlets with the same title: *Van den grooten eclips der sonnen, anno 1654. den 12. Augusti* (Amsterdam, 1654).

²⁰ Albertus Linemannus, *Deliciae calendario-graphicae, das ist die sinnreichsten und aller künstlichsten Fragen und Antwort: darinnen die edelsten Geheimnisse der Physic, Astronomi, Astrologi, Geographi etc. etc. bester Massen, Gelehrten und Ungelehrten zum Besten, anmutig und verständlich, außgeführt und verabschiedet werden* (Königsberg, Gedr. bey Pasche Mensen, 1654); Albertus Linemannus, *Prognosticatie, van ... Albertus Linemannus, ... Op de son eclips vanden 12 augusti, anno 1654 ... Vertaelt uyt zijn Prognosticon Astrologicum, op 't jaer 1654* (S.l.: s.n., 1654).



Fig. 2 – Van den grooten eclips der sonnen, anno 1654. den 12. Augusti. Amsterdam, 1654. An eclipse observation in Amsterdam.

Furman's preface to the calendar of 1654 was among them.²¹

The English astrologers, William Lilly (1602–1681)²² and Nicholas Culpeper (1616–1654) published their prognoses for the solar eclipse of 1652. Culpeper linked the solar eclipse in September 1652, the conjunctions of the Sun and Mars in May 1652, the Sun and Saturn in July 1652, Saturn and Mars in August 1652, the lunar eclipse in March 1653, and the conjunction of Saturn and Mars in 1654. According to Culpeper's forecast, the eclipse of September 17, 1652 would induce wars, sea storms, and other catastrophes.²³

²¹ Hermann de Werve; Johann Magirus; Caspar March; David Herlitz; Abdias Trew; Stephan Fuhrmann; Johann Crusius; Marcus Freund; Eberhard Welper; Andreas Goldmayer; David Frölich; Johann Majer; Johann Georg Götz; Nicolaus Schmidt Küntzel; Martin Horky; Vitus Stieffenberger; Johann Gabriel Sperber; Johann Michael Lihn; Georg Galgemair; Nicolaus Joly; Christian Pangratz. *Astrologische Beschreibung von der anno 1654 den 2., 12. August i vorfallenden grossen, sichtbaren und nachdencklichen Sonnen-Finsternuß* (Nürnberg: Endter, 1654), 16, 18.

²² William Lilly, *Annus tenebrosus, or The dark year Or astrologically iudgements upon two lunar eclipses, and one admirable eclips of the sun, all visible in England, 1652. Together with a short method how to judge the effects of eclipses* (London: Printed for the Company of Stationers, 1652).

²³ Nicholas Culpeper, *Catastrophe magnatum, or, The fall of monarchie a caveat to magistrates, de-*

Culpeper also analyzed the eclipse in a calendar for 1654, noting that the positions of Saturn and Mars in the sign of Leo would result in severe consequences. Mars's influence was expected to contribute to fires and other dangerous situations.²⁴ The English astrologer John Vaux (1575–1651) noted that the celestial event of 1654 would provoke the renovation of the world, but not doomsday.²⁵ Both the eclipses of 1652 and 1654 inspired not only a discussion, but also pamphlets against astrologers.²⁶

The eclipse of August 12, 1654 was described in various prognostics for 1654 and became the reason for discussions in Germany, France, England, and Holland. The sources mentioned above demonstrate how astrological information spread across Europe from country to country. Statements about the fatality of the event and the end of the world in 1656 led to critical publications against such forecasts.²⁷ Even later, the eclipse of 1654 aroused interest. The German poet and Jesuit Jacob Balde (1604–1668) mentioned it in his work of 1662. The astrological characteristics were based on Argoli's work.²⁸ Generally, Balde criticized astrology.²⁹ In a book against astrology, the Lutheran theologian Tobias Wagner (1598–1680) provided an addendum with a description of the eclipse of 1654. He stressed that not all astrological predictions would be realised.³⁰

The aforementioned works demonstrate that the eclipse was actively debated by European astrologers. This study, however, cannot provide a thorough analysis of the response in every country. Instead, this overview highlights broader trends in the eclipse literature to contextualize the reactions within the Crown of Poland and the Grand Duchy of Lithuania.

Criticism of forecasts for the eclipse of 1654 by the clergy in Polish–Lithuanian Commonwealth

The Crown of Poland and the Grand Duchy of Lithuania maintained intellectual and educational contacts with European countries, including in the field of astrology. From the 15th century onwards, the representatives of these countries studied astrology in Bolo-

duced from the eclipse of the sunne, March 29, 1652, with a probable conjecture of the determination of the effects (London: Printed for T.Vera and Nath. Brooke, 1652), 8, 57–58, 67–74.

²⁴ Nicholas Culpeper, *An Ephemeris for 1654* (London: Printed by John Macock for the Company of Stationers, 1654), 14–24.

²⁵ Capp, *Astrology and the Popular Press*, 171.

²⁶ Burns, "The Terriblest Eclipse That Hath Been Seen in Our Days," 137–152.

²⁷ Labrousse, *L'Entrée de Saturne au Lion*, 7–12.

²⁸ Jacobo Balde, *De Eclipsi Solari Anno MDCLIV Die XII. Augusti, in Europa, A pluribus spectata Tvbo Optico*. (Monachii: Wagner, 1662).

²⁹ Labrousse, *L'Entrée de Saturne au Lion*, 21.

³⁰ Tobias Wagner, *Astrologia genethliaca, ex theologicis ac physicis principiis, ipsisque Astrologorum penetrabilibus* (Stuttgart: literis & impensis Johann-Wyrichii Rösslini, 1656), 128–134; Thorndike, *A history of magic and experimental science*, vol. 8, 307–309.

gna and Padua. The works of Guido Bonatti, Cardano, and other astrologers were held in private libraries.³¹ Information about new publications reached intellectual circles fairly quickly. Argoli was a well-known author. Therefore, within this context, an interest in intellectual trends was a natural phenomenon for the Crown of Poland and the Grand Duchy of Lithuania.

The discussion of the eclipse in the Polish-Lithuanian Commonwealth occurred in various circles, including among the clergy, the nobility, and the court of the Polish Queen and Grand Duchess of Lithuania, Marie Louise Gonzaga (Polish: Maria Ludwika Gonzaga). Astrology played a significant role at Marie Louise Gonzaga's court. Abbot Marolles noted in 1643 that this topic was discussed there.³² Three French astrologers, Pierre des Noyres (1608–1693), Ismaël Boulliau (1605–1694), and Jean Baptiste Morin (1583–1656), were members of the astrological circle of the Polish Queen and Grand Duchess.³³ Pierre des Noyres served as a secretary at the Gonzaga's court. The Gdansk astronomer Johannes Hevelius (1611–1687) was in communication with this circle. He carried out the astronomical observation of the 1654 solar eclipse and printed his results. The work was dedicated to Pierre des Noyres³⁴. The correspondence between Hevelius and Noyres sheds light on the discussions surrounding the solar eclipse and its significance for the intellectuals of the time. Hevelius observed that Boulliau's work accorded little significance to this particular eclipse.³⁵ Hevelius and Boulliau discussed the celestial event. Boulliau

³¹ Significant scholarly works have been devoted to contacts with European countries. It is pertinent to highlight several foundational studies. Leszek Hajdukiewicz, *Biblioteka Macieja z Miechowa* (Wrocław: Ossolineum, 1960); Leszek Hajdukiewicz, *Księgozbiór i zainteresowania bibliofilskie Piotra Tomickiego na tle jego działalności kulturalnej* (Wrocław: Ossolineum, 1961); Marian Chachaj, *Zagraniczna edukacja Radziwiłłów od początku XVI do połowy XVII wieku* (Lublin: UMCS, 1995), etc.

³² Karolina Targosz, *Uczony dwór Ludwika Marii Gonzagi (1646–1667): z dziejów polsko-francuskich stosunków naukowych* (Wrocław; Gdańsk: Zakład Narodowy im. Ossolińskich, 1975), 45–46; Grell Chantal, "Astrologie et politique au milieu du XVII^e siècle: les "nativités" et "révolutions" de Boulliau et de des Noyers," *Dix-septième siècle* 266, 1 (2015): 46.

³³ Steven Vanden Broecke, "Catholic Spirituality and Astrological Self-Care in Seventeenth-Century France: Jean-Baptiste Morin's *Astrologia Gallica* (1661)," *Lias. Journal of Early Modern Intellectual Culture and its Sources* 47, 2 (2020): 123.

³⁴ *Johannis Hevelii Epistolæ II: Prior: De Motu Lunæ Libratorio, in certas Tabulas redacta* (Gedani: typis Andreae Molleri, 1654), 58–61; Targosz, *Uczony dwór Ludwika Marii Gonzagi (1646–1667)*, 295; Adam Szanser, "Johannes Hevelius (1611–1687) – Astronomer of Polish Kings," *Quarterly Journal of the Royal Astronomical Society* 17 (1976): 488–498; Derek Jensen, "The science of the stars in Danzig from Rheticus to Hevelius" (PhD diss. University of California, 2006), 174–295.

³⁵ *Correspondance de Johannes Hevelius, t. III, Correspondance avec Pierre des Noyers, secrétaire de la Reine de Pologne, 1646–1686*, éd. par Chantal Grell (Turnhout: Brepols, 2020), 81–82, 250–257.

also observed it and made calculations.³⁶

This event did not go unnoticed by the Queen's astrological circle. It was a topic of discussion in many exchanges, although no predictions were published on this occasion. And if this discussion occurred in the astrological circle, it probably reached the Queen. Later, military events compelled the royal court to consult astrologers.

Within religious circles, there was a strong reaction to the prediction of the end of the world, as presented in the work of Pseudo-Argoli. Overall, the clergy tended to hold a negative attitude towards such predictions, since officially everything was believed to be in the hands of God.

The reaction to the publication of Pseudo-Argoli was immediate. One of the earliest responses is probably Friderik Alembek's work (?–1673), a canon from Przemyśl. He was the son of the Lviv (Polish: Lwów) Councilor Jan Alembek, who authored various theological works, some of which have survived in handwritten form, whereas the printed edition of his work may not have survived, since it has never been found.³⁷ According to textual evidence, the work was likely printed in 1652 in Wrocław. A synopsis of Alembek's work is included in a manuscript by Goliński Marcin (1608–1673), a chronicler of events that covers various political documents from the seventeenth century, with a notable emphasis on astrology. Alembek's work is just one example of literature related to astrology. Based on the synopsis, the title may have been the next: "Rewolutia prognostiku uczinienego od matematikow Andreasa Argoliusa y Israelia Hutnera³⁸ w Niemcech na rok 1654 y na rok 1656 przez i. m. x. Friderika Alembeka, sacrae theologiae doctora, kanonika przemiskiego" (The revolution of the forecast made by the mathematicians Andreas Argolius and Israelia Hutner in Germany for the years 1654 and 1656 by Friderik Alembek, doctor of theology, canon of Przemyśl).³⁹

In Alembek's work, Argoli is depicted as an intelligent and distinguished mathematician, renowned professor from Padua and author of substantial scholarly works. An as-

³⁶ Boulliau, Ismaël, "Texte manuscrit d'Ismaël Boulliau, Paris, 12 août 1654," *Bibliothèque numérique – Observatoire de Paris*, consulté le 23 avril 2024. <https://bibnum.obspm.fr/ark:/11287/2ZjcJ>.

³⁷ Karol Estreicher, *Bibliografia polska*, cz. 3, t. 1 (Kraków: Akademia Umiejętności, 1893), 107; Sadok Barącz, "Wiadomość o Alembekach," *Przegląd Biblijograficzno-Archeologiczny* I (1881): 371–375; Jan Kwolek, "Alembek (Alnpech) Fryderyk," in *PSB*, 1 (Kraków: Polska Akademia Umiejętności, 1935), 73–74; Janina Bieniarzówna, "Goliński Marcin," in *PSB*, 8 (Kraków: Polska Akademia Umiejętności, 1958), 226–227.

³⁸ As for the name Israel Hulner there was a mistake. During this time there existed an astrologer called Israel Hebner (?–1668). He was an author of astrological works and calendars, who also addressed the topic of eclipses in his calendar for 1654. These errors in surnames further show that the clergy's reaction was rather addressed to the work of Pseudo-Argoli than to the authentic edition. Israel Hiebner, https://www.presseforschung.uni-bremen.de/dokuwiki/doku.php?id=hiebner_israel.

³⁹ Goliński, *Silva rerum*: (1648–1665), 317–324.

trological description of the eclipse was included in the text. The main arguments were against the theory dating from the end of the world in 1656, as well as against the belief in the fatal significance of the eclipse. It is noteworthy that astrologers were particularly concerned with the conjunctions of Saturn and Jupiter, as well as of Saturn and Mars. Religious arguments prominently feature in the reasoning, aimed at alleviating panic and bolstering the authority of the church.⁴⁰ While there is not sharp criticism of astrology, the intention is to address concerns and promote stability.

The next work appeared in Polish was called *Straszna dziwnych skutkow rzeczy po wszyskim swiecie metamorphosis z gwiazd y planetow niebieskich na rok panski 1654, 1655, 1656 przez Andrzeia Argola primarum astrologum paduanum et Israelem Hulnerum matheologum opisana* (The terrible strange effects of things all over the world, the metamorphosis of the celestial stars and planets in 1654, 1655, 1656 described by Andrea Argoli, one of the primary astrologers of Padua, and the mathematical theologian Israel Hutnerum). It is worth noting that this work was written under a foundational influence of Alembek's work.

The terrible strange effects... is kept in the private archive of the Belsky castellan Jakub Michałowski (1612–1663) as a part of the collection of documents dating from 1602 until 1658.⁴¹ This work was submitted for publication in 1653, but no book with this title was found. It was likely never printed, but only distributed in manuscript form. The work offers both astrological and religious interpretations of the event. It provides information on the planets' influence on humans, plants, and weather, as well as the mechanism of eclipses according to Pliny. The consequences due to the appearance of a comet in 1618⁴²—which brought wars to many countries—are mentioned, with the author stating that one of them would have been a defeat for Moscow.⁴³ In our opinion, the information about the defeat of the Moscow state is included intentionally. This was because the eclipse of 1654 was expected to provoke a war with that country. The text notes that such an astrological event should not always be a problem for the Polish-Lithuanian Commonwealth; it could instead become a problem for the enemy. At the same time, the author emphasized that, according

⁴⁰ Ibid., 317–324.

⁴¹ Biblioteka Narodowa, rkps BN III 6634, kk. 208r–215r.

⁴² Astrologers were often involved in predicting events related to comets. In 1618, three comets appeared. This event drew the attention of both astrologers and astronomers, leading to numerous publications. Mario D'Addio, *The Galileo Case: Trial, Science, Truth* (Rome: Nova Millennium Romae, 2004), 74–79; Patrick J. Boner, *Kepler's Cosmological Synthesis: Astrology, Mechanism and the Soul* (Leiden; Boston: Brill, 2013), 105–134; Miguel A. Granada and Patrick J. Boner, *Michael Maestlin's Manuscript Treatise on the Comet of 1618: An Edition and Translation of Manuscript WLB Stuttgart, Cod. Math. 4° 15b, Nr. 8* (Leiden: Brill, 2022).

⁴³ In 1618, the Truce of Deulino was signed, which was the greatest success of the Polish-Lithuanian Commonwealth in the war with the Muscovite state. The truce ended the war that had begun in 1609. Andrzej Adam Majewski, *Moskwa. 1617–1618* (Warszawa: Wydawnictwo Bellona S.A., 2006).

to religion, comets do not have power over a person's will but can affect their body.

The terrible strange effects... was religious in nature, quoting the Bible, the Gospel, and the works of saints to emphasize that everything falls within God's will. Various predictions of the end of the world were mentioned, none of which came to pass. The text was intended to counter the spread of information portraying the eclipse as a terrible omen linked to an expected apocalypse in 1656.

Stryjewicz Sebastian Fabian (1621–1681), the theologian and professor of astronomy of the Cracow Academy, also published a response to an anonymous work about the eclipse of August 12, 1654 (Fig. 3). He was familiar with astrology and astronomy. His publication devoted to the movement of Mercury noted the position of the planet according to the Ptolemaic system.⁴⁴ Stryjewicz was even the author of astrological calendars in the second half of the 1650s.⁴⁵ He made a synopsis of Johannes de Sacrobosco's work *Tractatus de Sphaera* in 1644. The manuscript was kept in Radzivil's library in Niasviž.⁴⁶

Stryjewicz did not give information about the title and place of publication of this Latin-language work. The professor presented the astrological characteristics of the eclipse and its horoscope. He disputed the negative effect of the event because of its position between Mars and Saturn. Stryjewicz pointed out that the eclipses of October 2, 1652; April 19, 1648; June 20, 1648 and others did not lead to the end of the world. He referred to Ptolemy's statement that the eclipse in the sign of Leo affected the countries that belonged to this sign. Stryjewicz noted that Poland was situated in the sign of Aries. It meant that the country was not under the influence of the eclipse. The author used the ephemerides of the German astronomer David Origanus (1558–1628/29). He stressed that the conjunction of the planets could be in an orb of 6 degrees, not more. Referring to the Italian astrologer Gerolamo Cardano (1501–1576), Stryjewicz concluded that the eclipse could not do much destruction. He also noted that the Paduan astrologer Argoli wrote nothing about the end of the world in his treatise.⁴⁷

⁴⁴ Stryewicz, *Qvæstio astronomica de orbibus & motibus mercurij*, Ad disputandum publice in Academia Cracoviensi. Sebastian Fabian Stryjewicz, *Qvæstio Astronomica De Orbibus & Motibus Mercurij*, Ad disputandum publice In Academia Cracoviensi (Cracoviae: Drukarnia Wdowy i Dziedziców Franciszka Cezarego, 1653).

⁴⁵ Mieczysław Barcik, "Stryjewicz (Stryjewic, Stryewic) Sebastian Fabian (1621–1681)," in PSB, 44 (Kraków: Wydawnictwo Towarzystwa Naukowego Societas Vistulana, 2006), 528–530; Małgorzata Krzysztofik, *Studium z dziejów krakowskich kalendarzy astrologicznych XVII wieku: Almanachy Stanisława Słowakowica jako podstawa uogólnień* (Kraków: Księgarnia Akademicka, 2010), 487.

⁴⁶ Нацыянальны Гістарычны Архіў Беларусі, ф. 694, воп. 1, спр. 455.

⁴⁷ Sebastian Fabian Stryjewicz, *Krotki Respons Na Praktykę wydaną na świat O Sądny Dniu, Przez Anonima Anno Domini 1652. Do którego przydane Opisanie Komety widzianego w Roku 1652. 19 Decemb z effectami, których się po nim spodziewać mamy* (Kraków: w drukarni wdowy y dziedziców Franciszka Cezarego, 1653).

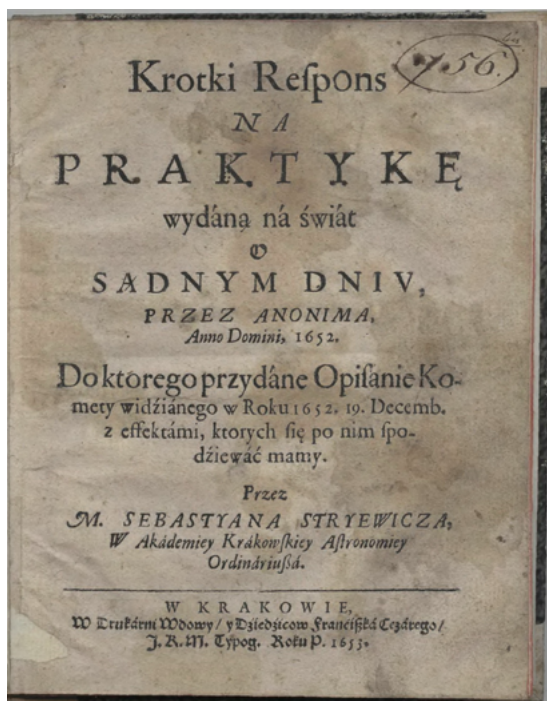


Fig. 3 – Sebastian Fabian Stryjewicz, *Krotki respons na praktykę wydaną na świat o sądnym dniu*, przez anonima anno domini 1652, Kraków, 1653.

The work was mainly directed against the assertion that the eclipse of 1654 would lead to Apocalypse. The arguments were based on the works of famous astrologers. It is worth noting that Stryjewicz treated astrologers with great respect, when referring to their works. His arguments were astrological, not theological. They had a professional character. The author showed a wide range of knowledge on astrology and commitment to this science, which was typical for a professional astrologer.

Of course, the eclipse did not mean the end of the world. This interpretation was used to propagandize and provoke panic. Critical works were necessary to pacify the situation. However, Stryjewicz could not avoid mistakes. From the astrological point of view the eclipse and the cycle of conjunction of Saturn and Mars provoked severe events. The Saturn and Mars cycle was also considered by astrologers as provoking conflicts, emergencies, and epidemics. As we saw it before, this cycle and the eclipse were included as the elements of the historical cycle of Saturn and Jupiter.

At the end of the work, there is also a text about a comet that appeared in Cracow on December 15, 1652. Here information is provided about the connection between the

appearance of comets and historical incidents. Stryjewicz also gave an astrological description of this phenomenon: in order to obtain the coordinates of the location of the comet, he used an astrolabe and a square. This demonstrates his astrological approach in explaining the appearance of comets and their consequences. Interestingly, this provoked the reaction of Professor of Cracow Academy Jan Tonsky, who published a special edition devoted to an astronomical description of the comet and criticized the astrological approach to the phenomenon.⁴⁸

In this context, the opinion of Lviv canon Tomasz Juzafowicz (1662–1728) is intriguing. The chronicles of Lviv for 1652 show that Alembek and Stryjewicz, in a Catholic manner, instigated a wave of panic about the end of the world due to the appearance of a comet that year. This panic was provoked by publications from Argoli and Hunter.⁴⁹ The author observed that Alembek used not only theological but also astronomical arguments in his assertions. Hunter was tagged as an unreliable and fraudulent astrologer.⁵⁰ The author spoke highly of the Catholic theologians's works, noting their correctness. He wrote nothing about Stryjewicz's astrological qualification. Juzafowicz based his work on documents from the Lviv magistrate, albeit much later than the incident itself. This explains the fact that he pointed out only the comet of 1652 and did not mention the eclipse.

The Vilna Academy (Vinius academy, Academia Vilmensis) entered the discussion in 1654 with a printed response to the prediction (Fig. 4).⁵¹ Belarusian linguist Mykola Ramanoŭsky translated the work into Belarusian. In the preface, he does not delve into the reasons behind the creation of the work, nor does he consider the astrological material. Instead, he emphasizes that this work is one of the first to oppose superstitions. It is worth noting that astrology cannot be strictly categorized as superstition. Nevertheless, Ramanoŭsky stresses that the prediction regarding the war in 1654 came true.⁵²

The brochure of the Vilna Academy contains no information about the author. It was written by "a certain Jesuit priest." Three errors in prognostics were submitted for consideration: 1) such an eclipse had never existed before; 2) the eclipse would be visible everywhere; 3) it would destroy the whole world, and after 1656 there would be no

⁴⁸ Ioannis Tonski, *De Cometa Dogma Mathematicum, Cometographo Anni proximè elapsi. 1652* (Cracoviae: apud Lucam Kupisz, 1653); Barcik, "Stryjewicz (Stryjewic, Stryewic, Stryewic) Sebastian Fabian (1621–1681)," 529.

⁴⁹ Juzafowicz recorded the surname as 'Hunter,' which appears to be the same error found in *The Terrible Strange Effects....* The correct name was Israel Hebner.

⁵⁰ Tomasz Józefowicz, *Kronika miasta Lwowa od roku 1634 do 1690: obejmująca w ogólności dzieje dawnej Rusi Czerwonej; a zwłaszcza Historyę arcybiskupstwa lwowskiego w tejsze epoce* (Lwów: Drukarnia Jezuitów, 1854), 158; Barącz, "Wiadomość o Alembekach," 374.

⁵¹ *Błędy jednego prognostyku z zaćmienia słonecznego roku pańskiego 1654 dnia 12 augusta około południa wystawione* (Wilno: Drukarnia Akademii Societ. Iesv, 1654).

⁵² "Аблуды аднаго прароцтва аб сонечным зацьменьні Году Гасподняга 1654," прадмова і пераклад Міколы Раманоўскага, *Спадчына* 5 (1993): 70–80.

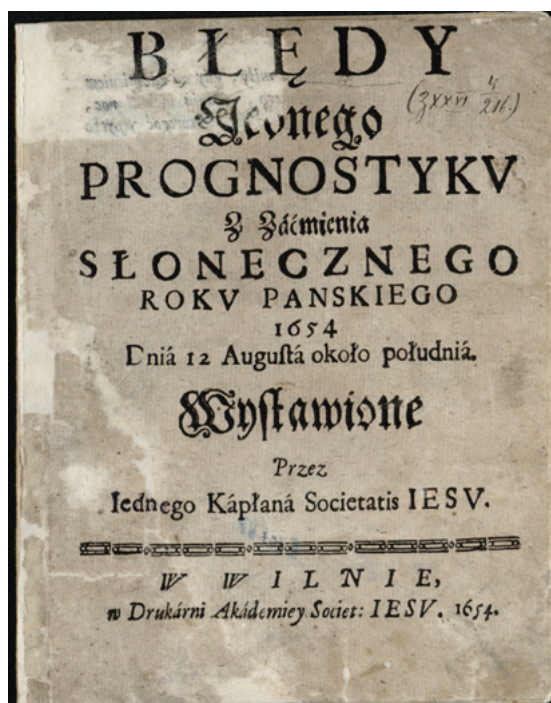


Fig. 4 – Anonymous, *Błędy jednego prognostyku z zaćmienia słonecznego roku pańskiego 1654 dnia 12 augusta około południa wystawione*, Wilno, 1654.

country. Astronomical information about the eclipse was presented as an argument. The list of eclipses for different years (393, 840, 957, 1415, 1485, 1560, etc.) showed that they did not lead to the Apocalypse. The information was taken from Plutarch, Johannes de Sacrobosco and other authors. A lot of citations in Latin were included in the text, thus showing wide knowledge in ancient literature and literature of that time. The nature of the celestial event explained the impossibility to observe it from all over the world. The thesis about doomsday and the destruction of countries was presented sceptically. Astrological description of the eclipse was also included. It was emphasized that Saturn and Mars did not make a conjunction in that period. They were 8 degrees away from the Sun.⁵³

Astrological data are present in the work, but there is no such astrological analysis of the phenomenon as in Stryjewicz's work. The text has a religious approach to the topic. It appears to be a critique of prognostication errors rather than astrology itself, aiming to address and alleviate fears related to an eclipse panic. The use of Latin language, while

⁵³ *Błędy jednego prognostyku z zaćmienia słonecznego roku pańskiego 1654 dnia 12 augusta.*

potentially complicating the text, suggests an attempt to reach an educated audience, although the work was intended for broader readership.

The emergence of this work at the Vilna Academy underscores the profound impact of the publicity surrounding the prediction. The participation of Catholic priests from the Polish-Lithuanian Commonwealth in the discussion regarding the eclipse is noteworthy. It raises an interesting question about whether their involvement stemmed from the chosen positioning of the local clergy or was rather driven by the Vatican. The autonomy of the Vilna Jesuits is also a matter of interest in this context. Despite their local presence, they maintained close ties with Rome, submitting reports and adhering to Vatican policies in their activities. The Jesuits were well-versed in astrology,⁵⁴ which was included in various courses of the Vilna Academy. The Jesuits aimed more at controlling knowledge and education than spreading it.⁵⁵

The correspondence between French Catholic priest Saint Vincent de Paul (1581–1660) and Charles Ozenne, superior in Warsaw⁵⁶ show the panic reaction to the eclipse. Saint Vincent de Paul wrote in his letter (Paris, July 10, 1654): “Although those extraordinary manifestations occurring there are not sure signs of some evil event, and ordinarily such portents should not be dwelt on, it is good, nevertheless, to redouble our prayers that God may be pleased to avert from His people the ills with which He may have planned to afflict them. We here are being threatened with a solar eclipse, the worst to take place for several centuries. It is supposed to occur on August 12, around nine or ten in the morning, so they say. Please find out if it will be visible in Poland and let me know the details.”⁵⁷ Indeed, the passage highlights the prevailing excitement within the community preceding the eclipse, as evidenced by the implementation of measures such as double prayers aimed at assuaging anxieties. Additionally, the author’s interest in the situation in Poland suggests a broader awareness of the socio-political context surrounding astrological events and their perceived significance within various regions.

After the event (September 11, 1654) Vincent de Paul returned to this issue. He noted

⁵⁴ Ribeiro, *Jesuit Astrology*; Natallia Sliž, “Astrology in exact sciences in Vilna Academy,” in *Vilnius ir jo bendruomenės: žvilgsnis į 700 metų miesto istoriją*, ed. by Jolita Sarcevičienė, Jurgita Šiaučiūnaitė-Verbickienė, Mindaugas Klovas (Vilnius: Lietuvos istorijos institutas, 2025), 385–408.

⁵⁵ Knowledge of the Copernican system, for instance, existed within the Academy but was disseminated through unofficial channels. Sliž, “Astrology in exact sciences in Vilna Academy,” 385–408.

⁵⁶ The first Missionaries arrived in Poland in November 1651, thanks to the petition of Queen Marie-Louise de Gonzaga. She knew St. Vincent personally and was familiar with the works of the Congregation of the Mission. Stanisław Rospond, “The Missionaries in Poland (1651–1697),” *Vincentiana* 2 (2011): 196–204.

⁵⁷ Vincent de Paul and Pierre Coste, *Correspondence, Conferences, Documents*, vol. V, ed. by Marie Poole, Julia Denton, and Elinor Hartman (USA: New City Press, 1995), 167–168.

that French astrologers assured people that they should not fear the eclipse. Monsieur Cassandieux (1592–1655), alias canon Pierre Gassendi, the famous astronomer and author of numerous works, gave him very pertinent arguments, noting that a solar eclipse was inevitable every six months. If the eclipse were so disastrous, people would see more often famine, plague, and the other calamities. Vincent de Paul regarded: “The experts in astrology are disturbed much by it, and even less those who are instructed in the school of Jesus Christ, who know that the wise man *dominabitur astris*.”⁵⁸

However, in this case, it is worth paying attention to the dates of these exchanges, it was July–September 1654. The ongoing war with the Moscow state likely intensified the atmosphere of fear and uncertainty during that time. In such a context, efforts to calm the public would have been particularly challenging, as people may have been concerned about the conflict and its potential consequences.

The wave of panic surrounding the eclipse was significant. The participation of the Catholic clergy in the discussion indicates a perceived need to address and manage the fears and concerns of the public, suggesting the magnitude of the event’s impact on society at large. These publications vividly show that the Catholic clergy in Polish-Lithuanian Commonwealth knew European astrological publications and questioned their influence on society. The learned ecclesiastics were well acquainted with astrology. While their primary arguments were grounded in religious texts, they also skilfully employed astrological principles to bolster their positions.

The eclipse of 1654 in the Grand Duchy of Lithuania

The information about the eclipse of 1654 was spread widely in the GDL thanks to Stefan Furman’s calendar. It was printed in Lubča (Polish: Lubcza, now Belarus⁵⁹) in December 25, 1653. This date is given in Jan Langius’ preface.⁶⁰ The settlement belonged to the Field Hetman of the Grand Duchy of Lithuania Januš Radzivil (Polish: Janusz Radziwiłł). The edition was dedicated to Januš’s relatives Baguslaū Radzivil (Polish: Bogusław Radziwiłł), the stableman of the GDL. Both Januš and Baguslaū belonged to Calvinism.⁶¹

In the Radzivils family it was common practice to support such publications. Thanks to Baguslaū Radzivil, a philosopher of Cracow Academy, Damian Pajęcki, published the

⁵⁸ Ibid., 181–183.

⁵⁹ I use official Belarusian Latin transliteration for spelling proper and geographical names in the cases attributed to the history of Belarus.

⁶⁰ Stefan Furman, *Xiąg rocznych część pierwsza to iest stary y nowy kalendarz... na Rok 1654* (Lubecz: typografia Jana Langiusa, 1653).

⁶¹ About the role of Radzivil in the Protestant movement see: Andrej Kotljarchuk, *In the Shadows of Poland and Russia The Grand Duchy of Lithuania and Sweden in the European Crisis of the mid-17th Century* (Huddinge: Södertörns högskola, 2006), 45–61.

calendar of 1651.⁶² Januš Radzivil inspired Furman's calendar of 1652.⁶³ In 1632, Jan Jonston's book was printed in Amsterdam under the patronage of Januš and Baguslaū Radzivils, Bogusław Leszczynski, Uładzisaū Monvid Daragastajski (Polish: Władysław Monwid Dorohostajski).⁶⁴ The publication contained encyclopaedic information about plants, metals, etc. However, it also included parts about the sky, the four elements (earth, air, water, and fire), comets and meteors. Based on different authors (Copernicus, Brahe, Ficino, etc.), the author provided astronomical and astrological information. Jan Jonston was a well known scientist and author of a number of works on natural science, medicine, etc. He came from Scotland, had received a good education and defended his dissertation in medicine.⁶⁵

The Radzivils belonged to the upper nobility and had great social influence. Moreover, they were interested in astrology. B. Radzivil was a well-educated person, having graduated from the Sluck gymnasium.⁶⁶ He possessed a good library, consisting mainly of Latin books on architecture, philosophy, medicine, etc. It included books by John Johnson, Vitruvius, Galen, and Francis Bacon.⁶⁷ They contained information on astrology. Celestial signs were considered important by B. Radzivil.⁶⁸ Januš Radzivil had occupied a number of posts in the state administration: the Field Hetman of Lithuania (from 1646) and the Grand Hetman of Lithuania (from 1654), the voivode of Vilna (from 1653), and others.⁶⁹ Januš and Baguslaū Radzivils were relatives. They were mentioned as being among the most influential people in the country according to the Italian espionage report from the

⁶² Damian Pajęcki, *Kalendarz z praktyką na Rok Pański 1651* (Kraków, 1650); Mariola Jarczykowa, *Książka i literatura w kręgu Radziwiłłów birżańskich w pierwszej połowie XVII wieku* (Katowice: Wydawnictwo Uniwersytetu Śląskiego, 1995), 71.

⁶³ Stefan Furman, *Nowy i Stary Kalendarz Świąt rocznych na Rok 1652* (Gdańsk: drukarnia Andrzeja Hünefelda, 1651).

⁶⁴ Ian Jonston, *Thaumatrographia naturalis* (Amsterdam: Blaeu, 1632).

⁶⁵ Tadeusz Bilikiewicz, "Jonston Jan (1603–1675)," in *PSB*, XI, (Wrocław, Warszawa, Kraków: Zakład Narodowy im. Ossolińskich; Wydawnictwo Polskiej Akademii Nauk, 1964–1965), 268–270.

⁶⁶ Elżbieta Bagińska, "Boguslavo Radvilos įvaizdis pagal jo privatų dienoraštį – pomėgiai, pramogos, sveikata," *Lietuvos Istorijos Metraštis* 2 (2019), 72–74.

⁶⁷ Martin Silvester Grabe, *Catalogus librorum quarumlibet facultatum, a (...) Boguslao Radziwil (...) Electorali, quae Regimonti Borussorum est. Bibliothecae legato donatorum 1668* (Regimonti: typis Friderici Reusneri, 1673).

⁶⁸ B. Radzivil wrote to Jan Merzensky (20 April 1665) about a comet and three suns in the sky. At the same time, he noted that even without such signs in the sky everything would be bad. Archiwum Główne Akt Dawnych, Archiwum Radziwiłłów, dz. IV, sygn. 60, k. 14r. I would like to thank Belarusian historian Aliaksandr Prudnikaŭ for paying attention to this source.

⁶⁹ About political situation in the GDL see: Robert Frost, *After the Deluge: Poland-Lithuania and the Second Northern War, 1655–1660* (Cambridge: Cambridge University Press, 1993); A. Kotljarchuk, *In the Shadows of Poland and Russia*.

middle of the seventeenth century.⁷⁰

Stefan Furman (German: Stephanus Fuhrmann, 1616–1683) hailed from the family of a Protestant clergyman and obtained his education at the University of Rostock. In 1642, he returned to his hometown of Lippstadt, where he was appointed as a pastor. In 1638 he commenced the publication of astrological calendars. Primarily written in German, these calendars were also translated into Polish and Swedish. Their designs varied, although their structures remained similar. German-language editions often featured his portrait on the cover (Fig. 5). The calendars were printed until Furman's death. Astrological information in them was based on Argoli's ephemerides. Biographical details about Furman mainly come from his calendars and a sermon delivered upon his death in 1683.⁷¹

The calendar for 1654 was published in both German and Swedish editions. The Swedish version was divided into two parts: one addressing the comet of 1652, and the other focusing on the eclipse of 1654.⁷² The German editions were dedicated to Queen Christina of Sweden. The preface, dated January 8 (18), 1653 (Fig. 6).⁷³ The aforementioned A. Argoli dedicated his work to her in 1652.⁷⁴ A gesture likely not made by chance. Christina was known to have an interest in astrology and sought advice from astrologers. Her own horoscope has also been preserved. Astrological calendars and horoscopes were topics of discussion at her court, a tradition that continued under her successor, Charles X Gustavus (Swedish: Karl X Gustav). An interesting correlation emerges between the Queen's

⁷⁰ This document meticulously outlines the structure of the Commonwealth's government, enumerating offices in the Grand Duchy of Lithuania and Poland and identifying key noble families. It furnishes detailed profiles for each elite figure, including personal attributes, networks with other councils, domains of activity and influence, and foreign connections. In doing so, it reveals the sophisticated espionage conducted within the political elite. Archivio di Stato Firenze, Miscellanea medicea, f. 101, insert 16, ff. 13v–14v.

⁷¹ "Fuhrmann, Stephan." *Biobibliographisches Handbuch der Kalendermacher von 1550 bis 1750*. https://www.presseforschung.uni-bremen.de/dokuwiki/doku.php?id=fuhrmann_stephan; Małgorzata Krzysztófik, "Aksjologia czasu antropologicznego w XVII-wiecznych kalendarzach gdańskich Stefana Furmana," *Respectus Philologicus* 26 (2014): 217–229.

⁷² Stephani Fuhrmans, *Kortta berättelse om thet ettusend sexhundrade femptijo och fierde åhrets beskaffenheet i gemeen; hwarvtinnan cometens som an. 1652. vppå himmelens firmamente lyste: jämwäl ock then stoore solenes an. 1654. föregående förmörkelses wärckande, then christiålskande läsaren troligen för ögon stält warder* (Stockholm: tryckt hoos Heinrich Keyser. 1654).

⁷³ Furman published calendars in German two editions of different formats. Stephan Fuhrman, *Jahr-Buchs Nach der erwünschten Menschwerdung Jesu Christi deß wahren Messiae und Heilandes der Welt M.DC.LIV* (Lüneburg: Gedruet und verlege durch Johann und Heirich denen Sternen Gehrudern, 1653); Stephan Fuhrman, *Jahr-Buch Nach der heilsamen Geburt unsers Herrn und Seligmachers Jesu Christi M.DC.LIV* (Lüneburg: Gedruet und verlege durch Johann und Heirich denen Sternen Gehrudern, 1653).

⁷⁴ Andrea Argoli, *De Diebus Criticis et agreorum decubitu libri duo* (Patavii: Apud Paulum Frambotum, 1652).



Fig. 5 – Stephani Fuhrmann, *Zeit-Buchs Ubers Jahr nach der Gnadenreichen Geburt Jesu Christi...* M.DC.LX, Lippstadt 1659. Stefan Furman's portrait.

biography and the solar eclipse. The influence of astrology and discussions surrounding the eclipse at court were also noted by the researcher Susanna Åkerman. Queen Christina abdicated in favour of Charles X Gustavus on June 14, 1654, after having announced her decision in February.⁷⁵ There is another noteworthy significant historical fact concerning Christina. Prior to the war, a Moscow embassy was sent to her court in 1653 to ascertain Swedish stance in the impending conflict. The outcome was very favorable: the planned military actions were not condemned, and the Queen agreed to supply twenty thousand muskets. This response was influenced by the tense relations between Sweden and the Polish-Lithuanian Commonwealth.⁷⁶ The Queen was aware of the impending war thanks to both the diplomatic channels and the astrological counsel. We can infer from these facts

⁷⁵ Susanna Kristina Åkerman, *Queen Christina of Sweden and her circle: the transformation of a seventeenth century philosophical libertine* (New York; Leiden: E.J. Brill, 1991), 157–177; Kjell Lekeby, “Astrology in the Early Modern Period in Sweden,” in *Western Esotericism in Scandinavia*, edited by Henrik Bogdan, Olav Hammer (Leiden: Brill 2016), 79.

⁷⁶ Заборовский, *Россия, Речь Посполитая и Швеция в середине XVII в.*, 33.



Fig. 6 – Stephani Fuhrmann, *Jahr-Buch Nach der heilsamen Geburt unsers Herrn und Seligmachers Jesu Christi M.DC.LIII*. Luneburg, 1653.

that Christina was aware of the forecast for 1654. The prevailing political situation and astrological discussions likely played a role in her decision to relinquish the throne.

Furman's calendars were translated from German into Polish. They were mostly printed in Gdańsk. Some of them were dedicated to the Polish King and Grand Duke of Lithuania Jan Casimir.⁷⁷ Since a lot of astrological calendars were published in Gdansk, the publication of Furman's calendars did not seem to be an extraordinary event for this city.⁷⁸ There are seven miniatures on the cover page of the 1654 calendar (Fig. 7).⁷⁹ It differs from the front pages of German and Swedish publications. The images of Saturn, Jupiter and Venus are on the top, Mercury and Mars on the left side, while the Sun and the Moon are on the

⁷⁷ Karol Estreicher, *Bibliografia polska*. t. XVI (Kraków: Uniwersytet Jagielloński, 1893), 374–376.

⁷⁸ Jensen, "The science of the stars in Danzig from Rheticus to Hevelius"; Piotr Paluchowski, "Kompozycja graficzna kart tytułowych gdańskich kalendarzy XVI–XVII wieku," *Rocznik Biblioteki Narodowej* LXIV (2013): 91–112.

⁷⁹ I would like to thank Magdalena Marcinowska and the Library of Polish Academy of Science in Gdansk for the photo of the cover page of the calendar of 1654.



Fig. 7 – Stefan Furman, *Xiąg rocznych część pierwsza to iest stary y nowy kalendarz... na rok 1654*. Lubcza, 1653.

right. The planets are presented with their zodiacs: Saturn with Aquarius and Capricorn, Jupiter with Pisces and Sagittarius, Mars with Aries and Scorpio, the Sun with Leo, Venus with Libra and Taurus, Mercury with Gemini and Virgo, and the Moon with Cancer. The same images were used in the herbal of 1568.⁸⁰ The anthropomorphic images of planets were usual in publications and employed as a decoration in Western and Eastern Europe.⁸¹

Quite exceptionally prognostications made the interpretation of the eclipse of 12 August 1654 their main focus, which was not common in such calendars. S. Furman noted that he had read a lot of German and Dutch calendars concerning 1654, which actively discussed the eclipse. However, the author was very surprised by “the unreasonable and inadequate fantasy” in these calendars. Like other astrologers⁸² he sharply criticized Pseudo-Argoli’s text. Furman did not agree that there would be a conjunction of Saturn and Mars during the eclipse. He criticized the idea that the eclipse was a bad omen because it involved the Sun and Moon, Saturn, Mars, the Tail of the Dragon and the star Regulus. He considered doubtful that an eclipse in the heart of the Leo had never happened before in history as well as the fact that it could provoke a fall of the upper strata. The author did not see astrological reasons for the end of the world in 1656. Furman mentioned that periodically astrologers (for ex., Regiomontanus) raised this question. Interestingly, Furman’s arguments were very similar to the arguments of the Dutch edition.⁸³ They derived more heavily from astrological premises than from religious doctrine. The astrologer knew the polemics about this eclipse in European publications. Furman used his calendar to take part in the discussion and present his position about it.

Furman’s forecast predicted the year as unfavorable because of indicators of the annual ingress of the Sun⁸⁴ and the eclipse. This eclipse would have brought strong changes for Germany and the Polish-Lithuanian Commonwealth. It would have also provoked epidemics and wars. The enemies would have come from the East and South. The main danger was expected from the Ottoman Empire. War, famine, and diseases would have affected representatives of all estates. The conjunction of Saturn and Mars would have led to military actions in September. The great danger would have been for the upper strata. The author also linked it to the consequences of the comet appeared in 1652.⁸⁵ The calendar

⁸⁰ Marcin Siennik, *Herbarz, to jest ziół tuteicznych, postronnych y zamorskich opisanie* (Kraków: drukarnia Mikołaja Szarffenberga, 1568), 428–431.

⁸¹ About astrology in art see: Jean Seznec, *The Survival of the Pagan Gods. The Mythological Tradition and Its Place in Renaissance Humanism and Art* (New York: Princeton University Press, 1953).

⁸² Labrousse, *L’Entrée de Saturne au Lion*, 17–19.

⁸³ *Klaer vertoogh van de eclipsis of sons-verdonckeringh. Die geschieden sal in’t iaer 1654. den 12 augusti* (Amsterdam: Webber, Zacharias, 1654).

⁸⁴ The Aries Ingress was pivotal for forecasting annual events (wars, plagues, etc.).

⁸⁵ Furman, *Xiąg rocznych część pierwsza to iest stary y nowy kalendarz... na Rok 1654*, A–E2.

presented other information about this year: the characteristics for each month (weather, Moon phases, etc.), aspects of the planets, harvest (expected to be average), the dates of fairs in Vilna, Kapył, Cracow, Warsaw, etc. This calendar differed from the German one in content and design.

Furman's calendar clearly demonstrates the Protestant approach in envisioning the event and in dealing with the situation in the GDL. The publication of this calendar, supported by the Radzivils, cannot be deemed as a coincidence. The calendar was addressed to the nobility and the royal court. It was an attempt to draw attention to external and internal problems through astrology. The Catholic clergy sought to reduce public panic. By contrast, although the Protestant calendar also downplayed the end of the world, its predictions specifically focused on an impending military conflict. In practice, these two modes of discussion were actually not in conflict with each other. Catholicism was the dominant religion at that time, and therefore several representatives of the clergy participated in the discussion. As for the Protestant side, the Furman calendar played a major role. We did not find any separate public publications issued by the Protestant clergy.

The discussion of the eclipse in the private documents of the nobility of the Grand Duchy of Lithuania

The sources provide a rare glimpse into the range of contemporary attitudes toward the 1654 forecast. Januš Radzivil's reaction, as outlined in his letter to Baguslaū Radzivil, provides insight into the diverse responses to astrological predictions during that period. Januš believed that the nobility relied too heavily on astrologers. He mentioned negative predictions made by Furman and Żórawski for the country in 1654.⁸⁶ Instead, he emphasized the importance of the Polish-Lithuanian Commonwealth's policies and actions in determining its future.⁸⁷ The letter was written on October 18, 1653, but the calendar was

⁸⁶ Mikołaj Żórawski also printed the calendar for 1654 (*Hemerologeion abo Kalendarz świąt rocznych na rok p. 1654... przez Mikołaja Żorawskiego... wyrachowany*, Kraków, 1654). But it has not been preserved. Żórawski was the son of a Lublin notary, a professor at the Cracow Academy, a physician to Sigismund III, Władysław IV and Jan Casimir. Marcin Goliński's contains the excerpts from his calendars of 1646, 1649, 1650, 1652, 1653, 1655, and 1656. The calendar for 1653 was established according to the information that it was sent to Hrodna to Jan Casimir. Żórawski forecasted a war and famine in 1655. It would be interesting to compare both the calendars, but Żórawski's publications have not been found in libraries. Later, Żórawski published calendars abroad. He was the author of the translation of Cicero. The book contains the horoscope for June 2, 1631. Józef Przyborowski, "Rok śmierci Fab. Seb. Kłonowicza," *Ateneum* I (1878): 318–319; Goliński, *Silva rerum: (1648–1665)*, 83–84, 89–91, 259–262, 315–316; Zakład Narodowy im. Ossolińskich, zespół 5, dz. 1, rkps 189, kk. 723r–726r, 786r.

⁸⁷ Edward Kotlubaj, *Życie Janusza Radziwiłła* (Wilno; Witebsk: M. Mindelsohn, 1859), 383–384; Jarczykowa, *Książka i literatura w kręgu Radziwiłłów*, 70.

published after December 25. It means that Januš Radzivil knew the content of Furman's calendar before its publication.

Radzivil's letter also delved into the political landscape of the country, expressing concerns about Moscow's aggression and the nobility's lack of attention to the need for defence. He complained about the lack of freedom of thought at the Sejm and the dominance of Catholicism, which hindered the emergence of alternative ideas and discussions.⁸⁸ In fact, he indicated a political crisis.

Radzivil, deeply involved in the political events of the time, was gravely concerned about the threat from the Muscovite state. The actions of this eastern neighbor provided compelling reasons for his apprehension. On October 11, 1653, the Zemsky Sobor decided to go off the war with the Polish-Lithuanian Commonwealth. Throughout 1653–1654, Moscow conducted active diplomacy across Western Europe. Its envoys were dispatched to various countries to gauge their support in the eventuality of an impending war as well as the strength of their anti-Moscow sentiment. The mobilized intelligence indicated that European powers were unlikely to intervene in the conflict.⁸⁹

As a military and political figure, Radzivil likely analyzed the situation from various angles and sources. He may have also considered astrology as one factor among many, but undoubtedly his attention was primarily focused on practical considerations and strategic decisions about the potential threats to the Commonwealth. Radzivil may have hoped to use the astrological forecast as a catalyst to mobilize forces for the potential conflicts. By highlighting the dangers represented by Moscow and advocating for greater vigilance and defense measures, he aimed to influence the course of events and mitigate potential risks for the Commonwealth's security and stability. Radzivil was not the only one concerned about Moscow's aggression, as evidenced by similar discussions among other contemporaries such as Jan Jonston.⁹⁰

The forecast for 1654 in the GDL and Poland matched with the outbreak of a bloody war between the Moscow state and the Polish-Lithuanian Commonwealth (1654–1667) and the Second Northern War (1655–1660). These conflicts led to widespread devastation, including epidemics, famine, and other negative consequences that affected all levels of society. The upper stratum faced significant danger, with estates being ravaged by enemy forces, peasants, and neighboring factions. Many representatives of the nobility were either taken prisoner or killed in the conflict, while peasants seized the opportunity to seek revenge against their oppressors.⁹¹

⁸⁸ About Radzivil's political activity before the war see: Milewski, "Między Moskwą a Szwecją: Jan Leszczyński i Janusz Radziwiłł," 169–191.

⁸⁹ Milewski, "Między Moskwą a Szwecją," 169–191; Заборовский, *Россия, Речь Посполитая и Швеция в середине XVII в.* 30–55.

⁹⁰ Bilikiewicz, "Jonston Jan (1603–1675)," 269.

⁹¹ About the war see: Bobiatyński, *Od Smoleńska do Wilna Wojna Rzeczypospolitej z Moskwą*

The war also revealed the unpreparedness at the eastern borders of the Grand Duchy of Lithuania, as Moscow's troops swiftly advanced into the country. The Battle of Škloū, which took place on August 12, 1654, and coincided with the day of the eclipse, saw Januš Radzivil leading the GDL army to victory, albeit with heavy casualties.⁹²

Radzivil understood well that the situation was catastrophic for the GDL.⁹³ Before the battle (August 6, 1654), he appealed for the nobility's intervention to defend the country. In his correspondence, Radzivil acknowledged the military superiority of the Moscow army and was aware of the daunting challenges ahead.⁹⁴ It seems that Radzivil strategically chose the date of the battle, so as to make it coincide with the eclipse, understanding the potential psychological impact this could have on both his own troops and the enemy forces. He probably tried to take advantage from the potential disarray or irrational behavior that this coincidence could provoke among the enemy ranks, particularly among those soldiers who, less educated, could be more susceptible to superstition or fear. Radzivil probably considered this tactic, because the Muscovite army was likely uninformed about the eclipse's astrological significance. No broad astrological discussion about the event occurred in the Muscovite state, despite Tsar Alexei Mikhailovich's personal interest in the subject and his later order to translate astrological calendars.⁹⁵

Some sources contain the next information about the battle. An unknown participant from the GDL army wrote the following (August 13, 1654): "Yesterday, 12 August, the severe and bloody battle of Škloū happened during that terrible eclipse of the Sun, but by the grace of God we gained the victory." The darkness caused by the eclipse contributed to confuse the Moscow ranks, so that the Moscow regiments attacked each other and the Lithuanian army won on the battlefield.⁹⁶

1654–1655; Kotljarchuk, *In the Shadows of the Poland and Russia*.

⁹² Maciej Vorbek-Lettow, *Skarbnica pamięci: pamiętnik lekarza króla Władysława IV*, oprac. Ewa Galos, Franciszek Mincer (Wrocław, Warszawa, Kraków: Zakład Narodowy im. Ossolińskich, 1968), 191–195; *Ojczyście wspominki w pismach do dziejów dawnej Polski; diaryusze, relacje, pamiętniki... Tudzież listy historyczne do panowania królów Jana Kazimierza i Michała Korybuta, oraz listy Jana Sobieskiego*, t. 1 (Kraków: J. Cypcer, 1845), 114–116; Wisner, *Janusz Radziwiłł 1612–1655*, 170–171; Bobiatyński, *Od Smoleńska do Wilna*, 35–62.

⁹³ About Radzivil's military activity see: Wisner, "Działalność wojskowa Janusza Radziwiłła, 1648–1655," 53–109; Kroll, "Kompania białoruska Janusza Radziwiłła 1654 r.," 43–64.

⁹⁴ *Korespondencja wojskowa hetmana Janusza Radziwiłła w latach 1646–1655: Dziariusz kancelaryjny 1649–1653*, oprac. Mirosław Nagielski, Konrad Bobiatyński, Przemysław Gawron, Krzysztof Kossarzecki, Piotr Kroll, Andrzej Adam Majewski, Dariusz Milewski (Warszawa: Wydawnictwo Neriton, 2019), 186–188.

⁹⁵ Вильям Фрэнсис Райан, *Баня в полночь. Исторический обзор магии и гаданий в России* (Москва: НАО, 2006), 578–580.

⁹⁶ *Ojczyście wspominki w pismach do dziejów dawnej Polski*, 114, 116.

A physician Maciej Vorbek-Lettow (1593–1663)⁹⁷ and the Deputy Cupbearer of Novagoradak Jan Cadroŭski (Polish: Jan Cadrowski) (1617–1682) had contrasting views on the significance of the solar eclipse of August 12, 1654. Both belonged to Protestant Radzivil's circle. Vorbek-Lettow's perspective, as a physician, focused on the physical aspects of the eclipse itself rather than its broader implications. He noted that the eclipse was not as severe as that predicted by Furman in his calendar,⁹⁸ considering that the prognostications of the eclipse rather described just the phenomenon itself. But Furman actually meant that the eclipse would have those consequences.

In contrast, Cadroŭski characterized the eclipse as “terrible,” likely emphasizing its broader impact, particularly in the context of the ongoing war and its aftermath. He underlined the significant losses among high-ranking individuals in the battle, such as Hrygory Mirsky, and he mentioned this event as one of the devastating consequences of the eclipse within the societal and political context.⁹⁹

Indeed, the publication of Furman's calendar in the GDL appears to have had a notable impact, particularly among the nobility. The prognostications contained in the calendar likely captured the attention of the educated elite, who were familiar with astrology and were then probably influenced by the predictions of dire consequences associated with the solar eclipse.

However, the most significant effect seems to be represented by the victory against the Moscow army on the day of the eclipse. In all likelihood this victory, narrated in association with the eclipse, contributed to a sense of divine intervention or providence among the populace. The darkness caused by the eclipse may have also played a role in sowing confusion or disarray among the Moscow troops, potentially aiding the GDL army in securing victory.

Overall, if the publication of the calendar may have heightened awareness and anticipation of potential threats, it was rather the tangible outcome of the battle on the day of the eclipse that in all likelihood had the most profound impact, solidifying the belief in the efficacy of astrology and the providential significance of celestial events.

⁹⁷ Maciej Vorbek-Lettow, a diarist, physician, and Lutheran activist, received a comprehensive education. He attended the Academic Gymnasium in Elbląg, the Academic Gymnasium in Gdańsk, and the Vilna Academy (1608–1610). He subsequently studied law at the University of Leuven (1611–1612) and medicine at the University of Padua (1612–1614) and the University of Bologna (1614–1616). In 1616, he established his medical practice in Vilna. Dariusz Champerek, “A Lutheran, Maciej Vorbek-Lettow (1593–1663) and his Studies in Padua,” *Perspektywy Kultury* 41, 2, 1 (2023): 63–74.

⁹⁸ Vorbek-Lettow, *Skarbnica pamięci: pamiętnik lekarza króla Władysława IV*, 188–189.

⁹⁹ Jan Cadroŭski studied with his twin brother Stefan in Königsberg and at the Cracow Academy. From 1637, he served at the court of Baguslaŭ Radzivil, accompanying the prince on extensive travels throughout Western Europe and executing various commissions for him. Аляксандр Коршунаў, *Помнікі мемуарнай літаратуры Беларусі XVII ст.* (Мінск: Навука і тэхніка, 1983), 132.

Conclusions

Argol's publication about the eclipse of 12 August 1654 significantly sparked public interest across various countries. The main sources of spreading astrological information were calendars, brochures, and publications for professional astrologers. The discussion touched upon many countries, from England to the Polish-Lithuanian Commonwealth. The eclipse was discussed by the clergy, nobility, and astrologers.

Interpretations of the celestial event varied widely. Forecasts of upcoming wars and epidemics fuelled the spread of apocalyptic ideas, with people viewing war as a true apocalypse. This was the reason for such a panicked reaction. At the same time, not all publications predicted the end of the world. Apocalyptic and panic-stricken moods began to spread throughout Europe, provoking publications created by the Catholic clergy.

This case clearly shows the importance of interpretation and the related problems. The categorical and harsh interpretations in forecasting provoked different reactions. This discussion vividly presented the mentality and the way of thinking at that time. This example also presents the manipulating of public opinion. Astrological calendars were related to popular literature, but they had a strong influence on mentality. People were afraid of the future. Fear was one of the strong emotions exploited by religion, government, etc. Prognoses were a good instrument for manipulating people, influencing the future, and preparing for events. Moreover, Catholic and Protestant approaches to the eclipse were presented in this case.

In the seventeenth-century GDL astrology was still on the Monarch's and the nobility's service. Astrological forecasts played an important role for them. The high strata took into account prognoses in their decision-making process. Of course, a war, as any extraordinary situation, pushed people turn to astrology and seek support. The practical use of astrological knowledge became equally important in this system.

In conclusion, the case of the 1654 eclipse touches upon a wide range of issues. It has shown the different kind of astrological publications, the level of development of astrology and its place in society, astrologers' work and the use of astrology by the upper strata, the spreading of astrological information in European courtiers, the reaction to forecasts and the mentality of the society of that time.

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Abbreviations

PSB= Polski słownik biograficzny

НГАБ= Нацыянальны Гістарычны Архіў Беларусі

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Paolo Falconieri: lettere a Vincenzio Viviani

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English title

Paul Falconieri to Vincenzio Viviani: Letters

Abstract

The article deals with a group of letters authored by Paul Falconieri, a man of letters and a trusted figure of the Medici family in Rome. Preserved in the Galilean Collection of the National Central Library of Florence, the letters address a wide range of topics, some of them concerning family matters. Among the figure appearing in the correspondence are the Dutch publishers Elzevier and Blaeu: the former published the *Discorsi e dimostrazioni matematiche* (1638), while the latter planned to produce an edition of Galileo's works edited by Viviani, although this project was never realised.

Keywords

Paul Falconieri, Vincenzio Viviani, The Medici Family, Galileo Galilei, Blaeu of Amsterdam

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Paolo Falconieri¹ e Vincenzio Viviani² furono amici ed ebbero rapporti epistolari soprattutto dal momento in cui Falconieri si trasferì definitivamente a Roma. I due dotti appartennero alle più importanti accademie a cominciare dall'Accademia della Crusca,³ passando per l'Arcadia,⁴ per finire all'Accademia del Disegno.⁵

Gli argomenti trattati nella corrispondenza sono molteplici, tra questi è degno di es-

- ¹ Paolo Falconieri (1634–1704), poeta, e Gentiluomo di Camera del granduca Cosimo III; su di lui, vedi DBI, XLIV, 1994, 388–390, voce di Mario Bencivenni. Sul Falconieri collezionista, vedi Dalma Frascarelli, *Paolo Falconieri tra scienza e arcadia. Le collezioni di un intellettuale del tardo barocco romano* (Roma: Campisano, 2012); vedi anche: *Paolo Falconieri (1634-1704) intenditore, collezionista e intermediario*, tesi di dottorato (nella scheda della BNCF, si legge: *Autori secondari, Aurigemma Maria Giulia*); la copia della tesi depositata in BNCF non è consultabile. Notizie su Paolo Falconieri anche in Elide Casali, “Messer Ottavio amatissimo ...” A proposito del carteggio di Lorenzo Magalotti con Ottavio Falconieri (1660–1674),” *Studi secenteschi* XXXI (1990): 87–111; notizie anche in *La festa delle Arti. Scritti in onore di Marcello Fagiolo per cinquant'anni di studi*, a cura di Vincenzo Cazzato, Vincenzo, Sebastiano Roberto e Mario Bevilacqua, I (Roma: Gangemi, 2014); Alessandro e Mirco Gallenzi, “Nuovi dati sulla vita e le opere di Peresio,” *Studi secenteschi* LVII (2016): 126–132. Spesso, Paolo viene confuso con il cugino Paolo Francesco Falconieri (1626–1696).
- ² Su Vincenzio Viviani (1622–1703), vedi *On the Life of Galileo. Viviani's Historical account and other Early Biographies*, Edited, translated and annotated by Stefano Gattei (Princeton and Oxford: Princeton University Press, 2019); DBI, C, 2020: 86–89, voce di Simon Dumas Primbault; vedi inoltre le mie ricerche: Alfonso Mirto, “Lettere di Stefano Gradi ai fiorentini: Viviani, Dati, Leopoldo e Cosimo III de' Medici,” *Studi secenteschi* XLIX (2008): 372–404; Id., “Lettere di Melchisédec Thévenot ai fiorentini: Leopoldo de' Medici, Cosimo III granduca di Toscana e Vincenzio Viviani,” *Galilaeana. Studies in Renaissance and Early Modern Science* XII (2015): 145–191; Id., “Lettere inedite di Francesco Bondicchi ad Antonio Magliabechi, Francesco Redi, Vincenzio Viviani e Giulio Benedetto Lorenzini,” *Atti e memorie dell'Arcadia* 9 (2020): 138–219.
- ³ Vincenzio Viviani entrò nella Crusca il 13 settembre 1661, con il nome di *Rinvigorito*, Paolo Falconieri, il 3 agosto 1633: *Catalogo degli Accademici*, a cura di Severina Parodi (Firenze: presso l'Accademia, 1983), 109–110 e 113. Sulla storia dell'Accademia della Crusca rimando alla Collana diretta da Massimo Fanfani (schede complete sul sito dell'Accademia).
- ⁴ Paolo Falconieri prese il nome di Fronimo Epirio, mentre Vincenzio Viviani assunse quello di Erone Geonio. Sull'Arcadia vedi Maurizio Campanelli, *Canoni d'Arcadia: il custodiato di Crescimbeni* (Roma: Edizioni di storia e letteratura, 2018) [relazioni del Convegno internazionale di studi: Roma 6–8 giugno 2018, Roma: Edizioni di storia e letteratura, 2019]; Maria Luisa Doglio, Manlio Pastore Stocchi, *Rime degli Arcadi I/XIV* (Roma: Edizioni di storia e letteratura, 2019) e naturalmente vanno consultati gli *Atti e Memorie dell'Arcadia*, pubblicati dall'Accademia dell'Arcadia.
- ⁵ Viviani dall'8 novembre 1648; Falconieri dal 30 agosto 1671 (*Gli accademici del disegno. Elenco cronologico*, a cura di Luigi Zangheri (Firenze: Olschki, 1999, 72 e 96); *Gli accademici del disegno. Elenco alfabetico*, a cura di Luigi Zangheri (Firenze: Olschki, 2000, 121 e 336). Sulle accademie fiorentine vedi *Dizionario storico delle Accademie toscane. Secoli XVI-XVIII. Vol. I Firenze*, a cura di Jean Boutier, Maria Pia Paoli e Claudia Tarallo (Pisa: Pacini editore, 2024).

sere segnalato quello della lettera del 19 giugno 1700, nella quale il Falconieri presenta al Viviani la preghiera che Antonia, vedova di Benedetto Landucci, rivolge allo scienziato fiorentino per avere un aiuto economico. Questo aspetto è stato studiato da Antonio Favaro,⁶ perciò non ci dilunghiamo più di tanto, evidenziamo solo che il Viviani, in ricordo del suo maestro, prestò soccorso alla pronipote di Galilei, inviandole del denaro.

La prima lettera, datata 16 febbraio 1669, proveniva da Lisbona, dove il Falconieri era giunto il 18 gennaio al seguito del gran principe Cosimo de' Medici e della comitiva faceva parte anche Lorenzo Magalotti;⁷ è una delle cinque lettere che non sono state inviate da Roma dove Paolo Falconieri operava per conto del cardinale Leopoldo e di Cosimo III. Nella corrispondenza si incontrano nomi importanti della cultura e del mondo della stampa e dell'editoria; tra questi ultimi sono da citare i Blaeu e gli Elzevier⁸ i quali in quegli anni dominavano il mercato librario europeo. Questi mercanti erano stati incontrati dal Falconieri durante il suo viaggio nei Paesi Bassi in compagnia del Magalotti; erano entrambi al seguito di Cosimo III, che visitò la Repubblica nell'inverno tra il 1667 e il 1668 e di nuovo nel 1669. Pieter Blaeu, riferendosi al primo viaggio degli ospiti toscani, scriveva a Magliabechi:

Il tempo mi manca di scrivere più, finisco dunque differendo il resto fin alla settimana prossima. In tanto ella ha da sapere che sono gionti qui li Sig.^{ri} Magalotti, e Falconieri. Forse sarà meglio ch'io scriva al Sig.^r Segni de' 4 suddetti pezzi, che nello scrivere mi viene nella mente, lo farò però l'ordinario prossimo.⁹

⁶ *Scampoli galileiani* (Padova: Tipografia G.B. Randi, 1886), vol. secondo, 16–21; Antonio Favaro ha pubblicato anche le lettere di Antonia Landucci al Viviani (ibid., 23–24) e riprodotte in *Atti e memorie della R. Accademia di Scienze ed Arti in Padova CCCLXIII* (1903–1904): 24–25.

⁷ Lorenzo Magalotti (1637–1712), diplomatico e scrittore nato a Roma da famiglia fiorentina, tra l'altro compilò i *Saggi di naturali esperienze* (1667); su di lui vedi *DBI LXVII* (2006): 300–305, voce di Cesare Preti–Luigi Matt; Stefano Miniati, Lorenzo Magalotti (1637–1712): rassegna di studi e nuove prospettive di ricerca, *Annali di storia di Firenze* 5 (2010): 31–47.

⁸ Per gli Elzevier resta fondamentale Alphonse Willems, *Les Elzevier. Histoire et Annales typographiques* (Bruxelles: Van Trigt, 1880 [ristampa anastatica: Nieuwkoop: De Graaf, 1991]).

⁹ BNCF, Fondo Nazionale, II.I.382, lettera del 18 novembre 1667, ora in Alfonso Mirto & Henk Th. van Veen, *Pieter Blaeu: lettere ai fiorentini Antonio Magliabechi, Leopoldo e Cosimo III de' Medici, e altri, 1660–1705* (Firenze: Istituto Universitario Olandese di Storia dell'Arte/Amsterdam, APA–Holland University Press, 1993), 168. I pezzi a cui il Blaeu faceva riferimento erano relativi ai *Globi terrestri*; scriveva il Blaeu: “Ho imbarcato ne sopranominato vascello Aletta Maria due botti con due Globi, una altra botte con una Sfera, e poi una cassa con Carte, sono in tutto quattro pezzi” (ibid.). Su Alessandro Segni vedi Mirto, *Alessandro Segni e gli Accademici della Crusca. Carteggio* (1659–1696) (Firenze: Accademia della Crusca, 2016); *DBI XCI* (2018): 756–758, voce di Alfonso Mirto.

E ancora:

Il Sig.^r Principe si ritrova adesso a Bruxelles come credo, mi ha onorato d'un anello avanti la sua partenza, ma di ciò e di tutto il resto le scriverò largamente fra otto giorni.¹⁰

Pieter Blaeu, infatti, con lettera del 3 febbraio 1668, avvertiva il Magliabechi che “Sarà forse qui di ritorno Ser.^{mo} Principe di Toscana fra pochi giorni.”¹¹ Lo stampatore olandese rimase in contatto epistolare con Cosimo de' Medici anche dopo che questi ebbe assunto il titolo di granduca, infatti, l'ultima lettera di Pieter Blaeu (2 marzo 1691) non conteneva argomenti relativi ai libri, ma trattava di politica europea e nello specifico della differenza di trattamento ricevuto dall'ambasciatore toscano rispetto a quello di Savoia.¹² La questione delle precedenza non riguardava solo i Savoia ma anche gli Este e i Farnese, con la “conseguenza di un isolamento dei Medici fra gli altri principi italiani.”¹³

Nel carteggio si parla anche di un progetto per un'edizione delle opere di Galilei da parte dei Blaeu, infatti, Paolo Falconieri scriveva:

La puntualità del Sig.^r Blaeu, et il suo affetto è inarrivabile; come ella vedrà dall'aggiunta lettera mi fa scuse di non essersi soddisfatto, per dir così, nella prima e replicandomi le stesse cose circa l'impressione delle opere del Sig.^r Galileo, vi aggiugne qualche cosa delle maiuscole di legno, e fregetti che desiderava promettendone le particolari notizie con altra sua.¹⁴

Questo disegno, però, non si realizzò e per avere un'edizione delle opere di Galilei bisognò attendere alcuni secoli. Come è noto, infatti, la prima edizione completa uscì tra il 1842 e il 1856, presso la Società editrice fiorentina.¹⁵

¹⁰ BNCF, Fondo Nazionale II.I. 382, c. 101r, lettera del 27 gennaio 1668, ora in Mirto & van Veen, *Pieter Blaeu: lettere ai fiorentini Antonio*, 169.

¹¹ BNCF, Fondo Nazionale II.I. 382, c. 102r, ora in Mirto & van Veen, *Pieter Blaeu: lettere ai fiorentini Antonio*, 170–171.

¹² Ibid., 311–312.

¹³ Furio Diaz, *Il granducato di Toscana. I Medici* (Torino: Utet, 1987), 237.

¹⁴ BNCF, Gal. 164, c. 321r; dell'ipotesi di una nuova edizione delle opere del Galileo, non si parla né nelle lettere di Pieter Blaeu a Magliabechi, né in quelle indirizzate a Leopoldo de' Medici (Mirto & van Veen, *Pieter Blaeu: lettere ai fiorentini*).

¹⁵ Una ristampa in 20 volumi fu eseguita a Firenze presso la tipografia Barbera nel 1968.

Lettere

Criteri di edizione

I documenti sono stati trascritti fedelmente eccetto qualche piccolo intervento nell'uso della punteggiatura e delle maiuscole.

1. BNCF, Gal. 163, c. 242r

Ill.^{mo} Sig.^{re} e Pron mio Oss.^{mo}

Con molta devozione, e con una buona caparra di ricevere grazie singolari me ne vo a S.^t Jago, mentre per così dire egli me le manda incontro sin qui in Lisbona, e mi fa cominciare il viaggio di Galizia con una fortuna tanto considerabile per me. Che altro poteva io chiedere Sig.^{re} Vincenzo mio, o che altro poteva ottenere, che l'occasione di far sovvenire a V. S. Ill.^{ma} di me, e assicurarla del desiderio mio incomparabile di servirla?

Questo ottengo da S.^t Jago il quale ha fatto trovar qui Santi¹⁶ suo servitore per ch'io gli dia una procura specialissima di riverirla, e pregarla insieme a non lasciarmi tornare anche questa volta a Firenze senza aver ricevuto alcun suo comandamento. Questa lettera mia dunque sarà lettera credenziale acciò che V. S. Ill.^{ma} creda per vere l'espressioni, che egli le farà per mia parte, ancor che la stimi superflua, non essendo bisogno, ch'io procuri fede da lei ad un suo servitore con mie lettere, né ch'io mi affatichi in persuaderla d'una verità così chiara, come è questa del mio desiderio di servirla, e che io sono di

Di V. S. Ill.^{ma}

Devotiss.^{mo}, et Oblig.^{mo} Ser.^{re} vero

Paolo Falconieri

Lisbona il 16 febbraio 1669

¹⁶ Sulla visita di Cosimo III di Santiago, vedi Miguel Taín Guzmán, *La ciudad de Santiago de Compostel en 1669. La peregrinación del Gran Príncipe de la Toscana Cosimo III de Medici* (Santiago: Teófilo Ediciones, and Consorcio, 2012).

2. BNCF, Gal. 164, c. 316r–v

Ill.^{mo} Sig.^{re} mio Oss.^{mo}

Dall'aggiunta lettera del Sig.^r Blaeu,¹⁷ V. S. Ill.^{ma} vedrà quanto io debba esserle obbligato per il merito che ella si è compiaciuta farmi acquistare con il nome gloriosissimo del Sig.^r Galileo, e per il godimento di essermi una volta fruttuosamente adoperato in servirla, dell'uno e dell'altro io sarei stato privo se la sua gentilezza non mi abilitava con il comandamento che si compiace qui di farmi. Bisogna ora che ella procuri di sapere chi possa avere la copia che egli accenna doverne essere costì, e quando questo non sia possibile la faremo venir di là. Stimo superfluo pregarla a tener segreto il tutto come desidera il Sig.^r Elzeviri¹⁸ per suo interesse, parendo a me che in questo si mischino il suo et il nostro.

Starò poi aspettando da lei, e la lettera del Sig.^r Blaeu, et i suoi comandamenti in ordine al rispondere in Amsterdam per poter compir l'opera cominciata così felicemente. Con che pregando il mio Sig.^r Vincenzo della continuazione della sua grazia, mi sottoscrivo

Di V. S. Ill.^{ma}

Devot.^{mo} Par.^{mo} Ser.^{re}

Paolo Falconieri

Pisa il 28 febbraio 1674

3. BNCF, Gal. 164, cc. 317r–318r

Ill.^{mo} Sig.^{re} e Pron mio Oss.^{mo}

Sig.^{re} mio

Ella vuole che messe da bando le cirimonie pigli con esso lei le forme più domestiche, et io che mi pregerò sempre di potermi vantare di avere avuta seco, e famigliarità, e domestichezza mi muto subito in così stimabile possesso come ella vede. Già gli espressi nella mia quanto dovevo a lei per avermi fatta conseguire la fortuna di servir lei, e di contribuire qualche piccola applicazione alla indennità, per dir così, della gloria del Sig.^r Galileo, onde lasciando anche in ciò i complimenti, le dico che non ho per anche risposto al Sig.^r Blaeu

¹⁷ Si tratta di Pieter Blaeu, che Paolo Falconieri aveva conosciuto personalmente durante i viaggi nei Paesi Bassi al seguito del gran principe Cosimo e, probabilmente, prima ancora quando il Blaeu soggiornò in Italia. Il futuro granduca visitò la Repubblica nell'inverno del 1667–1668 e nuovamente nel 1669, Godefridus Johannes Hoogewerff, *De twee reizen van Cosimo de' Medici, prins van Toscane, door Nederland (1667-1669)* (Amsterdam: Joannes Müller, 1919).

¹⁸ Si tratta di Daniel Elzevier (1626–1680), esponente di una delle più note famiglie di stampatori olandese, in contatto con Carlo Roberto Dati, su cui vedi Mirto, Carlo "Roberto Dati e Vincenzio Viviani: Carteggio (1659-1672)," *Studi secenteschi* LI (2010): 291–346.

perché stimerei bene che ella fusse certa prima, che il Sig.^r Panciatichi¹⁹ o altri anno le lettere di cui si tratta perché qualche cosa dee sapere l'Elsevirio²⁰ parla con tanta asseverazione, e quando veramente si potessero trovar qui sarebbe per ogni conto un grande vantaggio, e risparmio di spesa, e per risparmio di tempo, e per risparmio di richieste al Elsevirio al quale è assai meglio avere a richiedere solamente che stampi le lettere a nostro modo, che questo, e di più che si pigli il pensiero di farne fare una copia, e d'aspettare che sia fatta venire qui, e poi tornata in la. Su questo sospendo di scrivere al Sig.^r Blaeu sin che mi venga la sua risposta intorno a ciò, né l'indugio è improprio mentre egli che è pratico di Firenze saprà benissimo che non si può andare a dirittura a mettervi su le mani.

V. S. mi ha toccato nel mio debole quando mi ha lodato nella sua per ingenuo, onde per confermarla in questa buona opinione, della quale mi pregio, mi sono lasciato portare a scriverle forse troppo ingenuamente, ma ella che se ne picca non meno di me ha con la medesima a considerare questo che le scrivo, e se non le pare a proposito con la medesima comandarmi nel modo che vuol essere servita, che io l'obbedirò con quella prontezza che dee chi si professa di essere

Di V. S. mio Sig.^{re}

Dev.^{mo} e Par.^{mo} Ser.^{re}

Paolo Falconieri

Livorno il 12 marzo 1674²¹

4. BNCF, Gal. 164, c. 321r

Ill.^{mo} Sig.^{re} e Pron mio Oss.^{mo}

Sig.^{re} mio

La puntualità del Sig.^r Blaeu, et il suo affetto è inarrivabile; come ella vedrà dall'aggiunta lettera mi fa scuse di non essersi soddisfatto, per dir così, nella prima e replicandomi le stesse cose circa l'impressione delle opere del Sig.^r Galileo, vi aggiugne qualche cosa delle maiuscole di legno, e fregetti che desiderava promettendone le particolari notizie con altra sua. Dopo che ella si sarà soddisfatta potrà rimandarmela e quando abbia fatte tutte le normali diligenze per ritrovare la copia delle lettere di f. Paolo²² che dice il Sig.^r Elzeviri doverci essere, mi favorirà avvisarmelo perch'io possa continuare a servirla essendo

¹⁹ Lorenzo Panciatichi (1635–1676), canonico, gentiluomo del principe Leopoldo de' Medici, poeta satirico e scrittore elegante, amico di Magliabechi; su di lui vedi *DBI LXXX* (2014): 692–694, voce di Paolo Rondinelli.

²⁰ Daniel Elzevier.

²¹ La data si trova in testa alla lettera.

²² Nel 1673, a Ginevra sotto falso nome di Verona, uscì un'edizione delle *Lettere* di fra Paolo Sarpi. Alcune lettere di Galilei a Sarpi e di quest'ultimo a Galilei sono pubblicate on line.

Di V. S. mio Sig.^{re}
 Devot.^{mo} e Par.^{mo} Ser.^{re}
 Paolo Falconieri
 Pisa il 20 marzo 1674²³

5. BNCF, Gal. 164, c. 325r

Sig.^{re} mio

Se domani averò tempo scriverò al Sig.^r Blaeu, gli scriverò del tenore che ella mi accenna, cioè che subito che sarò a Firenze, che sarà tra giorni, procurerò d'aver notizie del manoscritto e che in caso che non si trovi starò aspettando da lui la cortesia promessami per impegnarlo così sempre più. Se poi non potrò, per sì poca dilazione aspetterò di farlo a Firenze dove concerterò seco la lettura dopo che averò fatto col P. Provinciale²⁴ tutte quelle diligenze che ella vorrà.

Ricevei la lettera del Sig.^r Scolari,²⁵ ma nella strettezza del tempo mi son preso la libertà di non rispondergli, perché per ancora non vi è stato tempo di negoziare, essendo adesso il Ser.^{mo} Principe²⁶ occupatissimo nelle cose di Livorno, e di questa città, e del Capitolo a suo tempo non lascerò di inviarlo per quanto potrò particolarmente per incontrare i sentimenti del mio Sig.^r Vincenzo, di cui mi sotto scrivo al mio solito

Devot.^{mo} e Par.^{mo} Ser.^{re}
 Paolo Falconieri
 Pisa il 30 Marzo 1674²⁷

6. BNCF, Gal. 164, c. 334r

Ill.^{mo} Sig.^{re} mio Oss.^{mo}

Essendosi contentato il Ser.^{mo} Gran Duca di comandarmi che per esercizio di questi giovani dell'Accademia della Scultura che faccia far qui il busto del Galileo, sapendo che V. S. ha procurato altre volte di farlo fare, mi persuado, che possa havere l'effigie della sua

²³ La data si trova in testa alla lettera.

²⁴ Difficile identificare questo personaggio.

²⁵ Potrebbe trattarsi di Virginio degli Scolari, notaio fiorentino e autore della *Scrittura intorno alla nobiltà del notariato* (Firenze: per Santi Franchi, 1681).

²⁶ Dovrebbe essere Leopoldo de' Medici, ma è strano che venga chiamato solamente col titolo di "Principe" e non "Principe Cardinale"; Francesco Maria, in questo momento, non aveva ancora quindici anni, essendo nato il 12 novembre 1660.

²⁷ La data si trova in testa alla lettera.

faccia, che mi pare, che fosse stata formata dopo la sua morte, ò pure qualche altra cosa, che ella si trovi, considerandosi che non possa riuscir bene l'opera quando non vi sia qualch'uno di questi aiuti. Che perciò prego V. S. a compiacersi di farmi giungere quello che ha in ordine a guisa; persuadendomi che sarà per farlo volentieri per l'affetto massimamente, che ha alla memoria di questo grand'huomo. E rassegnando dell'affettuosa mia osservanza, resa con particolar desiderio di farmi conoscere qual mi professo

Di V. S.

Aff.^{mo} e Parzialissimo

Paolo Falconieri

Roma 30 Giugno 1674

Scusi se non scrivo di mio pugno per scarsezza do tempo, del resto non lo fo per l'incomodo che havevo perché so che ella contribuisce volentieri questa sua applicazione alla gloria di quel grand'uomo, che almeno così sarà collocata la sua effigie nella Galleria.

7. BNCF, Gal. 165, c. 6r

Ill.^{mo} Sig.^{re} mio Oss.^{mo}

Ho riconosciuto negli augurij di felicità che V.S.Ill.^{ma} s'è compiaciuta farmi nella passata solennità del Natale i soliti effetti della sua singolar cortesia verso di me, onde adesso sono a rendergliene le dovute grazie desiderando di poter testificarle maggiormente coll'opere il vivo riconoscimento, che ne tengo. A tal effetto supplico V.S.Ill.^{ma} ad honorarmi de suoi stimati comandamenti, ed intanto augurando parimenti a lei ogni più prospero avvenimento nel corso di quest'anno santo, mi confermo

Di V.S.Ill.^{ma}

Devotiss.^{mo} et Oblig.^{mo} Ser.^{re}

Paolo Falconieri

Roma 5 Gennaio 1675

8. BNCF, Gal. 256, c. 91r

Ill.^{mo} Sig.^{re} mio Oss.^{mo}

Mando a V. S. Ill.^{ma} la risposta del Sig.^r D. Carlo Buragna²⁸ la quale le scriverà a riprova della prontezza con la quale ella è stata servita del recapito della sua.

Mi trovo impegnato di mandare a diversi virtuosi di Napoli²⁹ de' termometri e perché sieno esquisiti bisogna che ricorra alle grazie del mio Sig.^r Vincenzo. La prego dunque, di scerne due 12^{ne30} di quelli di 50 a suo comodo, et il loro prezzo potrà farglielo pagare dal Sig.^r Andrea del Rosso.³¹ Scusi la prego questo incomodo il quale mi son fatto lecito dargliele, perché son sicuro che ella lo soffrirà volentieri per le persone alle quali vanno, o per obbligar anche maggiormente me che sono

Di V. S. Ill.^{ma}

Aff.^{mo} e Par.^{mo} Ser.^{re}

Paolo Falconieri

Roma il 25 Aprile 1676

9. BNCF, Gal. 165, c. 281r

Ill.^{mo} Sig.^{re} mio Oss.^{mo}

Nuovi attestati de suoi favori si compiace V.S.Ill.^{ma} inviarmi ne suoi annunci cortesi, e perché un reciproco sentimento di simili auguri, al di lei merito sublime, dovea da me esprimersi in sì gloriosa festività, in testimonio del mio debito; questo io le presento con isperanza che le sia tanto più gradito, quanto da essa quasi richiesto con atto così gentile dalla mia obbligazione, e perché io non dubito che ella non sia persuasa della mia passione in dimostrarle con gli effetti quanto io esprimo sinceramente, attenderò il contento, che vorrà

²⁸ Carlo Buragna (1634–1679), “huomo di somma erudizione, e non inferiore di sapere all’istesso Signor Lionardo [di Capoa]”: *Addizioni copiose di Lionardo Nicodemo alla Biblioteca napoletana del dottor Niccolo Toppi* (In Napoli: Per Salvator Castaldo Regio Stamp., a spese di Giacomo Raillard, 1683), 151; su di lui vedi *DBI* XV (1972): 367–368, voce di Lovanio Rossi.

²⁹ Uno dei virtuosi era senz’altro Antonio Monforte (1644–1717), matematico, su cui vedi *DBI* LXXV (2011): 647–651, voce di Federica Favino. Di lui si conserva una lettera indirizzata al Viviani (BNCF, Gal. 252, c. 262).

³⁰ Sul verso della carta 91, il Viviani annotava: “Adì 16 maggio 1676: consegnato alla banca del S. del Rosso una scatoletta con le due dozzine di termometri, dove pagai al Gonfia lire ventiquattro, acciò l’invii a Roma all’Ill.^{mo} Sig. Paolo Falconieri pel procaccio. Adì 26 detto, ho scritto al medesimo Sig. Paolo di questa consegna.” Gonfia era il soprannome di Jacopo Mariani uno dei costruttori di termometri, Paolo Galluzzi, *Scienziati a corte: l’arte della sperimentazione nell’Accademia Galileiana del Cimento* (Firenze: Sillabe, 2001), 47.

³¹ Andrea del Rosso (1640–1715), senatore dal 1700.

somministrarmene la sua bontà con i suoi comandi, per testificare maggiormente quanto io sono

Di V.S.Ill.^{ma}

Aff.^{mo} e Oblig.^{mo} Ser.^{re}

Paolo Falconieri

Roma 17 Dicembre 1678

Sono sempre ammiratore della sua virtù, e desiderosissimo di portare servendola, con gusto totale la qualità di [...].³²

10. BNCF, Gal. 256, c. 244r

Ill.^{mo} Sig.^{re} e Pron mio Oss.^{mo}

Non mi sono riuscite meno inutili le diligenze fatte qui di quelle d'Asterdam.³³ Ogni uno conosce il valore della penna di V. S. Ill.^{ma} ogni uno applaude all'opera, ma ogni uno si spaventa della qualità de' tempi. Mi sono ristretto alla sola traduzione d'Archimede³⁴ sperando che questa possa aver meno disgrazia per essere toscana, e per non si trovar l'opera di questo autore, e conseguentemente più sicuro lo spaccio, ne ho travato ancora né sì, né no intero.

Per facilitare, la supplico d'avvisarmi che volume sarà, cioè quanti fogli, e che quantità di figure, e vedere con queste notizie di fare gl'ultimi sforzi. Io ho pena d'esserle così inutile, ma spero ch'ella riconoscerà esser colpa della sua disgrazia non mia. Forse muteranno i tempi, e quello che non s'è fatto si farà, e sempre in ogni caso abbiamo il Blaeu, che non l'ho mai liberato. Mi favorisca di darmi ottime nuove della sua salute e di vedermi

Di V. S. Ill.^{ma}

Par.^{mo} et Oblig.^{mo} Ser.^{re}

Paolo Falconieri

Roma il 19 Settembre 1682

³² La scoloritura dell'inchiostro non fa decifrare l'ultima parola.

³³ Amsterdam.

³⁴ Sulle traduzioni delle opere di Archimede, vedi Antonio Favaro, *Amici e corrispondenti di Galileo Galilei* (Venezia, Tip. Ferrari: 1894), 136–138.

11. BNCF, Gal. 256, c. 248r

Ill.^{mo} Sig.^{re} mio Oss.^{mo}

Mai mi sono pentito tanto di avere trascurati gli studi della geometria, ancor che li lasciassi per altre tanta necessità di quanta obbliga V. S. Ill.^{ma} ad abbandonarla, quanto che nel leggere la sua gentilissima del 5 stante, considerando a quanto onore mi avrebbe sollevato s'io fusse capace di poterle dire: Sig.^r Vincenzo, sono qui io per fare quella manifattura intorno alle sue opere, che la scarsezza del tempo toglie di fare a lei. Ma questo non ha rimedio, quanto a me, onde m'ingegnerò di offerirglielo d'altri. Il Sig.^r Santini³⁵ lo veggo pieno di cordialità, e di amore verso di lei, delle opere sue. Ella sa ch'egli possiede in queste scienze. Sa in grado è il suo Archimede, sa l'intera fede che può avere in me, circa la sicurezza d'esse, e sa che se le figure sono capace di insegnarle io lo farò volentierissimo. Dentro a questi termini vegga quello che si può fare per servirla, e disponga con ogni autorità di chi si professa

Di V. S. Ill.^{ma}

Aff.^{mo} et Oblig.^{mo} Ser.^{re}

Paolo Falconieri

Roma il 14 ottobre 1682

12. BNCF, Gal. 167, c. 112r

Ill.^{mo} Sig.^{re} mio Oss.^{mo}

Mando a V.S.Ill.^{ma} la Roma del Nardini,³⁶ ch'è la medesima che riserbò per sé il poveretto di mio fratello,³⁷ perché questa è stato capace il mio cameriere di trovarla, ma non già

³⁵ Vincenzo Santini, matematico, cfr. Antonio Favaro, *Documenti inediti per la storia dei manoscritti galileiani nella Biblioteca Nazionale di Firenze* (Roma: Tipografia delle scienze matematiche e fisiche, 1886), 41–42, 84, 88; nella Biblioteca Riccardiana (*Riccardiano* 2304, cc. 46r–47v), di Vincenzo Santini, è conservata una lettera, datata Roma 14 settembre 1683, senza indicazione di destinatario, ma ad uno della famiglia d'Ambra, dove si parla di cannocchiale.

³⁶ *Roma antica di Famiano Nardini alla Santità di N. S. Alessandro VII* (In Roma: per il Falco, 1665), a spese di Biagio Diversino, e Felice Cesaretti.

³⁷ Si tratta di Ottavio Falconieri (1636–1675), erudito e archeologo, su cui vedi *Lettere di Ottavio Falconieri a Leopoldo de' Medici*, a cura di Laura Giovannini, prefazione di Mina Gregori (Firenze: Edam, 1984); *Giuseppe Campani "Inventor Romae," an Uncommon Genius*, by Silvio A. Bedini, edited by Cristiano Zanetti (Leiden–London: Brill, 2021); Annalisa Laganà, *Lettere d'artista. Invenzione di un patrimonio nell'Italia del nation-building* (Napoli: Federico II University Press, 2024); *DBI* XLIV (1994): 385–388, voce di Matteo Sanfilippo.

il libro *de Aquaeductibus* di M.^r Fabbretti,³⁸ che farò trovare dal Sig.^r Ab. Falconieri³⁹ subito, che capiterà qui da me, e glielo manderò, e veggo se vi è appresso di me altro che sia di suo servizio, che glielo manderò subito.

Il panno per due ferraoli è stato pagato Canne 4, e panni 6^{1/2} che a ragione di giuli 32 la canna fanno

d. 15.40,

e più un giulio di tela incollativa per i baucchi

– .10 –

d. 15.50 –

Il conto del sarto è trovato, come ella vedrà nello stesso conto in ducati [scudi] 4:88.⁴⁰ Per mero suo incomodo basterà, che il danaro lo mandi in mano di Bartolomeo mio cameriere, che averà egli il pensiero di pagarlo a chi va, e riportare le ricevute. Si prevalga liberamente di chi a somma ambizione di servirla professandosi serenamente

Di V.S.Ill.^{ma}

Devot.^{mo} et obblig.^{mo} Ser.^{re} vero

Paolo Falconieri

Di Casa il 3 del 1690

13. BNCF, Gal. 257, c. 98r–v

Ill.^{mo} Sig.^{re} mio Oss.^{mo}

Dal nostro Sig.^{re} Luigi del Riccio⁴¹ avevo già avuta notizia della bella opera data da V. S. Ill.^{ma} alle stampe,⁴² e da questa era nata in me quel desiderio, ch'ella può immaginarsi dalla necessaria stima, che sia d'avere, da parte del suo ammirabile ingegno particolarmente da me, ch'alla stima dell'opera aggiugno l'amore distintissimo all'autore d'esse. Doppo di questo non vi è bisogno di esprimerle, con qual godimento io me ne vegga favorito per mezzo del Sig.^r Andrea del Rosso, e quante obbligazzioni ne professi alla di lei cortesia, che me ne arricchisce, senza alcun mio merito, né contenta di tanto, v'aggiugne l'onore de suoi bramati

³⁸ *De aquis et aquaeductibus veteris Romae dissertationes tres* (Romae: Typis Ioannis Baptistae Busotti, 1680). Sull'argomento vedi Elena Vaiani, "Raffaele Fabretti, il 'Signor censore': una polemica antiquaria sui medaglioni di Gaspare Carpegna," *Studi secenteschi* XLVI (2005): 211–228.

³⁹ Si dovrebbe trattare di Paolo Francesco Falconieri.

⁴⁰ Per il ducato, vedi Martinori, *La moneta*, 135–136; per lo scudo, ibid., 467; per il Giulio, ibid., 183–184.

⁴¹ Luigi del Riccio (1637–1718), senatore, accademico della Crusca con il nome di *Sostenuto*.

⁴² Dovrebbe trattarsi dell'opera *Al serenissimo principe di Toscana Formazione, e misura di tutti i cieli con la struttura esatta dell'intero, e delle parti di un nuovo cielo ammirabile, e di uno degli antichi delle volte regolari degli architetti. Curiosa esercitazione matematica di V. V. ultimo scolare del Galileo* (In Firenze: nella stamperia di Piero Matini, 1692).

comandamenti per obbligarmi quel più ne l'ordine di distribuire l'altri 8 esemplari come ò di già cominciato a fare, e dell'una, e dell'altri gli ne rendo le grazie più devote, et a suo tempo mi prevarrò della libertà, che si compiace darmi di supplicarla d'altri esemplari per appagarne la curiosità d'altri, che la possino avere.

Spero che tra quell'altre opere, che m'avvisa aver finite vi possa essere quella, che fece qui intorno a varie osservazioni all'acque condotte [parola non leggibile], che goderci sommessamente d'indovinarla, vedendo che fusse giovevolissima, e se non vi è per che vi sia V. S. Ill.^{ma} se ne venga a passare il prossimo inverno a Roma, già che vi trova vantaggio per la sua salute, e quando alla voglia darò a me il merito col mondo galantuomo, di aver parte in conservargli la persona di cui egli à più da gloriarsi, l'esibisco cordialmente una bella veduta qui nella mia casetta, la quale servirà di svago alle fatiche, et alle speculazioni, né vi sarà cerimonie, che glielie disturbino. Io per me mi figuro che il P^{ne} Ser.^{mo},⁴³ che conosce bene il suo interesse, non averà difficoltà di permetterglielo perch'ella si conservi, e si liberi da codest'aria, che tanto le sendo inutile la sua persona costà.⁴⁴ Ma quando la mia poca fortuna non mi permetta tanto, mi abiliti ella almeno a contribuire nella sua salute col vino, se può esserne mezzo, avvisandomi di qual sorta ella li beveva, che gliene manderò a suo tempo la quantità necessaria, e se ella mi ammirerà, oltre al luogo, la qualità che ànno d'avere, o di dolce, o di asciutto, li farò fare espressamente, e con particolar diligenza. Sig.^r Vincenzo mio, degl'uomini grandi, ne nascan pochi, e però bisogna cercar di conservar que' pochi, e però è obbligata a conservarsi per gl'altri, se non lo vuol per sé, e mi faccia meritare qualche cosa a me col mondo contribuendovi.

Mi ricordi cordialmente al suo Sig. Abb. suo nipote,⁴⁵ et ella sia persuasa, ch'io sono, e sarò sempre

Di V. S. Ill.^{ma}

Par.^{mo} et Oblig.^{mo} Ser.^{re}

Paolo Falconieri

Roma il 30 Agosto 1692

⁴³ Cosimo III de' Medici.

⁴⁴ Intendi: *Essendo inutile la sua persona costà*.

⁴⁵ Jacopo Panzanini, figlio della sorella e successore del Viviani nella cattedra di matematica nello Studio di Firenze (*Notizie degli aggrandimenti delle scienze fisiche accaduti in Toscana nel corso di anni LX del secolo XVII raccolte dal dottor Gio. Targioni Tozzetti*, tomo primo (In Firenze: 1780)), si vende da Giuseppe Bouchard Libraio in Mercato Nuovo, 124); appartenne all'Accademia Fiorentina [*Fasti consolari dell'Accademia Fiorentina di Salvino Salvini* (In Firenze: Nella Stamperia di S.A.R.). Per Gio: Gaetano Tartini, e Santi Franchi, 1717, *passim*] e a quella del Disegno (*Gli Accademici del Disegno. Elenco alfabetico*, 243).

14. BNCF, Gal. 257, c. 102r

Ill.^{mo} Sig.^{re} mio Oss.^{mo}

Intendo assai bene il senso della umanissima lettera di V. S. Ill.^{ma} perché a presso a poco so chi è quello, che à in concetto d'operazioni peccaminose, quelle delle quali abbiamo parlato, e mi dispiace assaissimo di non essere in grado di poterlo disingannare, che almeno, mi pare, ch'io avrei le ragioni da farlo. Ma se non è praticabile un rimedio, non trascuriamo l'altro. Se questi versi più sottili, e meno ritrosi possono esser buoni a mitigare il suo male, voltiamoci a questo. Né voglio, che ciò le costi altra applicazione o fastidio che dirmi bianco o rosso, dolce o asciutto, et il paese particolare, se ella lo sa, e poi lasci fare a me.

Mi sovviene benissimo, che V. S. Ill.^{ma} essendo qua scriveva le riflessioni e le speculazioni fatte sopra queste acque condottate, ma non sapevo se vi si fusse soddisfatta in modo di non aver bisogno di farvi su altra manifattura, e mi rallegro di sentire che questa sia opera finita, e che ella pensa di darne fuori dell'altre, ciò che desidero sommamente per l'interesse, ch'io prendo, nell'interesse di quel mondo, e nella di lei gloria per la qualità di cui mi pregio d'essere

Di V. S. Ill.^{ma}

Par.^{mo} et Oblig.^{mo} Ser.^{re}

Paolo Falconieri

Roma il 13 Settembre 1692

15. BNCF, Gal. 257, c. 110r–v

Ill.^{mo} Sig.^{re} mio Oss.^{mo}

Se bene il nostro gentilissimo Sig.^r Luigi mi aveva già cortesemente sgomberato l'animo d'ogni sorta d'apprensione del male di V. S. Ill.^{ma}, il vedere il suo carattere, et avere con esso una certezza maggiore della sua salute, mi è stato d'un giubbilo infinito: sia laudato Dio, che si è compiaciuto di non privare quel mondo, ch'ella sa, del bel lume che l'adorna, e piaccia alla sua misericordia, di conservarglielo per lungo tempo con grandissimo ozio e salute, perché l'arricchisca di tante belle cose, che la soverchia occupazione l'obbliga a tenerle racchiuse in quelle sue cassette.

A me fa V. S. Ill.^{ma} un grandissimo favore dandomi occasione di servire il Sig.^r Steigenthal,⁴⁶ soggetto da stimarsi infinitamente, mentre si è saputo acquistar così altamente l'amore e la stima d'un suo pari. Ma non credo che egli dirà già così doppio che mi averà riconosciuto

⁴⁶ Johann Georg Steigenthal (1666–1740), medico del re Giorgio I; dell'amicizia con Viviani se ne parla nella corrispondenza di Christiaan Huygens (*Oeuvres complètes. Tome X. Correspondance*, n. 2756, 292: *Christiaan Huygens à J. G. Steigenthal*, 9, juin 1692; citato dal pdf).

tanto diverso da quello che me l'avrà figurato il suo cortese impegno, di volermi fare apparire qualche cosa, quando non son nulla. Questo gl'averà da voltarselo seco, che l'ha fatto restare al fallimento, meco, quando vedrà che non son più di tanto, bisognerà ch'ella abbia pazienza. È ben vero, che quel poco che sono, sarà tutto suo per riguardo di lei e di lui. Non lo affretti a venire in qua, sentendovi delle stravaganze dell'aria. Egli è in un paese assai frequentato dalle tramontane, aspetti che queste nel passar di Siena, lo conduchino qua.

Mi informerò qual occasione sia più propria per mandare di questi suoi libri in Amsterdam al Blaeu, mio grandissimo amico, perch'egli li distribuisca in Olanda, Fiandra, et Inghilterra nel modo accennatomi, e gli toccherò qualche cosa della versione in latino, che non l'ò per difficile a conseguire, e subito che ne averò qualche risposta gliela parteciperò. Intanto si abbia buona cura, mi comandi, e mi voglia bene per quello infinito che voglio a lei, di cui sono, e sarò sempre

Di V. S. Ill.^{ma}

Par.^{mo} et Oblig.^{mo} Ser.^{re}

Paolo Falconieri

Roma li 11 Ottobre 1692

16. BNCF, Gal. 167, c. 183r–v

Ill.^{mo} Sig.^{re} mio Oss.^{mo}

Non resi grazie a V.S.Ill.^{ma} la settimana passata di tutti quegli favori fattimi col Sig.^r Ab. Alessandro Aldobrandini⁴⁷ prima costà con farle concepire così buona opinione di me, poi col sottoporlo all'incomodarlo di esser qui da me a darmi la fortuna a poterle servirla in qualche cosa nella sua dimora in questa città, perché le molte occupazioni di sabato passato, mi obbligarono ad usar seco questa confidenza, adatta tuttavia alla sua propria cortesia, et alla bontà ch'ella à per me. Gliele rendo adesso a misura di tanti favori, quanti sono quegli che contiene la sua lettera, e procurerò di farlo meglio coll'opere, servendola nella persona del suddetto signor Abate quanto mi sarà possibile, perch'egli non abbia cagioni di escludere da quel mondo galantuomo, ella per quello che gli à detto di me; e me per non aver fatto quello che debbo verso di lui.

Non è mai comparso qui da me quel Sig.^{re} di Luneburg,⁴⁸ ma qualche cosa, che ò sen-

⁴⁷ Alessandro Aldobrandini (1667–1734), abate fiorentino, su cui vedi *DBI* II (1960): 101–102, voce di Elena Fasano Guarini; notizie in Sara Bonechi, “Dediche tortuose. *La Geometria morale* di Vincenzo Viviani e gli imbarazzi dell'eredità galileiana,” *Noctua* VI, nn. 1–2 (2019): 75–181, *passim*.

⁴⁸ Forse Johann Georg Steigerthal Luneburgese, su cui scrive Viviani a Paolo Falconieri. “Il medesimo Sig.^r Giovanni Giorgio il di cui gentil tratto e le soavi maniere so che legheranno, consegnerà a V.S.Ill.^{ma} n. 12 esemplari di quella mia operuccia ...” (BNCF, Gal. 253, c. 135r–v); vedi

tito dire, debba essere uno, ch'è stato in Roma nel tempo, ch'io ero a Frascati, e che passato a Napoli, di dove tornando può essere ch'egli mi recapiti que' libretti suoi, e mi dia occasione di servirlo in qualche cosa.

Mi rallegro ch'ella si sia liberata del suo male e voglio sperare, che il timore ch'ella à non sia givoso,⁴⁹ ma solo effetto di convalescenza, che le fa ancora soffrire qualche cosa del male passato. Si abbia buona cura in una stagione, che le rende più contraria codesta aria, e se in qualche cosa posso contribuire alla sua salute mi comandi, perché oltre al debito di servirla, mi parrebbe di far peccato mortale, quando potessi operare alla conservazione di sì grand'uomo, e non lo facessi. Mi onori di riverire il suo Sig.^r Nipote, et augurare a lei nella vicinanza delle S.^{te} feste, tutte quelle medesime felicità, che auguro a lei di cui sono immutabilmente

Par.^{mo} et Oblig.^{mo} Ser.^{re}

Paolo Falconieri

Roma il 6 dicembre 1692

17. BNCF, Gal. 167, c. 188r

Ill.^{mo} Sig.^{re} mio Oss.^{mo}

Mi è stato di somma consolazione leggere nella umanissima carta di V.S.Ill.^{ma} in data del 16, il suo cortese aggradimento del mio ossequio che non è altro, che un atto di giustizia verso della sua virtù, e d'interesse della mia vanità, e ch'ella se la passi con buona salute, desiderata da me, quanto ella sa. Il di più m'à afflitto parendomi, e non pare a me solo, che le cose rare, e perciò preziose, vadano custodite, e tenute con grandissima gelosia, e riguardo, e così farei certo se stesse a me, e se mi fusse dato luogo di farlo, et ella sa che ò cercato di certo dirne qualcuna. Al particolare del felice augurio ch'ella m'invia anche in nome del Sig.^r Abate suo Nipote⁵⁰ corrispondo, come debbo, alla qualità del favore, tanto più stimato, quanto creduto più cordiale, e più ingenuo. Io per me sono persuaso che ella abbia qualche cagione di interessi meco, essendo certo che in me vive il maggior veneratore della sua virtù, e quello ch'à maggiore ambizione d'obbedirla, e che cono maggior sincerità si professa

Clara Silvia Roero, "La matematica tra gli "affari di Stato" nel Granducato di Toscana alla fine del XVII secolo," *Bollettino di Storia delle Scienze Matematiche* XI, 2 (1991): 85–142; l'opera a cui si fa riferimento è *Formazione e misura di tutti i cieli. Con la struttura, e quadratura esatta dell'intero, e delle parti di un nuovo cielo ammirabile, e di uno degli antichi delle volte regolari degli architetti. Curiosa esercitazione matematica di Vincenzio Viviani ultimo scolare di Galileo* (Firenze: Nella stamperia di Pietro Matini, 1692). L'opera fu recensita sul *Giornale de' letterati di Modena* VIII (1692): 279–282.

⁴⁹ Gravoso.

⁵⁰ Jacopo Panzanini.

Di V.S.Ill.^{ma}
 Par.^{mo} e Obblig.^{mo} Ser.^{re}
 Paolo Falconieri
 Roma il 20 Dicembre 1692

18. BNCF, Gal. 167, c. 199r

Ill.^{mo} Sig.^{re} mio Oss.^{mo}

Non è egli più sicuro a credere, che i principi s'ingannano colla medesima facilità degl'altri uomini, ch'è quello che per verità dico io, che quello che dee V.S.Ill.^{ma} per farmi grazia, cioè che si danno occhi ne' grandi, che vedono con po' di lume. È certo che poco può aver veduto in me la S.^{ta} di N. S.^{re},⁵¹ che l'abbia possuto⁵² muovere a farmi l'onore, che mi à fatto, onde o bisogna ridurlo ad inganno, o ad un atto d'una generosità da sovrano, che à pensato solo a sodisfar se stessa beneficando. Comunque si sia ella mi dà nuovo motivo di professare maggiori obbligazioni alla S.^{ta} S.^a, poiché la grazia fattami da lui, mi fa godere queste tanto stimate della sua penna ne contenta di queste, mi da speranza di farmene dell'altra assai più stimabili di questa fattami dal Papa. Dell'uno, e dell'altro favore, ne professo a V.S.Ill.^{ma} vivissime obbligazioni, e vorrei vedermi una volta tanto felice di poterla persuadere con altro, che colla penna, o colla lingua di quanto io sono dal suo Sig.^{re} Nipote e

Di V.S.Ill.^{ma}
 Par.^{mo} et Obblig.^{mo} Ser.^{re} vero
 Paolo Falconieri
 Roma il 6 Giugno 1693

19. BNCF, Gal. 167, c. 210r–v

Ill.^{mo} Sig.^{re} mio Oss.^{mo}

Mi aveva appunto il Sig.^r Abate Aldobrandini significati i favori di V.S.Ill.^{ma}, et il suo passato male, quando mi è comparsa la sua gentilissima lettera in data del 29 scorso, nella quale ella considerando più la sua propria bontà, et il suo cortese impegno di favorirmi e che la scarsezza del mio merito, soddisfa più a se stessa, che alla giustizia, e che al vero, nello augurarmi felicità nel presente nuovo anno. Come pregai il suddetto Sig.^r Abate a renderle

⁵¹ Innocenzo XII, Antonio Pignatelli, papa dal 1691 al 1700 (*I papi da Pietro a Francesco* (Roma: Istituto della Enciclopedia), III, 2014, 396–404, voce di Renata Ago); *DBI* LXII (2004): 495–500, voce di Renata Ago.

⁵² Leggi: *potuto*.

devote grazie per quelle che mi venivano per il suo mezzo, così gliene rende da me stesso, per questo che si compiace farmi adirittura, forse con suo gravissimo incomodo, e puol esser sicuro che le corrispondo abundantemente, e per genio, e per giustizia, e per debito, e che le desidero felicità, e lunghezza di vita per interesse pubblico, e per mio privato, conoscendo benissimo, che il mio nome non può vivere, che per la sua penna, e conseguentemente, che non posso non desiderare con vemenza che ella goda, così lunga e sì perfetta salute da poter soddisfare la sua cortese intenzione, accennatami più volte, e manifestare a secoli futuri la stima grandissima, e dico tantissima, ch'io ò sempre avuta di lei, e della sua virtù, e la sincera professione, ch'io ho fatta d'essere

Di V.S.Ill.^{ma}

Par.^{mo} et Obblig.^{mo} Ser.^{re} vero

Paolo Falconieri

Roma il 9 Gennaio 1694

20. BNCF, Gal. 167, c. 297r–v

Ill.^{mo} Sig.^{re} mio Oss.^{mo}

Dal nostro gentilissimo Sig.^{re} Pandolfo Pandolfini⁵³ ò ricevuto quelle notizie che la cortesia di V.S.Ill.^{ma} gli consegnò da comunicarmi, a seconda del suo cortesissimo impegno di farmi grazie, e della giustizia dell'animo suo, riconoscendomi per uno di que' servitori, che s'interessano più seco perché la venerano, e più la stima di sì grato avviso per tutte le sue circostanze avvantaggiosissime per la sua persona, ne resi allora al suddetto cavaliere le grazie maggiori, ch'io seppi, come intendo anche di far ora con esso lei, e rallegrarmi seco vivissimamente e con Firenze, e molto più col Re di Francia.⁵⁴ Con lei che lo riceve, con la

⁵³ Pandolfo Pandolfini (1656–1731), avvocato e senatore fiorentino (*Il senato fiorentino o sia notizia de' senatori fiorentini dal suo principio fino al presente data in luce da Domenico Maria Manni*, seconda ed. (In Firenze: Per lo Stecchi, e il Pagani, 1771), 91–92); appartenne all'Accademia del disegno, dove fu immatricolato il 4 giugno 1706 (*Gli accademici del disegno. Elenco alfabetico*, 243) e ad altre accademie fiorentine e nella sua abitazione di via San Gallo, ogni sabato, soleva tenere conversazioni letterarie (*Dizionario storico delle Accademie toscane*, 349, scheda di Jean Boutier); la famiglia Pandolfini possedeva una fornita "Libreria," che tuttavia fu snobbata dal Magliabechi che non la ritenne degna di essere menzionata nella relazione che fece al padre Aprosio sulle biblioteche fiorentine, mentre si soffermò su quella del Viviani (Genova Biblioteca Universitaria, Ms., E.VI.15, cc. 283v–301v. Di questo documento ha parlato Achille Neri ("Alcune librerie in Firenze nel Seicento," *Rassegna Nazionale* vol. XII, V (1883): 527–536; poi in *Giornale ligustico* XV (1888): 444–458).

⁵⁴ Luigi XIV erogò una pensione al Viviani fin dal 1664; sull'argomento, vedi Antonio Favaro, "Documenti inediti per la storia dei manoscritti galileiani nella Biblioteca Nazionale di Firenze," *Bullettino di bibliografia e di storia delle scienze matematiche* XVIII (1885): 23 e *passim*.

nostra città per avere prodotto un tale ingegno atto ad invaghire l'ambizione d'un sì gran monarca a desiderarlo, ancorché cento di tante vittorie, o di tanti fatti egregij, et il re perché volle e seppe sodisfarla. E per verità, non so come la Maestà Sua avrebbe potuto essere contento di sé, se dopo aver fatte e sì diverse opere grandissime, et acquistatavi tanta gloria si fusse veduto costretto a sollevare, che con giustizia si vantasse Firenze a possedere in sé sola nella persona di V.S.Ill.^{ma}, quello ch'egli non aveva nella grandezza de suoi regni, e delle sue conquiste. Ella dunque, s'io non mi inganno, è stata più generosa con Sua Maestà, concedendosegli, avendogli concesso in sé quello che mancava di più lucido allo splendore⁵⁵ della sua gloria, di quello che Sua Maestà è stato seco in conferirle il consaputo onore. Me ne rallegro di nuovo seco che abbia saputo rendersi colla sua virtù desiderabile ad un sì gran re. Con esso lui che la seppe acquistare e sommamente meco di godere la fortuna di essere di chi à tanta stima da un Re sì grande.

Di V.S.Ill.^{ma}

Par.^{mo} et Obblig.^{mo} Sr.^{re}

Paolo Falconieri

Roma il 30 Gennaio 1700

21. BNCF, Gal. 167, c. 313r⁵⁶

Ill.^{mo} Sig.^r mio Oss.^{mo}

Una tale Antonia Landucci,⁵⁷ vedova del Sig.^r Benedetto, del medesimo cognome, è stata più volte qui a pregarmi che io trasmetta a V.S.Ill.^{ma} l'annessa lettera con speranza che la di lei somma gratitudine, alla memoria del gran Galileo Galilei l'abbia a muoverla a pietà della sua miseria. Io hebbi pensiero veramente di non accettarla, ma considerata poi la bontà di V.S.Ill.^{ma} il suo amore verso di quel buon vecchio, e che con una lettera poteva sbrigarsi di questo negozio, quando non havesse intenzione di pigliar questo impegno, mi risolsi a pigliarla, per autenticare alla suddetta donna il concetto della di lei bontà. Con questi motivi pretendo di scusare appresso di V.S.Ill.^{ma} l'incomodo, che le arredo, e godere l'avvantaggio, di ricordarle che io sono

Di V.S.Ill.^{ma}

Par.^{mo}, et Obblig.^{mo} Ser.^{re}

Paolo Falconieri

Roma 19 Giugno 1700

⁵⁵ Nel documento: *spendore*.

⁵⁶ Edita in *Scampoli galileiani*, 24–25.

⁵⁷ Di Antonia Landucci sono conservate tre lettere indirizzate al Viviani, la prima del 7 giugno, la seconda del 18 giugno, la terza del 31 luglio 1700 (BNCF, Gal. 167, cc. 308r–309r; 315v). I figli di Benedetto e Antonia Landucci erano: Olimpia, Niccolò, Ignazio e Marco.

22. BNCF, Gal. 167, c. 314r

Ill.^{mo} Sig.^r mio Oss.^{mo}

Pagai alla Sig.^{ra} Antonia Landucci li scudi sei di questa moneta, come V.S.Ill.^{ma} si compiacque comandarmi. Ella hoggi mi dà l'annessa,⁵⁸ perché glie⁵⁹ la invij, e vi aggiunga le mie suppliche, che ella si compiaccia di non abbandonarla. Il suo desiderio particolare sarebbe di assicurare il buono indirizzo della maggiore, già grande, con metterla in un monastero, il quale ha già trovato e già vi sarebbe dentro se non si rimanesse, per difetto di denaro, che vuol dire una doppia al mese in tutto. Questa assistenza la vorrebbe da V.S.Ill.^{ma}, parendole che sia dovuta alle lodi della medesima giovane datele in una sua lettera, responsiva nella quale la ragguaglia di tutto quello che passò tra ella et il Sig.^r Landucci, essendo qui in Roma, fra le quali loda sommamente la vivacità di questa ragazza. A questa ragione s'aggiunge quella della sua necessità, che non è piccola, e per ultimo la somma di una doppia per quanto pare a lei è assai discreta. Pongo sotto gli occhi della sua pietà tutte queste riflessioni, dalle quali parmi che essa ne habbia a ricevere qualche forza per sollevar la miseria di questa povera donna e con questa occasione ricordandole il vivo desiderio di servirla mi confermo

Di V.S.Ill.^{ma}

Par.^{mo} et Obblig.^{mo} Ser.^{re} vero

Paolo Falconieri

Roma 31 Luglio 1700

23. BNCF, Gal. 167, c. 318r

Ill.^{mo} Sig.^{re} mio Prone Oss.^{mo}

Già che dal tempo mi conviene attendere le occasioni per tendere a V.S.Ill.^{ma} gl'atti della mia divozione, in mancanza di quelle de suoi comandi, sono ad annunziarle felicissime le feste del S[anto] Natale, pregando V.S.Ill.^{ma} del suo cortesissimo gradimento verso questo riverente ufficio, e di credermi sempre costante nell'osservanza con cui mi sottoscrivo.

Di V.S.Ill.^{ma} la quale prego a credere sincerissimo questo ossequio del mio cuore, et interessatissimo nella conservazione di un uomo del quale ò una infinita stima e ch'è gloria della nostra patria, e d'Italia. Non si può negare ad una povera donna il servizio d'includere una sua lettera in questa come fo, e conseguentemente spero, ch'ella sarà contenta della libertà, che mi prendo e resto suo

Par.^{mo} et Oblig.^{mo} Ser.^{re} vero

Paolo Falconieri

Roma 18 Dicembre 1700

⁵⁸ BNCF, Gal. 167, c. 315r-v.

⁵⁹ Nel documento: *gle*.

Abbreviazioni:

Ab., Abb. = Abate; Dev.^{mo} = Devotissimo; Ill.^{mo} = Illustrissimo; Oblig.^{mo}, Obl.^{mo} = Obbligatissimo; Oss.^{mo} = Osservantissimo; Par.^{mo} = Parzialissimo; Pron = Padrone; Ser.^{re} = Servitore; Sig.^r = Signor; Sig.^{re} = Signore; V. S. Ill.^{ma} = Vostra Signoria Illustrissima.

BNCF = Biblioteca Nazionale Centrale di Firenze.

Gal. = Manoscritto Galileiano.

DBI = Dizionario Biografico degli Italiani, Roma, Istituto della Enciclopedia Italiana, 1960–2020.

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— OBITUARIES —





Maurice Clavelin

(12 septembre 1927–11 février 2024)

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De son entrée à l'École normale supérieure en 1948, jusqu'à la chaire de philosophie des sciences de La Sorbonne, qu'il a occupée de 1971 à 1995, Maurice Clavelin a consacré l'essentiel de ses travaux de recherche à la compréhension de l'œuvre de Galilée, dont il a été, en France et à l'étranger, l'un des spécialistes les plus reconnus et respectés.¹ Ce centrage galiléen de son œuvre ne découle nullement d'un manque d'intérêt pour d'autres pensées ou domaines de l'histoire de la philosophie, comme en témoignent plusieurs publications portant sur l'empirisme logique et le Cercle de Vienne, ou encore la liste des thèses soutenues sous sa direction à l'université de Paris-Sorbonne.² Toutefois s'il s'est principalement consacré, toute sa carrière durant, à l'œuvre du savant florentin, manifestant un intérêt non nul mais très modéré pour les approches psychologique, historique, socio-culturelle ou socio-politique de cette œuvre,³ c'est d'abord en raison de l'objet même que constitue

¹ Clavelin, dont le travail a été très vite discuté au-delà des cercles académiques francophones, a donné plusieurs contributions importantes à la revue *Galilæana* : « Galilée, Descartes, et la nouvelle vision du monde », *Galilæana. Journal of Galilean Studies*, 1 (2004), 3–28 ; « Duhem et Tannery lecteurs de Galilée », *Galilæana. Journal of Galilean Studies*, 3 (2006), 3–17. Signalements encore : « Des nouveautés célestes aux textes sacrés », in M. Bucciattini, M. Camerota et F. Giudice, *Il caso Galileo. Una rilettura storica, filosofica, teologica. Convegno internazionale di studi, Firenze, 26-30 maggio 2009*, Florence : Olschki, 2011, 17–32. Voir la bibliographie complète établie par M. Fichant, « Maurice Clavelin, 1927–2024 », *Revue de métaphysique et de morale*, 3 (2024), 403–408.

² Liste accessible dans le répertoire en ligne : *theses.fr*.

³ On pourra consulter le très substantiel compte-rendu de l'ouvrage de Mario Biagioli : « Galilée homme de cour : sur un ouvrage de Mario Biagioli, *Galileo Courtier* », *Revue d'histoire des*

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à ses yeux la science galiléenne, et parce que Clavelin n'a jamais douté, contre toutes les formes de relativisme historique contemporaines, que Galilée fût réellement l'instaurateur et proto-fondateur de la science classique dans un sens que Husserl lui-même n'eût pas désavoué. Il faut donc tenter d'expliquer ici les motivations de ce parti pris résolument rationaliste, qui peut paraître aujourd'hui presque désuet ou relever d'un genre herméneutique en voie de disparition.

Publié en 1968 dans une collection dirigée par Jules Vuillemin et Gilles Gaston Granger – ce qui n'est pas anecdotique – le livre issu de la thèse d'État, *La philosophie naturelle de Galilée* intervient dans un contexte philosophique où la question de l'objet même de l'histoire de la philosophie, et la question connexe de la méthode, ont pris un relief particulièrement saillant. Avec Canguilhem, avec Gueroult, Granger, Vuillemin, et même paradoxalement avec Foucault, Clavelin est convaincu que Galilée inaugure un nouveau type de discours « détachant le discours sur le monde de tous les repères cosmologiques du sujet qui le tient » ; un discours « qui se supporte lui-même à partir de sa validité intrinsèque », comme le dit Foucault au sujet de Galilée, dans les années où Clavelin lui-même rédigeait sa thèse.⁴ On reconnaît certes dans ce propos caractéristique une des lignes de force interprétatives de Koyré, convaincu que la science classique repose sur la « destruction » ou « dissolution » du cosmos. Il fallait entendre par là, selon Koyré dont Clavelin a pris sur ce point les analyses au pied de la lettre,⁵ que la science galiléenne n'a plus pour objet la nature comprise comme ce qui est immédiatement donné par la sensation, mais comme un objet *posé* par et dans la pensée elle-même. Cet objet n'est rien de naturel, et n'est rien d'autre que l'espace tridimensionnel dont le *Dialogo* entendra fournir d'emblée une démonstration géométrique, ce qu'aucune science ne fait de son objet, celui-ci lui

sciences, 51, 1 (1998), 115–126 ; compte-rendu qui permet à Clavelin de contre-distinguer, avec d'intéressantes nuances, son approche « conceptualiste » du relativisme dont Biagioli fournit l'exemple le plus sophistiqué et le plus intéressant, même si, en dernière analyse, Clavelin estime que la négligence et l'oubli de l'effort de « légitimation épistémologique » de la science du mouvement « déforme... jusqu'à le rendre difficilement reconnaissable » le visage du philosophe qu'il a été.

⁴ Michel Foucault, *Le discours philosophique* [1966–1967], éd. O. Irrera et D. Lorenzini, sous la responsabilité de F. Ewald, Paris : Seuil/Gallimard, 2023, 80–82.

⁵ Sur ce point précis, car au demeurant le livre de 1968 s'emploie à récuser la thèse koyréenne du « platonisme » de Galilée (428 ss.). Cette critique, toutefois, ne rend pas pleinement justice à Koyré, dont le concept de « platonisme » renvoie beaucoup plus à celui des post-kantiens (Lotze) et néo-kantiens (Cohen) qu'à Platon lui-même. Ce qui rebute Clavelin dans cette invocation d'un Galilée platonicien, est surtout le recours à une psychologie de la connaissance platonicienne (la théorie de la réminiscence, l'innéisme) ; mais ce que Koyré avait en vue à travers ce supposé « platonisme » de Galilée, était surtout sa compréhension de la « philosophie naturelle » non comme un savoir acquis sur la base de l'expérience, mais comme une eidétique de la nature corporelle.

étant toujours, en principe, *donné* par une science antérieure à elle. D'où l'importance de cette *Première journée* du *Dialogue*, à laquelle Clavelin consacre une étude magistrale lors du Congrès international d'études galiléennes (Pise, Venise, Padoue et Florence, 1983).⁶ En conséquence de quoi, comme Clavelin y insistait, Galilée ne doit pas tant être regardé comme un inventeur génial doublé d'un expérimentateur rigoureux, que comme l'instaurateur d'une « conception nouvelle de l'intelligibilité scientifique », donc d'une manière fondamentalement différente de penser le rapport de la science à son objet, ou, pour utiliser les termes qui sont aussi ceux de l'époque (Granger, Vuillemin, Foucault) de la « raison » et du « réel ». Si Clavelin s'est intéressé aux condamnations de 1277 ou à l'essor de la physique parisienne, ce n'est nullement pour y retrouver, comme Duhem avait cru pouvoir le faire, autant de « précurseurs » de Galilée, mais pour comprendre comment Galilée, par contraste, « substitue un modèle explicatif à un autre modèle explicatif » — entreprise qui est censée demeurer indépendante des découvertes scientifiques faites dans ce cadre lui-même. On est tout près de ce que Foucault appelle l'*épistémè*, et cette première recherche sur Galilée relève moins de l'enquête historique que de ce que le même Foucault appelait, dans sa propre terminologie, une *archéologie*.

Avec la publication, en 2004, de *Galilée copernicien. Le premier combat (1610-1616)*,⁷ Clavelin prolongeait et approfondissait cette enquête, en même temps qu'il livrait au lecteur francophone une anthologie de textes — lettres et opuscules rédigés entre la publication du *Sidereus nuncius* et le décret de mars 1616 — dont la plupart n'avaient encore jamais connu de traduction et d'édition française, en particulier les *Lettres sur les taches solaires*, rédigées par Galilée en réponse aux *Tres epistolae* du jésuite Scheiner. Le titre de l'ouvrage, « Galilée copernicien » ne révèle pas un secret, l'engagement copernicien de Galilée, débouchant sur le *Dialogo*, n'étant un mystère pour personne. Mais derrière l'apparente banalité de ce titre apparaît bientôt toute la profondeur d'un problème, et l'originalité de la thèse soutenue par Clavelin. D'une part, sur le plan historique et doxographique, Clavelin bat en brèche une idée largement répandue chez les historiens de la science galiléenne (en premier lieu desquels Stilman Drake et William Wallace) selon laquelle le Padouan se serait détourné de Copernic entre 1604 (date des leçons perdues sur la *nova* apparue la même année) et 1610 (date du fracassant retour de son engagement copernicien avec la publication du *Messenger céleste*). Comme le montre Clavelin en s'appuyant d'ailleurs sur les travaux indispensables de Massimo Bucciattini, il s'agit là d'une illusion historiographique due au manque de documents écrits, mais rien n'autorise à penser que l'engagement copernicien

⁶ Le texte d'abord publié dans les actes du colloque de 1983, *Novità celesti e crisi del sapere*, a cura di P. Galluzzi, (1984), est ensuite réédité en France en appendice à *Galilée copernicien* (2004): « Le "Dialogue" ou la conversion rationnelle », 527–552.

⁷ Paris, Albin Michel, 2004. C'est chez le même éditeur qu'avait été réédité l'ouvrage de 1968 quelques années auparavant (Albin Michel, 1996).

des années 1597–1604 ait le moins du monde faibli par la suite.⁸ On peut en outre risquer une hypothèse, que Clavelin ne semble pas avoir envisagée, mais qui abonderait dans son sens : le problème physique que posent les *novae* et les autres corps du même genre (i.e. les étoiles) est de savoir si ces corps émettent une lumière propre, ou s'ils ne sont que des corps réfléchissants, comme les comètes et les planètes. Or si l'on admet que les étoiles ont une luminosité propre, comme autant de soleils (ainsi que Galilée annonce l'avoir *démontré* dans sa lettre à Giuliano de Medici de février 1611, cf. *Galilée copernicien*, 144–146), la destruction de la sphère des fixes et l'infinetisation de l'univers, à la manière de Bruno, semblent en être la conséquence nécessaire et inévitable. Si Galilée ne fait donc pas état de ses positions cosmologiques et de son ralliement à Copernic dans ces années, c'est parce qu'il est en fait écartelé entre deux copernicianismes incompatibles et antagonistes : celui, mathématique et finitiste de Kepler, et celui, naturaliste et infinitiste, de Bruno.

Mais l'intérêt de Clavelin pour l'engagement copernicien de Galilée ne se résume pas à cette question, à la limite doxographique : il vient de ce que l'hypothèse héliocentrique apparaît selon lui comme la seule qui soit « vraiment compatible avec le nouveau savoir sur la *nature* des corps célestes ». En d'autres termes (qui ne sont pas ceux de Clavelin, mais qui expriment bien sa pensée), il est très tôt apparu à Galilée que l'astronomie (copernicienne) jouait le rôle d'un *principium cognoscendi* vis-à-vis de la philosophie naturelle et de la découverte des lois géométrisées du mouvement des corps terrestres. Ainsi Galilée copernicien ne faisait-il, en un sens, que documenter et illustrer *in concreto* la thèse centrale de *La philosophie naturelle* selon laquelle l'astronomie copernicienne constitue une voie d'accès à la mécanique rationnelle de la science classique, ce qui donne à la séquence historique Copernic-Galilée-Newton une forme d'intelligibilité, voire de nécessité historique sans précédent et sans pareille. Qu'il y ait un peu d'optimisme téléologique dans cette représentation de l'histoire de la science classique, que les erreurs de Galilée (e.g. sur les comètes), ou son ignorance des lois de Kepler soient présentées comme des accidents inessentiels à l'intelligibilité globale de l'œuvre, pourra certes laisser certains historiens d'aujourd'hui perplexes ou dubitatifs, mais cela n'ôtera rien à l'insigne mérite d'un philosophe des sciences qui a ramassé la flèche et l'a lancée loin devant, pour le plus grand profit des générations futures.

⁸ Clavelin renvoie à M. Bucciattini, « Galileo e la nova de 1604 », in *La diffusione del copernicanismo in Italia 1543-1610*, 237 ss. Ayant achevé la rédaction de son ouvrage vers la fin de l'année 2002, Clavelin n'a pas encore pu prendre connaissance de l'*opus magnum* de Bucciattini : *Galileo e Keplero* (2003 ; tr. fr. *Galilée et Kepler. Philosophie, cosmologie et théologie à l'époque de la Contre-Réforme*, Paris : Les Belles Lettres, 2008). On notera que cette référence aux travaux de M. Bucciattini n'a pas été référencée dans l'*index nominum* de son *Galilée copernicien*. L'ouvrage de 2004 comporte par ailleurs quelques coquilles ou imprécisions historiques, peu significatives, mais que des travaux plus récents (en particulier l'édition du *De Revolutionibus* copernicien par M.-P. Lerner, A.-P. Segonds, J.-P. Verdet et al., Paris : Les Belles Lettres, 2015) permettraient aujourd'hui de compléter et corriger.



Ricordo di Gianni Micheli (1936–2025)

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Nelle pagine che seguono rievocherò brevemente il contributo di Gianni Micheli alla didattica e alla ricerca della storia della scienza partendo dalla mia esperienza personale.¹ La recente scomparsa di Gianni ha infatti riattivato una trama fitta di ricordi che rendono difficile separare il profilo scientifico da quello umano. La sua attività, ampia e articolata, ha attraversato campi tematici e periodi storici molto distanti tra loro; ne è derivata una pluralità di esperienze formative, talora differenti, ma quasi sempre intense e profondamente positive, per le generazioni di studenti e studentesse che hanno lavorato con lui. L’invito dei direttori di *Galilæana* mi ha offerto l’occasione per riflettere sul debito intellettuale che ho contratto nei confronti del suo magistero e sul modo, spesso non del tutto consapevole, in cui esso ha orientato il mio percorso scientifico.

Ho incontrato Gianni per l’ultima volta pochi mesi prima del congedo definitivo. Nonostante gli acciacchi che lo affliggevano da tempo, mi risultava difficile accettare l’idea della sua scomparsa. La commozione che ho provato alla notizia della sua morte ha fatto riaffiorare una miriade di ricordi. In queste pagine voglio condividerne alcuni, scegliendo tra quelli che più fanno emergere, dietro al contributo scientifico e didattico, la sua grande ed eccentrica personalità.

Devo a Gianni la mia vocazione di storico della scienza, ed è stato per me un maestro nel senso più pieno del termine e, con il tempo, un amico. Pur essendo persona riservata e di grande cultura, non instaurava mai una distanza gerarchica con gli studenti. Si capiva che amava insegnare ed era convinto che la funzione dell’insegnamento universitario

¹ Per una ricostruzione puntuale e più oggettiva dell’opera di Gianni rimando al contributo recentemente pubblicato dall’amico Elio Nenci, “Gianni Micheli (1936–2025),” *Scientia* 3, 2 (2025): 199–215.

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dovesse essere, in primo luogo, una missione: “l’insegnante è al servizio degli studenti e non viceversa, come accade spesso nelle università italiane. Verso gli studenti occorre avere pazienza, rispetto ed essere sempre cortesi e disponibili; il docente ha il dovere di preparare le lezioni al meglio delle sue possibilità, senza barare in alcun modo.”² E Gianni, anche se talvolta severo, non ha mai barato. Non faceva distinzioni tra gli studenti e sosteneva, con quella che potremmo chiamare una concezione autenticamente democratica dell’insegnamento, che la soddisfazione più grande di un docente consistesse nel prestare maggiori attenzioni verso coloro che faticavano di più. Mi sono iscritto a filosofia alla Statale di Milano nel 1980 e non mancavano docenti dotati di straordinarie abilità oratorie e conoscenze profonde sulle materie filosofiche, tanto che per uno studente del primo anno c’era solo l’imbarazzo della scelta. Il corso di storia della scienza non era molto frequentato e Gianni non brillava per la sua eloquenza, ma gli studenti che restavano a seguirlo per due-tre settimane non l’abbandonavano più, perché intuivano che le sue lezioni non erano mere esposizioni descrittive di testi classici o l’espressione retorica e ammiccante della propria erudizione ma l’estratto di una ricerca coerente e a lungo ponderata. Gli studenti poi erano chiamati a partecipare in modo attivo e a preparare delle relazioni scritte su argomenti o autori che solo tangenzialmente toccavano i temi trattati sul corso. Durante le lezioni, infine, Gianni esprimeva in grande libertà giudizi molto netti che poi giustificava nel dettaglio mostrando come l’attività dello storico dovesse essere insieme critica e rigorosa.

Un’altra qualità, insieme politica e morale, che lo ha reso unico è stata la sua coraggiosa onestà intellettuale: anche quando sentiva la necessità di assumere una posizione storiografica divergente rispetto a quelle dominanti, pur rimanendo aperto alla discussione e al confronto dialettico con tutti, non abdicava mai alle proprie convinzioni, disinteressandosi delle conseguenze negative che tali posizioni potevano avere sulla sua carriera accademica. Per Gianni, infatti, il confronto libero e aperto tra opinioni diverse non costituiva soltanto una rivendicazione personale, ma rappresentava un principio metodologico fondamentale da trasmettere a chi studiava con lui. Estraneo a qualsiasi forma di dogmatismo scolastico, incoraggiava i propri studenti a intraprendere percorsi di ricerca autonomi, anche radicalmente diversi dai suoi. Fu lui, per esempio, a indirizzarmi a studiare storia della chimica anche se non era un tema che considerava particolarmente promettente. Non credo che con il tempo abbia cambiato idea, ma ha sempre seguito con grande curiosità le mie ricerche ed è sempre stato disponibile a riconoscere l’interesse di un approccio storiografico diverso dal suo. La sua posizione si irrigidiva diventando più severa solo quando non vedeva rispettati i requisiti deontologici della nostra professione. Per Gianni l’etica del lavoro costituiva una premessa necessaria anche se non sempre sufficiente dello storico della scienza e, più in generale, di qualsiasi lavoro intellettuale.

Tantissimi sono i suoi insegnamenti che mi sono rimasti impressi indelebilmente. Il

² Gianni Micheli, *Mezzo secolo di storia della scienza a Milano* (Milano: Mimesis, 2016), 17.

primo che ricordi fu il costante invito a prendere contatto fisico con le fonti originali. In numerose circostanze Gianni accompagnava noi studenti alla Biblioteca Braidense e ci guidava innanzitutto a leggere i cataloghi manoscritti dei fondi antichi, primo tra tutti quelli della biblioteca di Albrecht von Haller di cui non era ancora disponibile il catalogo a stampa, in seguito pubblicato da Maria Teresa Monti. Ci illustrava poi le differenze, a volte sostanziali, tra l'edizione originale di un testo classico e la sua traduzione moderna. Tale insegnamento, soprattutto in un dipartimento di filosofia, sarebbe stato istruttivo anche per dei ricercatori maturi ma per degli studenti del primo o del secondo anno universitario costituiva una vera e propria rivelazione perché evocava, attraverso l'esposizione di fonti originali, la distanza dal passato e l'importanza di acquisire una coscienza storica del contesto in cui erano venute alla luce certe idee, scoperte e teorie. Gianni adottava questa innovazione pedagogica anche durante i suoi corsi perché era solito portare a lezioni le fotocopie dei testi originali che leggeva e che traduceva in classe così da trasmettere, indipendentemente dalle nostre conoscenze linguistiche, la complessità dell'assimilazione degli autori classici. La sua convinzione sull'importanza di accostarsi sempre alle fonti originali era associata all'idea che con un buon dizionario fosse possibile accedere a qualunque testo. A questo proposito raccontava spesso le vicissitudini che lo spinsero a tradurre il *Leviathan* di Thomas Hobbes, un lavoro durato diversi anni e coronato alla sua conclusione dagli apprezzamenti della critica filosofica. Questo lavoro, apparentemente distante dalla storia della scienza, nasceva "da un tentativo di comprensione del meccanicismo in generale, in quanto Hobbes presenta[va] lo Stato come un corpo meccanico."³ Consapevole che il processo di assimilazione di una metodologia così faticosa richiedesse tempo ed esperienza Gianni soleva spesso sottolineare durante i suoi corsi che, diversamente da quanto accade in matematica e nelle scienze esatte, il percorso per diventare un buon storico è molto lungo e i migliori lavori si pubblicano in età matura, normalmente dopo i quarant'anni. Forte di questa convinzione Gianni era sempre molto critico con chi prendeva scorciatoie e, pur di pubblicare, finiva per produrre lavori superficiali. Tali ammonimenti furono particolarmente frequenti durante i primi corsi che seguì nei primissimi anni '80 quando da poco era stato pubblicato l'Annale 3 della *Storia d'Italia* Einaudi curato da Gianni e dedicato alla scienza e alla tecnica, un'opera che aveva innescato, come vedremo, accese polemiche.

Un altro insegnamento fondamentale sul quale Gianni ritornava spesso era la diffusione del plagio, sia tra gli autori classici sia tra gli storici contemporanei. Riguardo a questi ultimi sosteneva che copiare non era solo un atto riprovevole, ma costituiva un vero e proprio tradimento della propria professione. Anche in questi giudizi sempre molto severi, Gianni non perdeva la sua naturale inclinazione alla curiosità e più che soffermarsi a condannare queste pratiche ne esaminava le varie modalità di esecuzione che, senza farci

³ Ibid., 38.

i nomi degli autori, ci illustrava nel dettaglio sia per ammonirci a seguire condotte appropriate sia per spiegare le cause principali delle incongruenze che spesso si riscontrano nella letteratura secondaria tra le citazioni e i riferimenti bibliografici.

Un altro punto che tengo sempre presente riguarda la visione di lunga durata sulla storia della scienza: Gianni sosteneva con forza che la tradizione costituisce una categoria fondamentale per comprendere lo sviluppo storico della scienza, ma l'inveterata inclinazione di molti storici italiani a considerare il pensiero scientifico come un corpus di sapere impermeabile ai condizionamenti della cultura in cui si è formato ne ha sistematicamente sminuito l'importanza. A questo riguardo faceva spesso l'esempio di Aristotele, la cui importanza, indipendentemente dall'opinione dello storico, era comunque assicurata dal fatto che le sue opere erano state discusse e commentate dai filosofi per oltre duemila anni. Era quasi impossibile, secondo lui, che un autore "dimenticato" da qualche secolo potesse rivestire l'importanza storica del precursore. Il motivo di questa attenzione discendeva dalla diffusa propensione della storiografia della scienza italiana ad attribuire a presunti precursori anticipazioni di scoperte o idee del tutto estranee al contesto storico da loro vissuto. Il caso di Leonardo veniva spesso preso ad esempio per illustrare le distorsioni di questo tipo di storiografia.

Gianni cambiava quasi ogni anno argomento dei suoi corsi e anche se la sua visione tendeva a coprire temi e periodi ampi non mancarono cicli di lezioni dedicati a singoli autori o addirittura a singole opere. Uno dei corsi che più mi rimase impresso fu quello dedicato alle *Meditazioni Metafisiche* di Descartes (suo autore prediletto), tenuto quando venne chiamato a sostituire per un anno Arrigo Pacchi nel suo insegnamento di storia della filosofia. Gianni affrontò la lettura delle *Meditazioni* offrendoci un esame estremamente analitico dell'opera tanto che, sottolineandone la coerenza e la concatenazione dei ragionamenti, aveva individuato 114 argomenti usati da Descartes per dare fondamento ontologico alla sua nuova filosofia naturale. La coerenza della filosofia cartesiana nella difesa del razionalismo critico fu per Gianni una specie di setaccio attraverso il quale esprimere un giudizio anche su altri autori del Seicento e in particolare Galileo e i suoi seguaci.

Sosteneva con molta convinzione che per essere un buon storico della scienza ci vuole più passione che intelligenza. Gianni, infatti, non disdegnava l'erudizione anche se il compito più alto dello storico era quello di individuare problemi e casi di studio storicamente rilevanti, una filosofia della storia che era sostenuta dalla sua ammirazione per Hegel e dall'indimenticata lezione di Mario Dal Pra con il quale si era laureato con una tesi dedicata alla biologia di Cartesio. Recentemente, nella sua ultima lezione tenuta il 26 marzo 2025 agli studenti dell'Università di Urbino, ha chiarito i principi del suo approccio metodologico e i risultati di cui andava più fiero:

Anzitutto, leggo i testi e i documenti dell'oggetto che esamino nella lingua in cui sono stati scritti, una o più volte, cercando di comprenderli nel modo più ampio possibile utilizzando

dizionari, repertori e facendo il controllo delle citazioni esplicite e implicite. Porto avanti questo lavoro, che è quello normale di ogni storico coscienzioso fino a quando non trovo l'elemento chiave di tutti gli scritti dell'autore che considero. Tale chiave è quella giusta se spiega tutti o gran parte dei risultati scientifici conseguiti dallo scienziato che sto esaminando. Credo che questa chiave si trovi sempre negli autori più rilevanti, ma che non sia facile trovarla. Se l'autore della ricerca non la trova, per varie ragioni, non avrà comunque fatto un lavoro inutile. Avrà comunque fatto un lavoro erudito descrittivo, pregevole in sé, anche se non esplicativo. Ecco le chiavi di lettura che ho trovato nei filoni di ricerca che ho svolto. Il modello meccanico mutuato dal metodo analitico della geometria antica, nella filosofia e nella scienza cartesiane. Riduzione della scienza a situazioni sperimentali con una metodica specifica ben delineata, ma privo di dimensione filosofica. È la concezione di Galilei, divenuta poi la norma della scienza italiana post-galileiana anche senza dimensione metodica specifica. Lazzaro Spallanzani è lo scienziato tipico della tradizione scientifica italiana a partire dalla seconda metà del secolo XVII. La delimitazione disciplinare come caratteristica della scienza antica con la conseguente mancanza di riflessione filosofica che è esercitata dai filosofi invece (Platone, Aristotele, Proclo). Scoperta dell'analisi concettuale come elemento portante della ricerca filosofica e storica, scoperta che ho fatto scrivendo alcune voci dell'*Enciclopedia Einaudi*. Le indicazioni che ho dato vanno viste in connessione con la premessa di ogni mia ricerca storica: l'assunzione come punto di riferimento del punto di vista dell'autore considerato.⁴

Oltre a essere un modello nell'insegnamento, Gianni è stato uno studioso eccellente e i suoi lavori, sia quelli storici sia quelli storiografici, si sono sempre distinti per la loro originalità. Prima di esaminarne qualcuno non si può tacere una caratteristica essenziale del suo lavoro storiografico: l'impegno critico e militante. Tale vocazione discendeva, oltre che dalla sua appartenenza al PCI, dall'assidua frequentazione di Ludovico Geymonat e dalla partecipazione al grande progetto della *Storia del pensiero filosofico e scientifico* al quale Gianni contribuì redigendo numerosi contributi dedicati alla scienza antica e moderna e curando, insieme alla moglie Pina Madami, le bibliografie di moltissimi capitoli⁵. Come è noto, l'obiettivo di questa grande opera, sostenuta dal suo straordinario successo editoriale, era quello di mostrare il valore fondamentale che la scienza aveva assunto nell'affermarsi, in epoca moderna, di una nuova forma di razionalità che apriva possibilità inedite di emancipazione politica e sociale. La posta in gioco era chiara: gli intenti dell'opera coordinata da Geymonat miravano a rinnovare una cultura filosofica ancora imbrigliata

⁴ Tratto dal testo della relazione che Gianni ha preparato per la lezione agli studenti e docenti dell'Università di Urbino. Ringrazio Davide Pietrini, della medesima Università, per avermi messo a disposizione il testo.

⁵ A Pina Madami era stato affidato il compito di redigere i capitoli dedicati alla storia della sociologia e delle scienze sociali.

nelle maglie dell'idealismo e dello storicismo crociano, rivendicando per gli intellettuali schierati a sinistra un impegno più rigoroso nei confronti dei progressi della scienza e delle sue relazioni concrete con la società.

Non meno importante fu per Gianni l'incontro con lo storico Ruggiero Romano avvenuto durante un lungo soggiorno a Parigi nel 1963, ma che solo un decennio dopo prese la forma di una collaborazione concreta con l'invito a collaborare a un altro importantissimo progetto collettivo, l'*Enciclopedia Einaudi* e, quasi contemporaneamente, a curare un annale della *Storia d'Italia* dedicato alla scienza e alla tecnica nella storia della cultura italiana.⁶ Entrambi questi progetti aspiravano a realizzare un profondo mutamento nella cultura italiana. Il compito, affidato a Gianni, di curare *Annale 3. Scienza e tecnica nella cultura e nella società dal Rinascimento a oggi* (1980) offriva un'opportunità inedita di affrontare la tradizione della storia della scienza italiana entro un orizzonte amplissimo e, sul piano storiografico, decisamente nuovo. In linea con il taglio dato da Romano e Vivanti alla loro *Storia d'Italia* Gianni si prefiggeva non tanto "di giudicare il valore in sé del contributo dato dagli scienziati italiani" ma piuttosto l'individuazione delle caratteristiche culturali che a partire dal Rinascimento hanno progressivamente marginalizzato la funzione della scienza nella società italiana. A dispetto della vitalità e dei contributi, talora notevolissimi, dei singoli, Gianni si interrogò sulle cause strutturali del declino della scienza italiana, un tema affrontato più volte dalla storiografia ma per lo più ricondotto semplicisticamente all'oscurantismo della Chiesa controriformista, alla condanna di Galileo, alla divisione politica dell'Italia pre-unitaria e alla sensibilità perennemente deficitaria dei nostri governanti per la ricerca. "Contrariamente a quanto si crede comunemente," scriveva, "l'emarginazione della scienza nella cultura italiana si è venuta svolgendo secondo modalità che non sono di rifiuto o di critica radicale del discorso scientifico, ma di liquidazione della sua componente pratica, operativa, strumentale. È da sottolineare, infatti, che in Italia la scienza nasce e si sviluppa con alcune caratteristiche peculiari che la differenziano da quella di altri paesi; su tutte spiccano il suo essere frammentata in settori specifici privi di articolazioni, con metodi strettamente legati agli enti e ai fenomeni che indaga, e il suo rinchiudersi in se stessa, con scarsa apertura verso questioni filosofiche."⁷ Al fine di chiarire con maggiore precisione e coerenza questo assunto, Gianni ricorreva al "criterio della tradizione," il quale gli permetteva di inquadrare sulla lunga durata quei caratteri che, pur nel mutare delle circostanze storiche, gli parevano rimanere costanti. Tra questi, quello che destò le discussioni più accese riguardava la critica alla debolezza della tradizione filosofica italiana che, secondo Gianni, aveva contribuito a emarginare, fin dal Rinascimento, la valenza filosofi-

⁶ Per la cronologia e i contributi specifici di Gianni a queste due opere rimando al saggio di Nenci citato in nota 1.

⁷ Gianni Micheli ed. *Annale 3. Scienza e tecnica nella cultura e nella società dal Rinascimento a oggi* (Torino: Einaudi, 1980), xvii.

ca delle scienze particolari, destinate così ad occupare un ruolo culturalmente periferico. Il consolidamento delle argomentazioni avanzate dai nostri principali filosofi, da Vico a Gentile, aveva finito per essere accettato non soltanto dalla tradizione filosofica ma anche dagli scienziati stessi, i quali solo raramente e con risultati per lo più velleitari cercarono di dare un fondamento teorico generale alla propria disciplina. Singolarmente, questa tesi, che attirò a Gianni più di una critica, era stata sostenuta nel 1949, pur in una prospettiva differente, da Leonardo Olschki. Nel suo fortunatissimo *The Genius of Italy*,⁸ tradotto per i tipi Mondadori nel 1953, Olschki metteva ripetutamente in evidenza come, a partire dal Medio Evo, “gli Italiani erano [stati] attratti verso gli aspetti pratici della scienza piuttosto che verso le sue basi teoriche e speculative” (p. 201), tanto che il risveglio per gli studi umanistici e la traduzione dei testi filosofici “non riuscirono a fare dell’Italia un paese di filosofi” (p. 267). Il fallimento delle dottrine di Vico e Croce a intercettare i cambiamenti intervenuti nella società moderna e contemporanea, dominate ormai dalla scienza e della tecnica, ne erano una dimostrazione (pp. 399–400). Non è forse un caso che l’opera di Olschki, molto apprezzata nei paesi anglosassoni, passasse quasi inosservata tra gli studiosi del Rinascimento del nostro Paese, segno dell’idiosincrasia della nostra storiografia nell’affrontare ricostruzioni sulla lunga durata.⁹ Il riferimento all’opera di Olschki, sconosciuta persino agli autori dell’Annale, è rivelatore: il fatto che Gianni fosse giunto indipendentemente alle stesse conclusioni dello studioso italo-tedesco conferma la solidità del suo approccio, fondato su un’indagine rigorosa delle fonti e sulla volontà di riconsiderare la storia della scienza italiana da una prospettiva del tutto nuova.

Mi sono soffermato sull’Annale non solo per l’importanza dell’opera in sé ma anche perché poco tempo dopo la sua uscita, quando decisi di laurearmi con Gianni con una tesi dedicata alla storia della chimica nel Settecento, ebbi modo di studiare il volume e di accorgermi che la tesi storiografica si applicava perfettamente al caso con cui ero chiamato a confrontarmi. I chimici e naturalisti italiani del Settecento, infatti, diedero contributi sperimentali molto importanti alla chimica, specialmente quella pneumatica, senza però riuscire ad afferrare il significato innovativo delle riforme teoriche proposte da Lavoisier e, ancora meno, la sua interpretazione di alcuni esperimenti cruciali come quelli sull’acqua che in Italia rimasero a lungo un mistero. Mi stupii dunque che di fronte all’efficacia di

⁸ Leonardo Olschki, *L’Italia e il suo genio* (Milano: Mondadori, 1953).

⁹ L’opera venne salutata con grande favore da Gaetano Salvemini (<https://www.nytimes.com/1949/09/04/archives/a-study-of-the-italian-renaissance-the-genius-of-italy-by-leonardo.html>) e da Paul Oskar Kristeller che era legato a Olschki da antica amicizia e dal comune destino della persecuzione nazista. Negli anni ’30, quando era ancora in Germania, Olschki aveva anche pubblicato un’importante opera di storia della scienza, rimasta incompiuta, dedicata al linguaggio delle scienze e delle tecniche dal Medioevo alla prima età moderna (*Geschichte der neu sprachlichen wissenschaftlichen Literatur*, 3 voll., Heidelberg: Winter, 1919–1927). Entrambe le opere godettero in Italia di scarsa fortuna.

un nuovo orizzonte storiografico entro il quale inquadrare la scienza italiana, l'Annale fu oggetto di critiche in più punti pretestuose. Benché la stragrande maggioranza degli storici della scienza fosse concorde che i tempi della celebrazione nazionalistica della tradizione italiana andassero sostituiti con categorie storiografiche nuove, a Gianni e a molti suoi collaboratori venne imputato di aver lasciato in secondo piano i “grandi,” di aver dato troppo peso, nell'esame dei rapporti tra scienza e cultura nell'Italia contemporanea, all'influenza nefasta di Giovanni Gentile.¹⁰ A rileggere oggi il tono e i contenuti che animavano molte delle recensioni critiche si capisce che i tempi non erano favorevoli a un dibattito aperto e costruttivo e che una discussione approfondita sull'identità culturale della scienza italiana poteva essere solo rimandata. Tuttavia, nonostante si siano moltiplicati i notevoli contributi dedicati a singoli aspetti o autori della storia della scienza del nostro Paese, non ci si è più cimentati nello sforzo di ricostruire con la medesima ampiezza di vedute gli aspetti principali e costitutivi della tradizione nazionale, relegando così la storia della scienza a un ruolo del tutto secondario rispetto ad altre discipline, come la storia moderna e contemporanea, la storia della letteratura, la storia dell'arte che, al contrario, si sono confrontate più volte negli ultimi 50 anni con il tema dell'identità culturale.¹¹ Non è chiaro se, come sostengono in molti, le condizioni per una disamina critica della tradizione scientifica italiana occorra attendere ancora che la storia della scienza maturi ulteriori e più sofisticati

¹⁰ Stupisce che la filosofia di Gentile non solo non sia stata messa da parte dalla cultura italiana ma abbia goduto negli ultimi 40 anni di crescente fortuna. In un recente documentario dedicato al centenario dell'Istituto Treccani, il suo direttore scientifico, Massimo Bray, ha dichiarato con orgoglio la continuità culturale che lega l'*Enciclopedia Italiana* voluta e diretta da Gentile con le attività che l'Istituto svolge attualmente. (vedi *L'enciclopedia del secolo. Treccani 1925–2025* <https://www.raiplay.it/programmi/lenciclopediaelasecolotreccani1925-2025>). Per capire in che misura i rilievi di Gianni cogliessero nel segno si leggano le pagine illuminanti di Gabriele Turi, *Giovanni Gentile. Una biografia* (Torino: Utet, 2006) e, dello stesso autore, *Il mecenate, il filosofo e il gesuita. L'Enciclopedia italiana, specchio della nazione* (Bologna: il Mulino, 2002).

¹¹ Non sono tuttavia mancati tentativi, di spessore ed ampiezza molto diversi, di riprendere il tema. Il primo, curato da Carlo Maccagni nel 1989 (*La Storia delle scienze* Busto Arsizio, Bramante), raccoglie un insieme di contributi dedicati alle storie disciplinari che l'introduzione di sole quattro pagine non intende minimamente collegare in una visione d'insieme omogenea. Anche se il volume era inserito in una *Storia sociale e culturale d'Italia*, quasi tutti i capitoli ricostruiscono le singole discipline sulla base di un approccio rigidamente internalista. Decisamente più articolato e ricco di suggestioni innovative è l'Annale 26 della *Storia d'Italia Einaudi: Scienze e cultura nell'Italia unita* curato nel 2011 da Francesco Cassata e Claudio Pogliano che copre tuttavia solo il periodo contemporaneo. Un tentativo di combinare storia istituzionale, culturale, epistemologica con un'attenzione particolare alle biografie si registra nel volume dedicato alle scienze de *Il contributo italiano alla storia del pensiero* (Roma: Istituto Enciclopedico Treccani, 2013) curato da Antonio Clericuzio e Saverio Ricci. Anche in questo caso però i curatori hanno rinunciato a tracciare una trama storiografica omogenea che guidasse i numerosi autori chiamati a contribuire all'opera.

strumenti di analisi.¹²

Con oltre 60.000 copie vendute, il volume di Gianni, nonostante le critiche, ebbe un grandissimo successo editoriale, segno che il tema era sentito ben al di là dei confini della comunità accademica degli storici della scienza.¹³

Le critiche, tuttavia, lasciarono il segno. Ritengo, ma si tratta di un'interpretazione personale, che Gianni abbia deliberatamente scelto, negli anni successivi, di concentrarsi su indagini più circoscritte, così da evitare polemiche altrettanto accese e logoranti. Con questo ovviamente non voglio dire che gli studi intorno alla storia della meccanica e alla diffusione e traduzione dei *Problemi meccanici* di Pseudo Aristotele nel tardo Rinascimento, ai quali cominciò a dedicarsi proprio quando io ero studente, non fossero per lui importanti. Infatti, dalle sue ricerche sui *Problemi* e su Bernardino Baldi (queste ultime svolte in collaborazione con Elio Nenci) nacquero progetti di ricerca di rilevanza internazionale e, tra le tante pubblicazioni, la monografia *Le origini del concetto di macchina* (1995). Anche in queste opere poi Gianni non ha rinunciato a situare il tema prescelto entro prospettive storiografiche di ampio respiro che lo riconnettevano in modo coerente ai suoi studi giovanili su Descartes e la diffusione del meccanicismo.

A dispetto della grande personalità, durante tutta la sua carriera Gianni ha privilegiato il lavoro collegiale su quello individuale. Questa predilezione, probabilmente maturata nelle feconde esperienze con Geymonat e Romano, non è soltanto desumibile dai suoi lavori ma anche dai numerosi seminari che in collaborazione con amici e colleghi organizzava periodicamente. Tra questi i seminari sulla scienza antica e la sua trasmissione, tenuti regolarmente negli ultimi venticinque anni presso Villa Feltrinelli a Gargnano, furono probabilmente i più riusciti e hanno dato vita a una ricca comunità di studiose e studiosi provenienti da diversi Paesi e formazioni, accomunati non solo da interessi scientifici condivisi ma anche dalla passione per le discussioni, il dialogo informale e gli allegri incontri conviviali.

La passione per l'insegnamento e per lo studio non lo hanno mai abbandonato e fino all'ultimo Gianni ha coltivato l'una e l'altra: nel marzo 2025 ha tenuto il suo ultimo semi-

¹² Cassata e Pogliano, nella loro introduzione all'Annale citato nella nota precedente, dopo aver brevemente richiamato il lavoro curato da Gianni, hanno espresso l'esigenza di un'agenda decisamente ridimensionata: "Trent'anni dopo, noi non disponiamo di una tesi così chiaroscurale, né purtroppo ci illudiamo di migliorare qualcosa della cultura di un paese che proprio in questi trent'anni si è sempre più smarrito." A questo hanno aggiunto l'invito agli autori a non "indulgere a stereotipi storiografici – quali ad esempio la cosiddetta egemonia idealistica nella cultura del Novecento, o la dimensione puramente 'applicativa' della scienza nel periodo fascista – non del tutto privi di verità, ma troppo spesso assunti acriticamente e sbrigativamente." : xviii.

¹³ Del resto, se in Italia l'opera aveva suscitato molte polemiche, su *Isis*, il più autorevole periodico internazionale di storia della scienza, veniva salutata come "an essential reference work for those interested in the development of science in Italy and, more generally, for those interested in the influence of national context on scientific development" (*Isis* 73, 2 (1982): 282).

nario all'Università di Urbino, dove era stato accolto calorosamente da Enzo Fano e dal suo gruppo. A breve usciranno anche una raccolta di articoli e gli atti di un convegno sulla storia della meccanica nel Rinascimento urbinato, ai quali stava lavorando insieme a Elio Nenci e ad altri amici e giovani colleghi. Altri studi, come quello dedicato a Comte, sono in corso di pubblicazione.

Nella ricerca come nell'insegnamento è impossibile separare i risultati che Gianni ha conseguito nel corso della sua lunghissima e operosa carriera dalla presenza concreta della sua eccezionale personalità. Era una persona spiritosa, autoironica, generosa e buona, e la sua presenza serena si offriva agli amici come uno spazio fecondo di idee da condividere e su cui discutere convivialmente. Fino all'ultimo ha amato con grande passione la vita, gli studi, la famiglia e gli amici.



Fig. 1 – Nella foto, scattata a Pavia nel settembre 2015, Gianni siede insieme ai suoi laureati dell'anno 1986. Da sinistra Elio Nenci, Gianni Micheli, lo scrivente e Maria Carla Gadebusch Bondio.



Mario Biagioli (1955–2025)

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Last year, the community of scholars devoted to Galileo and his world lost one of its most influential participants with the passing of Mario Biagioli. Born in Prato, where he received his Diploma di Maturità Classica from the Liceo Ginnasio Cicognini (1974), Mario followed Galileo’s path by entering the Italian university system, studying computer science at the University of Pisa for three years before his father insisted that he work in the family business. *Industrie Biagioli* continues to produce *tessuti non tessuti* (non-woven fabrics) and reflects the enduring role of textiles in the Pratese economy but Mario’s intellectual passions eventually called him back to more academic pursuits. Mario never wrote a poem against wearing academic robes, but shared Galileo’s iconoclastic love of institutions of learning. As his UC Davis colleague Daniel Stolzenberg put it, he had a very Realpolitik view of science and academia, which was the heart of his brilliant historical insights, while being utterly devoted to scholarship and academic life in a genuine and uncynical way.

Mario moved to the US to study photography and visual studies at the Rochester Institute of Technology (MFA, 1984), where his interest in the history and philosophy of science grew, leading him to study in this field with his mentor John Heilbron and philosopher Paul Feyerabend at the University of California, Berkeley (PhD, 1989). There he encountered the new interdisciplinary work being done on Italian Renaissance patronage and cultural production under Randolph Starn’s mentorship. For those of us who went to graduate school with Mario – and in Paula Findlen’s case recall him sketching the basic idea of *Galileo, Courtier: The Practice of Science in an Age of Absolutism* (1993) on a

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blackboard during Heilbron's Scientific Revolution seminar in 1986 under John's somewhat skeptical gaze – this stimulating intellectual environment helped incubate Mario's delightfully unorthodox and powerful vision of Galileo as a scientific actor navigating and shaping the world of politics and knowledge. It seemed fitting that when he first began to teach at UCLA, as Visiting Assistant Professor in 1988–89 and then on the regular faculty after receiving his PhD, he found an apartment near Venice Beach. Galileo would have approved.

Navigation was not only a metaphor for Mario, but a way of life as a sailor. During his doctorate, he successfully filed a patent for the masting of sailboats (EP0321425A1), accompanied by twenty-two diagrams of the invention. This patent presaged his later interests in intellectual property, material culture and the visual studies of science, which form some of the key themes of *Galileo's Instruments of Credit: Telescopes, Images, Secrecy* (2006). This second book on Galileo explained how the Italian philosopher moved between different courtly, academic, religious and artisanal economies of credit, mirroring Mario's uncanny ability to reinvent himself as an STS scholar, a professor of law, or an expert on computer science in Russia.

Al Van Helden recalls hearing Mario give an early paper about how court culture influenced Galileo's science and then inviting him to Rice University after *Galileo, Courtier* appeared in 1993, winning over everyone with his talk. In the morning, Mario "rode" with Al and his friends on their daily ten-mile breakfast bike ride, resourcefully, on in-line skates. Always fun, Mario was ever something of a streetfighter. Around this time, he wanted to move to Massachusetts to be near his son Gabriel. His application for a position in the History of Science at Harvard encountered certain obstacles: a contagious skepticism about his entire approach and harangues devoted to particularly controversial arguments. Ever the strategist, he was nimble enough to counter, contain, and charm away most of this resistance, and he got the job. Did he get the job done? Yes! While *Galileo, Courtier* still has its critics, Mario's approach to the practice of science in an absolutist culture transformed the field. We remain indebted to him and wish he were still with us.

Jessica Riskin's first memory of Mario is at a conference. From the back of the room where he stood leaning nonchalantly against the wall, he called out showstopping questions in his gorgeous accent with a wry smile and set the room buzzing. In edited volumes, such as *Nature Engaged* (2012), which he co-edited with Riskin and dedicated to John Heilbron, he used humor and the lightest touch to cajole and inspire the contributors, turning the collective project into a lively conversation. At dinner among friends and colleagues, he lent a dimension of elegance and warmth just by his presence (his experimental cooking in graduate school is well remembered for its delicious successes and occasional technicolor failures – a vibrant beet sauce better photographed than ingested!). Many a graduate student learned elegant and simple Italian cooking as well as Bourdieu from him. Mario knew how to be brilliant, rigorous, and funny all at once, and – most importantly

– he was always gracious: even when Massimo Mazzotti ground his sailing boat into a sandbank off the coast of Boston. The boat barely made it back, but Mario could not stop smiling, as he found the whole thing hilarious. Throughout his career, he showed us that one could be a serious and committed scholar while retaining an ironic detachment and a sprezzatura that would always put things in perspective. Former students fondly recall his fierce advocacy which, in Dániel Margócsy's case, ensured that he did not sleep under a bridge in Utrecht when Harvard forgot to disburse his scholarship! To the field of history of science, his scholarship performed an essential public service, striking bold positions that got people talking and debating. Mario was the life of the party, and the party feels sadly quiet without him.

Among Mario's tricks was a kind of effortless action-at-a-distance. Take, for instance, his influence on a seminar room ca. 1995, the semi-dark setting for some pre-PowerPoint lecture on early modern art and astronomy that Eileen Reeves attended. There had been a little ritualized mudwrestling about exactly what any aesthetic effort might disclose, distort, or even contest about an evolving scientific discipline, to whom, and why. Suddenly someone asked if the speaker's argument was not an illustration of "what Mario Biagioli taught us in *Galileo, Courtier*." Alas, in his excitement the interlocutor referred either to "Mario Bígolo" – rhyming it with "gígolo" – or maybe to the apocryphal *Galileo Courtesan*. Something strange and smutty hung in the air, and yet no one laughed: it was a pertinent query, and one which the audience could and did pursue. Mario was not present at that gathering, and he isn't with us now, but his lessons are. His willingness to identify with precision the stakes and stakeholders of any contest, his insistence that obscurity, distance, delay and even ineptitude were potential resources, and above all the antic iconoclasm with which he treated our cherished Galilean myths, tidy theoretical models, and outworn dichotomies animate us still.

His vivacity was driven not by vanity but generosity; as Nick Wilding recalls, his mind was never at rest, but always playing, probing and exploring, shuttling ideas between disciplines and smuggling them over boundaries. This was the great challenge of *Galileo, Courtier*: to invite us to think freely, productively, heterodoxically; poaching, improvising and bricolaging ideas, methodologies, insights, into strange, new, occasionally enraging intellectual contraptions. Mario guided his students to treat the intellectual enterprise playfully, and not too self-seriously, as well: Elizabeth Yale remembers a formative moment in a seminar when, discussing the work of another brilliant historian of science, Mario declared that when he grew up, he wanted to write like that historian. The old adage that academics resemble their subjects has perhaps never been truer than Mario and Galileo, and Mario's Galileo was far more human and weird than the dogged and defiant hero he first encountered. He gave us an engaged and engaging experimenter, brokering, facilitating and meddling in sometimes vicious debates, manufacturing the conditions to test the tensile strength of social levers, learning by doing, failing, reconfiguring, adapting, lathing

his own tools of self-fashioning within unstable systems. And like Galileo, Mario's easy *ingegno* rendered him an instinctive gift-giver, spontaneous, and sparkling.

Unlike Galileo, Mario's gifts were not transactional, or rather, the currency in which he thrived was not political power but intellectual stimulation, which meant social life. There was something uncannily alchemical about his ability to transmute a student's footnote into the outline of a book, to breathe brilliance into the dim embers of colleagues' faltering efforts, with nothing owed in return but membership in a community of pleasure and refreshingly serious ideas. Fittingly for a scholar of intellectual property and authorship, his contributions are not limited to the works that bear his name, but are felt in the tendrils that bind together once disparate individuals wherever he encountered and engendered interest. This is qualitatively different from mere networking. Few thinkers could operate simultaneously in so many domains, not opportunistically but with serious wit, to show, say, how contemporary problems of scientific falsification illuminate and are rooted in early modern practice, or how crucial translation is to media and systems. Not just the dance of an ambitious courtier navigating a stagnant system, his unorthodox career moves worked to fuse, bridge, alloy and ally individuals, groups and fields into new forms, capable of making new kinds of knowledge. He also freely imparted to his students what he learned from the course he charted through academia, offering strategies for sailing through choppy seas and around submerged dangers as they built their careers, even for many years after they graduated.

Mario's long career teaching history of science, STS, and law began in the History Department at UCLA where he completed his first major study of Galileo. He subsequently taught in the Department of the History of Science at Harvard University, with a brief stint at Aberdeen. His colleague Katy Park fondly recalls Mario as a person of real integrity who made their time together in the Harvard History of Science Department rewarding, both personally and professionally. Mario then moved to the University of California, Davis where he founded their Center for Science and Innovation Studies and held a joint appointment in the School of Law, Science and Technology Studies, and History. Mario returned to UCLA in 2019, as Distinguished Professor of Law and Communication.

While Mario's scholarship and teaching expanded beyond the early modern period in the second part of his career, as he became more interested in contemporary questions of science, law, and intellectual property, he never left Galileo behind. As Matt Jones recalls, it was precisely his deep sense of the historical contingency of questions of genre, authorship, property and propriety in early science enabled and animated his extraordinary and transformative scholarship in intellectual property law. Indeed, his long-gestating article on Galileo's geometric and military compass, "Replicating Mathematical Inventions," appeared in *Perspectives on Science* in 2022 (30/3: 437–462). Many of us are grateful that we were able to spend time with him in February 2024 during a two-day workshop at the Stanford Humanities Center on Galileo's correspondence. The forthcoming book edited

by Paula Findlen and Hannah Marcus, *Reading Galileo's Letters: Experiments in Friendship, Knowledge, and Community* (University of Chicago Press, 2026) is dedicated to the memory of two passionate Galileo scholars, John Heilbron and Mario Biagioli.

Mario is survived by his wife, Kriss Ravetto-Biagioli, and his sons Luka and Gabriel. Testimony to his intellectual generosity and sociability is the extensive community of friends and scholars that also remain, a community that he delighted in teasing into being through conferences, publications and brilliant dinners around the world - spanning age groups, languages and specializations to bring people together with kindness, curiosity, humor and wit. Those ties still endure, binding his many students and colleagues, now infiltrated into journals, learning institutions, and the different cultures in which he moved both inside and outside of academia, which continue to come together in the constantly evolving spirit of his uniquely irreverent mixture of thinking, learning and scholarship. He would find this praise excessive, if not ridiculous, but we collectively bestow it upon him with affection, knowing that he enjoyed laughing as much as Galileo ever did.

In the spirit of Mario's own sense of playful seriousness, we conclude with an excerpt of a poem that Randy Starn wrote for Mario's 66th birthday:

Eppur si muove for Mario Biagioli

The tree fell,
 Truth to tell,
 None too soon,
 Because he could see it
 Coming down
 In Bishop Berkeley's town
 When he turned
 Galileo's cut-off finger
 into a middle finger's zinger
 for hagiography's Pure Science.
 As Galileo stopped the sun,
 The Biagio charmed mentors
 With Galileo's moves and moons
 For his patrons' swoons –
 And job lessons opportune.
 If Galileo had skis and bikes
 For darting and dodging
 He might have kept the pace;
 Better for him that for his crashes

He only served Arcetri-time in place.
Eppure Mario, even screwed tight bionic
Muove still, the tonic,
That he always took
Every few years or so
Across continents and seas,
Disciplines and dumpster deans,
Unconverted at Harvard,
Unmooing at Davis,
Mafia clean at Aberdeen,
Under cover at St. Pete
With Metrics everywhere to beat.